

RADIO LABELING FOR BARYCENTRIC SUBDIVISION OF PATHS AND INCENTRIC SUBDIVISION OF SPOKEWHEEL GRAPHS

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ABSTRACT

A radio labeling of a graph G is an injective function $f:V(G) \to N \cup \{0\}$ such that for every $u, v \in V$, $|f(u) - f(v)| \ge diam(G) + 1 - d(u, v)$.

In this paper, we find the radio number of Barycentric Subdivision of path, Incentric Subdivision of Spokewheel graphs and Radio mean number of Incentric Subdivision of Spokewheel graphs.

Keywords: Radio labeling, Radio number, Radio mean labeling, Radio mean number, Subdivision, Barycentric Subdivision, Incentric Subdivision, Spoke

1. INTRODUCTION

Radio labelling, or multilevel distance labeling, is motivated by the channel assignment problem introduced by Hale (1980) [5]. Chartrand et al. investigated the upper bound for the radio number of path P_n . The exact value for the radio number of path was given by Liu and Zhu [6]. A wireless network is composed of a set of stations (or, transmitters) on which appropriate channels are assigned. The task is to assign a channel to each station such that the interference which is caused by the geographical distance between stations is avoided.

Let G = (V (G), E(G)) be a simple connected graphs. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter of G.

Definition 1.1: [2] A radio labeling is an injective function $f: V(G) \to N \cup \{0\}$ such that

 $|f(x) - f(y)| \ge 1 + diam(G) - d(x, y)$ holds for every two distinct vertices x and y of G. The span of a labeling f is the greatest integer in the range of f. The minimum span taken over all radio labelings of the graph is called radio number of G, denoted by rn(G).

Definition 1.2: [1] Let G= (V, E) be a graph. Let e=uv be an edge of G and w is not a vertex of G. The edge e is subdivided when it is replaced by edges $e^1 = uw$ and $e^{11} = wv$

2. RADIO LABELING FOR BARYCENTRIC SUBDIVISION OF PATH

Definition 2.1 [1]: If every edge of graph G is subdivided then the resulting graph is called barycentric subdivision of graph G. In otherwords barycentric subdivision is the graph obtained by inserting a vertex of degree two into every edge of original graph.

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D. S. T. Ramesh¹, K. Sunitha^{*2} / Radio labeling for Barycentric Subdivision of Paths and Incentric... / IJMA- 6(6), June-2015.

Consider barycentric subdivision of path and join each newly inserted vertices of incident edges by an edge. It is denoted by $BS(P_n)$

Existing Result 2.1 [4]

For $i = 1, 2, 3, ..., p - 1, d(x_i, x_{i+1}) \le \begin{cases} n^2 + 2n - 1 & \text{if } n \text{ is odd} \\ n^2 + 3n - 1 & \text{if } n \text{ is even} \end{cases}$

Theorem 2.1: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices. Then $rn(BS(P_n)) = n^2 + 1$ if n is odd.

Proof: Let f be an optimal radio labeling for $BS(P_n)$ where $0 = f(x_1) < f(x_2) < f(x_3) < ... < f(x_p)$

Define the radio labeling $f: V(BS(P_n)) \to N \cup \{0\}$ and satisfying the radio condition

$$f(x_{i+1}) \ge f(x_i) + d + 1 - d(x_{i+1}, x_i) \text{ for all } 1 \le i \le p - 1$$

$$(2.1)$$

Let n = 2a + 1 and $a = \left\lfloor \frac{n}{2} \right\rfloor, a \ge 1$

In this case diameter d = 2a + 1 and p = 2a + 1

Using the radio condition defined in (2.1) and summing up p-1 inequalities we get $rn(BS(P_n)) = f(x_n)$

$$\geq (p-1)(d+1) - \sum_{i=1}^{p-1} d(x_i, x_{i+1})$$
$$= 2n(n+1) - (n^2 + 2n - 1) = n^2 + 1$$

$$\therefore rn(BS(P_n)) \ge n^2 + 1$$

Arrange the vertices of the graph $BS(P_n)$ as follows

Starting from v_i^j , $1 \le i \le n, 1 \le j \le 2$ and ending with v_{n+1}^1 . Then rename the vertices by

$$v_i^j = \begin{cases} v_{L(a+1-i),}^j & 1 \le i \le a, & 1 \le j \le 2\\ v_c^j, & i = a+1, & 1 \le j \le 2\\ v_c^{j+2}, & i = a+2, & j = 1\\ v_{R(i-(a+1)),}^j & a+2 \le i \le 2a+1, & j = 2\\ v_{R(i-(a+2)),}^1 & a+3 \le i \le 2a+2, & j = 1 \end{cases}$$

So for $BS(P_n)$, $V(BS(P_n)) = \{v_i^j, v_{n+1}^1 : 1 \le i \le n, 1 \le j \le 2\}$

Label the vertices $x_1, x_2, ..., x_p$ as in the following procedure $v_c^2 \rightarrow v_{Ra}^2 \rightarrow v_{La}^2 \rightarrow v_c^3 \rightarrow v_{La}^1 \rightarrow v_{Ra}^1$ $v_{L1}^2 \rightarrow v_{R(a-1)}^2 \rightarrow v_{L1}^1 \rightarrow v_{R(a-1)}^1 \rightarrow v_{L2}^2$ $v_{R(a-2)}^2 \rightarrow v_{L2}^1 \rightarrow v_{R(a-2)}^1 \rightarrow ... \rightarrow$ $v_{L(a-1)}^2 \rightarrow v_{R1}^2 \rightarrow v_{L(a-1)}^1 \rightarrow v_{R1}^1 \rightarrow v_c^1$

Define a function
$$f: V(BS(P_n)) \rightarrow N \cup \{0\}$$
 by
 $f(x_1) = 0$ and for all $1 \le i \le p - 1$
 $f(x_{i+1}) = f(x_i) + d + 1 - d(x_i, x_{i+1})$

$$(2.2)$$

Then the labeling $f: V(BS(P_n)) \to N \bigcup \{0\}$ as $V: x_1, x_2, ..., x_p$

 $f(v): 0, 5, 6, ..., n^2 + 1$ is clearly a radio labeling satisfying (2.2) with span $n^2 + 1$.

$$\therefore rn(BS(P_n)) \le n^2 + 1$$

Hence $rn(BS(P_n)) = n^2 + 1$ if n is odd

Example 2.1: In Table 1, Figure 1 and Figure 2, an ordering of the vertices, renamed version and radio labeling of $BS(P_9)$ are shown



Theorem 2.2: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices. Then $rn(BS(P_n)) \ge n^2 - n + 1$ if n is even.

Proof: Let f be an optimal radio labeling for $BS(P_n)$, where $0 = f(x_1) < f(x_2) < f(x_3) < ... < f(x_n)$

Define the radio labeling $f: V(BS(P_n)) \to N \cup \{0\}$ and satisfying the radio condition $f(x_{i+1}) \ge f(x_i) + d + 1 - d(x_{i+1}, x_i)$ for all $1 \le i \le p - 1$ (2.3)

Let n = 2a and $a = \left\lfloor \frac{n}{2} \right\rfloor, a \ge 1$

In this case diameter d = 2a and p = 2n+1

Using the radio condition defined in (2.3) and summing up p-1 inequalities we get $rn(BS(P_n)) = f(x_n)$

$$\geq (p-1)(d+1) - \sum_{i=1}^{p-1} d(x_i, x_{i+1})$$
$$= 2n(n+1) - (n^2 + 3n - 1)$$

 $\therefore rn(BS(P_n)) \ge n^2 - n + 1$ if n is even

Theorem 2.3: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices. Then $rn(BS(P_n)) \le n^2 + 1$ if n is even.

Proof: For $BS(P_n)$ define $f: V(BS(P_n)) \to N \cup \{0\}$ by $f(x_1) = 0$ and for all $1 \le i \le p - 1$ $f(x_{i+1}) = f(x_i) + d + 1 - d(x_i, x_{i+1})$ as per following ordering of vertices:

Arrange the vertices of the graph $BS(P_n)$ as follows

Starting from $v_i^j, 1 \le i \le n, 1 \le j \le 2$ and ending with v_{n+1}^1

Then rename the vertices by
$$v_i^j = \begin{cases} v_{L(a+1-i)}^j, & 1 \le i \le a, & 1 \le j \le 2\\ v_c^1, & i = a+1, & j = 1\\ v_{R(i-a)}^j, & a+1 \le i \le 2a, & j = 2\\ v_{R(i-(a+1)),}^1, & a+2 \le i \le 2a+1, & j = 1 \end{cases}$$

So for $BS(P_n), V(BS(P_n)) = \{v_i^j, v_{n+1}^1 : 1 \le i \le n, 1 \le j \le 2\}$ be the vertices $x_1, x_2, ..., x_p$ as in the following procedure

$$v_{L1}^{2} \rightarrow v_{Ra}^{2} \rightarrow v_{L1}^{1} \rightarrow v_{Ra}^{1} \rightarrow v_{L2}^{2}$$
$$v_{R(a-1)}^{2} \rightarrow v_{L2}^{1} \rightarrow v_{R(a-1)}^{1} \rightarrow \dots \rightarrow$$
$$v_{La}^{2} \rightarrow v_{R1}^{2} \rightarrow v_{La}^{1} \rightarrow v_{R1}^{1} \rightarrow v_{c}^{1}$$

Thus it is possible to assign labeling to the vertices of $BS(P_n)$ with span n more than the lower bound.

Hence $rn(BS(P_n)) \le n^2 + 1$ if n is even

Example 2.2: In Table 2, Figure 3 and Figure 4 an ordering of the vertices, renamed version and optimal radio labeling of $BS(P_{10})$ are shown

Table-2 $v_{L1}^{2} \rightarrow v_{R5}^{2} \rightarrow v_{L1}^{1} \rightarrow v_{R5}^{1}$ $v_{L2}^{2} \rightarrow v_{R4}^{2} \rightarrow v_{L2}^{1} \rightarrow v_{R4}^{1}$ $v_{L3}^{2} \rightarrow v_{R3}^{2} \rightarrow v_{L3}^{1} \rightarrow v_{R3}^{1}$ $v_{L4}^{2} \rightarrow v_{R2}^{2} \rightarrow v_{L4}^{1} \rightarrow v_{R2}^{1}$ $v_{L5}^{2} \rightarrow v_{R1}^{2} \rightarrow v_{L5}^{1} \rightarrow v_{R1}^{1} \rightarrow v_{c}^{1}$



3. RADIO NUMBER FOR INCENTRIC SUBDIVISION OF SPOKE WHEEL GRAPHS

Definition 3.1: A vertex of degree 3 is called rim vertex [1]. A vertex which is adjacent to all the rim vertices is called the central vertex.

Definition 3.2: The edges with one end incident with the rim and the other incident with the central vertex are called spokes [1].

Definition 3.1: If every edge inside a wheel graph is subdivided then the resulting graph is called in centric subdivision of spoke wheel graph. In other words, in centric subdivision is the graph obtained by inserting a vertex of degree two into every edge inside the original graph.

Definition 3.3: The graph $SS(W_n)$ [6] is obtained from the wheel W_n by subdividing each spokes by a vertex. Let $W_n = C_n + k_1$ where $C_n = v_1 v_2 \dots v_n v_1$ and $v(k_1) = \{u\}$ and the spokes are subdivided by the vertices u_i $(1 \le i \le n)$

Note that the diameter of $SS(W_n)$ is 4.

Thereom 3.1: $rn(SS(W_n)) = 4n + 2$ for $n \ge 8$

Proof: Let *u* be the centre vertex.

The cyclic vertices are $v_1, v_2, ..., v_n$ and the vertices of incentric subdivision of spokes are $u_1, u_2, ..., u_n$.

Clearly $SS(W_n)$ has 2n+1 vertices and $diam(SS(W_n)) = 4$.

Define radio labeling $f: V \to N \bigcup \{0\}$ satisfying the condition $d(u,v) + |f(u) - f(v)| \ge 1 + diam(SS(W_n)) = 5$, for every pair of distinct vertices u and v.

Case-1: Label u with 0.

That is f(u) = 0

Label the vertices of cycle and vertices of subdivision which are at distance 3, alternatively.

Then $f(v_i) = 4i$ and $f(u_{3+i}) = 4i + 2$, i = 1, 2, ..., nWhere $u_{n+1} = u_1$, $u_{n+2} = u_2$ and $u_{n+3} = u_3$

Hence rn(f) = 4n + 2© 2015, IJMA. All Rights Reserved D. S. T. Ramesh¹, K. Sunitha^{*2} / Radio labeling for Barycentric Subdivision of Paths and Incentric... / IJMA- 6(6), June-2015.



Case-2: Label the cyclic vertices with 0

Let $f(v_1) = 0$

Then label the vertices of subdivision and cyclic vertices alternatively, which are at distance3

Then $f(v_i) = 4i - 3, i = 2, 3, ..., n$ and $f(u_{3+i}) = 4i - 1, i = 1, 2, ..., n$ Where $u_{n+1} = u_1, u_{n+2} = u_2$ and $u_{n+3} = u_3$

Since $d(u_3, u) = 1$, f(u) = 4n + 3

Hence rn(f) = 4n + 3



Figure-6: $rn(SS(W_8)) = 35$

Case-3: Label the vertices of subdivision with 0

Let $f(u_3) = 0$

Since $d(u_3, u) = 1$ and $d(u, u_1) = 1$, f(u) = 5 and $f(v_1) = 8$

Now we label the vertices of subdivision and cyclic vertices alternatively with distance 3.

Then $f(u_{3+i}) = 4i + 6, i = 1, 2, ..., n$ and $f(v_i) = 4i + 4, i = 1, 2, ..., n$ where $u_{n+1} = u_1$, $u_{n+2} = u_2$ and $u_{n+3} = u_3$

Hence rn(f) = 4n + 4



Figure-7: $rn(SS(W_8)) = 36$

Hence $rn(SS(W_n)) = \min\{rn(f)\}\$ = $\min\{4n+2, 4n+3, 4n+4\}\$ = 4n+2

Example 3.1: The radio number of some incentric subdivision of spokewheel graphs are given in Fig.8



Figure-8: (*a*) $rn(SS(W_9)) = 38$

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4. RADIO MEAN NUMBER OF INCENTRIC SUBDIVISION OF SPOKE WHEEL GRAPHS

Definition 4.1: A radio mean labeling is a one to one mapping f from V(G) to N Satisfying the condition

$$d(u,v) + \left\lceil \frac{f(u) + f(v)}{2} \right\rceil \ge 1 + diam(G) \text{, for every } u, v \in V(G)$$

The radio mean number of G is denoted by rmn(G)

Theorem 4.1: $rmn(SS(W_n)) = 2n+1, n \ge 8$

Proof: Let u be the centre vertex

The cyclic vertices are $v_1, v_2, ..., v_n$ and the vertices of subdivision of spokes are $u_1, u_2, ..., u_n$.

Clearly $SS(W_n)$ has 2n+1 vertices and $diam(SS(W_n)) = 4$

Define the radio mean labeling $f: V \rightarrow N$ satisfying the condition

$$d(u,v) + \left| \frac{f(u) + f(v)}{2} \right| \ge 1 + diam(SS(W_n)) = 5, \text{ for every pair of distinct vertices } u \text{ and } v.$$

Consider the rim vertices v_i $(1 \le i \le n)$

Define the radio mean labeling $f: V \to N$ by

$$f(v_i) = \begin{cases} 1 & \text{if} \quad i = 1 \\ n - i & \text{if} \quad i = 2 \\ n - i + 2 & \text{if} \quad i = 3 \\ 2 & \text{if} \quad i = 4 \\ n & \text{if} \quad i = 5 \\ i - 3 & \text{if} \quad i = 6, 7, \dots n \end{cases}$$

$$f(u_i) = n + i, i = 1, 2, ..., n$$
 and $f(u) = 2n + 1$

Now we check the radio mean condition for the above labeling f.

Case-1: Consider the pair (u_i, u_j)

Subcase-(a): Verify the pair (v_1, v_i) .

It is easy to verify that the pairs (v_1, v_i) , i = 4, 6, 7, 8

For $i \notin \{4, 6, 7, 8\}$ we have

$$d(v_1, v_i) + \left\lceil \frac{f(v_1) + f(v_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{1+6}{2} \right\rceil \ge 5.$$

Subcase-(b): Verify the pair (v_4, v_i) .

The pairs $(v_4, v_6), (v_4, v_7)$ satisfy the radio mean condition.

For
$$i \neq 1, 6, 7, d(v_4, v_i) + \left\lceil \frac{f(v_4) + f(v_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{2+5}{2} \right\rceil \ge 5.$$

Subcase-(c): Check the pair (v_i, v_j) , $i, j \notin \{1, 4\}$

$$d(v_i, v_j) + \left\lceil \frac{f(v_i) + f(v_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{3+4}{2} \right\rceil \ge 5$$

Case-2: Check the pair (u_i, u_j)

$$d(u_i, u_j) + \left\lceil \frac{f(u_i) + f(u_j)}{2} \right\rceil \ge 2 + \left\lceil \frac{n+1+n+2}{2} \right\rceil \ge 13$$

Case-3: Check the pair (v_i, u_j)

$$d(v_i, u_j) + \left\lceil \frac{f(v_i) + f(u_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{1 + n + 1}{2} \right\rceil \ge 7$$

Case-4: Examine the pair (u, u_i)

$$d(u, u_i) + \left\lceil \frac{f(u) + f(u_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n + 1 + n + 1}{2} \right\rceil \ge 16$$

These cases establish the radio mean condition.

Hence $rmn(SS(W_n)) \le 2n+1$.

Since the number of vertices is 2n+1 and the labels are unique, it follows that $rmn(SS(W_n)) \ge 2n+1$.

Hence $rmn(SS(W_n)) = 2n+1$

Example 4.1: The radio mean number of some incentric subdivision of spokewheel graphs are given in Fig.9



Figure-9:(b) *rmn* $(SS(W_{10})) = 21$

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