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THE HOMOMORPHISM THEOREMS IN A REGULAR INCLINE

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ABSTRACT

In this paper, we have proved that if R is a regular incline, I and J are ideals of R such that I⊆J⊆R then J/I is an ideal of a regular quotient incline R/I and studied the homomorphism between the regular inclines. Further we have proved that Π is the natural homomorphism between the regular inclines R and R/I and Kernel of Π is an ideal of R. In particular we have shown Kernel of Π equals I and we have proved the homomorphism theorems in the context of regular incline.

Key words: *inclines, subinclines, homomorphism, kernel, ideals.*

Mathematics Subject Classification: *16D25, 16Y60.*

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1. INTRODUCTION:

Inclines are additively idempotent semirings in which products are less than (or) equal to either factor. The concept of incline was introduced by Cao and later it was developed by Cao, et.al, in [2]. Recently a survey on incline was made by Kim and Roush [3]. Incline algebra is a generalization of both Boolean and fuzzy algebras and it is a special type of a semiring. It has both a semiring structure and a poset structure. It can also be used to represent automata and other mathematical systems, to study inequalities for non-negative matrices of polynomials.

In [1], S.S. Ahn, Y.B. Jun and H.S. Kim, have introduced the equivalence relation in R with respect to an ideal I of R and if J is an ideal of R such that I⊆J⊆R then they have proved that the set of all equivalence classes forms an incline called quotient incline R/I, under certain operations and proved that J/I is an ideal of R/I and have obtained the structure of incline algebras. In our earlier work [7], we have developed the results on equivalence relation in R, with respect to I and observed that the results found in [1] are valid only for a regular incline R. In particular, the set of all equivalence classes of R with respect to I denoted as R/I is a regular incline, under suitable operations, where I is an ideal of a regular incline R.

In this paper, by using the results of [7] we have studied the mappings between the inclines and proved the homomorphism theorems in the context of a regular incline. In section 2, we present the basic definitions, notations and required results of an incline R. In section 3, we have proved that if R is a regular incline, I and J are ideals of R such that I⊆J⊆R then J/I is an ideal of a regular quotient incline R/I and studied the homomorphism between two regular inclines. Further the basic properties of the kernel of homomorphism are obtained such as the kernel of a homomorphism is an ideal of the domain of the homomorphism and we have proved the homomorphism theorems in the context of regular incline with respect to the ideals of the inclines.

2. PRELIMINARIES:

In this section we present some definitions, notations and required results in an incline.

Definition 2.1: An *incline* is a non-empty set R with binary operations addition and multiplication denoted as +, · defined on R x R → R such that for all x, y, z ∈ R,

$x+y = y+x, \quad x+(y+z) = (x+y)+z,$
$x(y+z) = xy+xz, \quad (y+z)x = yx+zx,$
$x(yz) = (xy)z, \quad x+xy = x,$
$x+x = x, \quad y+xy = y.$

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An incline R is said to be *commutative* if xy = yx, for all $x, y \in R$.

In [2], Authors have considered the incline as commutative incline.

Definition 2.2: An element 0_R in an incline R is the *zero element* of R if $x + 0_R = 0_R + x = x$ and $x \,.\, 0_R = 0_R \,.\, x = 0_R$, for all $x \in R$.

Lemma 2.3[5]: An incline R is *regular* if and only if $x^2 = x$, for all $x \in R$.

In [2], Authors have proved in Proposition 1.1.1 that "Every Distributive lattice is an incline. An incline is a Distributive lattice (as semiring) if and only if $x^2 = x$, for all $x \in R$". But in [5], Authors have proved "In a commutative incline R, R is regular if and only if R is a Distributive Lattice".

Example for regular incline:

Let D ={a, b, c} and $\wp(D)$ be the power set of D and R ={ $\wp(D), \cup, \cap$} is an incline. Here for any $x \in \wp(D)$, $x^2 = x \cap x = x$ and xy = yx. Hence x is idempotent and x is regular, by Lemma 2.3, R is a commutative regular incline.

Definition 2.4: $(R, \leq)$ is an incline with order relation '$\leq$' defined as, $x \leq y$ if and only if x+y = y, for $x, y \in R$. If $x \leq y$ then y is said to dominate x.

Property 2.5: For x, y in an incline R, $x+y \geq x$ and $x+y \geq y$.

Property 2.6: For x, y in an incline R, $xy \leq x$ and $xy \leq y$.

Definition 2.7: A *subincline* of an incline R is a non–empty subset I of R which is closed under the incline operations addition and multiplication.

Definition 2.8: A subincline I is said to be an *ideal* of an incline R if $x \in I$ and $y \leq x$ then $y \in I$.

Definition 2.9 [1]: A subincline I of an incline R is said to be a *k-ideal* if $a \in I$ and $x \in R$ such that $a+x \in I$, then $x \in I$.

In [1], it is proved that Definition 2.8 and definition 2.9 are the same.

Definition 2.10 [1]: For any $x, y \in R$, a *relation* '~' on R is defined by x ~ y with respect to an ideal I of R if and only if there exist $i_1, i_2 \in I$ such that $x + i_1 = y + i_2$.

In [1], it is proved that the relation '~' is an equivalence relation on R.

NOTATION:
For any $x \in R$, $[x]_I = \{ y \in R / x \sim y$, with respect to I$\}$ denotes the equivalence class of x, with respect to an ideal I of R . In an incline R, for $x, y \in R$, x ~ y if and only if $[x]_I = [y]_I$ and $x \nsim y$ if and only if $[x]_I$ and $[y]_I$ are disjoint.

Definition 2.11: If R and R′ are inclines then an incline *homomorphism* is a mapping $\Phi : R \to R'$ such that

$\Phi(x + y) = \Phi(x) + \Phi(y)$ for all x, y in R
$\Phi(xy) = \Phi(x)\,\Phi(y)$ for all x, y in R.

Definition 2.12: An incline *isomorphism* is a bijective incline homomorphism.

Definition 2.13: If $\Phi : R \to R'$ is an incline homomorphism then the *kernel* of Φ is defined as
$\text{Ker}(\Phi) = \{ x \in R / \Phi(x) = 0_{R'} \}$ where $0_{R'}$ is the zero element of R′.

In the sequel we shall make use of the following results from our earlier work found in [7]:

In [7], we have proved that if R is a Regular incline then R/I is an incline under the certain operations. Hence we have assumed R to be a regular incline.

Proposition 2.14: Let x be any element of R. Then $x \in I$ if and only if $[x]_I = I$. In particular, $[0]_I = I$, where 0 is the zero element of R.

Theorem 2.15: In a regular incline R, if I is an ideal of R, then the set of all equivalence classes, with respect to I, denoted by R/I is a regular incline under the operations, (A.1) $[x]_I + [y]_I = [x+y]_I$.

(A.2) $[x]_I \,.\, [y]_I = [xy]_I$, for any $x, y \in R$.

And $[0]_I$ is the zero element of R/I.

3. HOMOMORPHISM THEOREMS IN A REGULAR INCLINE:

In this section, we prove that if R is a regular incline, I and J are ideals of R such that $I \subseteq J \subseteq R$ then J/I is an ideal of R/I and Π is the natural homomorphism between the regular inclines R and R/I and Ker(Π) is an ideal of R. In particular we have shown Ker (Π) = I. Further, we have proved the homomorphism theorems in the context of regular incline. Throughout this section, R is a regular incline with zero element '0'.

Lemma 3.1: If I and J are any ideals of a regular incline R and $I \subseteq J$, then $J/I = \{[x]_I \,/\, x \in J\}$ is an ideal of the quotient incline R/I.

Proof: Since $I \subseteq J$, $I = [0]_I \in J/I$ (by Proposition 2.14) and J/I is non-empty.

Let $[x]_I, [y]_I \in J/I$, for some $x, y \in J$.

Now, $[x]_I + [y]_I = [x+y]_I$ (by Theorem 2.15)
$= [z]_I$, $z = x+y \in J$ (by Definition 2.8)
$\in J/I$.

Also, $[x]_I \,.\, [y]_I = [xy]_I$ (by Theorem 2.15)
$= [p]_I$, $p = xy \in J$ (by Definition 2.8)
$\in J/I$.

Therefore, J/I is a subincline of R.

Now if $[x]_I \in J/I$ and $[a]_I \in R/I$ such that $[a]_I \le [x]_I$

$\Rightarrow [a]_I + [x]_I = [x]_I$
$\Rightarrow [a+x]_I = [x]_I$ (by Theorem 2.15)
$\Rightarrow a+x \sim x$, with respect to I
$\Rightarrow a+x+i_1 = x+i_2$, for some $i_1, i_2 \in I$

Since $I \subseteq J$, $x+i_1$, $x+i_2 \in J$ and by Definition (2.9) $a \in J$ and $[a]_I \in J/I$.

Thus, J/I is an ideal of R/I.

Lemma 3.2: If I is an ideal of a regular incline R, then

(i) $\Pi : R \to R/I$ is the natural homomorphism.
(ii) Ker (Π) is an ideal of R.

Proof: The natural homomorphism is a mapping $\Pi : R \to R/I$ defined as $\Pi(x) = [x]_I$, for all $x \in R$ and I, an ideal of R.

(i) Let $x, y \in R$. We have,

$\Pi(x + y) = [x + y]_I$ (by Definition of Π)
$= [x]_I + [y]_I$ (by Theorem 2.15)
$= \Pi(x) + \Pi(y)$

Also, $\Pi(xy) = [xy]_I$ (by Definition of Π)
$= [x]_I.[y]_I$ (by Theorem 2.15)
$= \Pi(x)\, \Pi(y)$

Therefore, Π is a homomorphism.

Now, let $[x]_I \in R/I$, then $\Pi(x) = [x]_I$, for some $x \in R$.

Therefore, Π is surjective.

Let Ker(Π) = $\{x \in R \,/\, \Pi(x) = [0]_I = I\}$ (by Proposition 2.15)

(i) To show that Ker(Π) is an ideal of R, let x, y ∈ Ker(Π)

$\Pi(x + y) = \Pi(x) + \Pi(y)$ (by Definition 2.11)
$= I + I$ (by Definition of Ker(Π))
$= I$
$\Rightarrow x + y \in$ Ker(Π)

and, $\Pi(xy) = \Pi(x)\, \Pi(y)$ (by Definition 2.11)
$= I \,.\, I$ (by Definition of Ker(Π))
$= I$
$\Rightarrow xy \in$ Ker(Π)

Therefore, Ker (Π) is a subincline of R.

Let x ∈ Ker (Π) and y∈R such that $y \le x$

$\Rightarrow y + x = x$ (by Definition 2.4)
$\Rightarrow \Pi(y + x) = \Pi(x)$
$\Rightarrow \Pi(y) + \Pi(x) = \Pi(x)$ (by Definition 2.11)
$\Rightarrow \Pi(y) + I = I$ (by Definition of Ker(Π))
$\Rightarrow \Pi(y) = I$ (by Definition 2.2)
$\Rightarrow y \in$ Ker(Π)

Thus, Ker (Π) is an ideal of R.

Lemma 3.3: If $\Pi : R \to R/I$ is the natural homomorphism and J/I is an ideal of R/I then $J = \{x \in R \,/\, \Pi(x) \in J/I\}$ is an ideal of R.

Proof: Let $\Pi : R \to R/I$ be the natural homomorphism and J/I be an ideal of R/I.

If x, y ∈ J then by our assumption $\Pi(x), \Pi(y) \in J/I$

Therefore, $\Pi(xy) = \Pi(x)\, \Pi(y)$ (by Definition 2.11)

$\in J/I$ (since J/I is an ideal of R/I)
$\Rightarrow xy \in J$ (since Π is the natural homomorphism)

Also, $\Pi(x+y) = \Pi(x)+\Pi(y)$ (by Definition 2.11)

$\in J/I$ (since J/I is an ideal of R/I)
$\Rightarrow x+y \in J$ (since Π is the natural homomorphism)

Hence J is a subincline of R.

If x∈J then by our assumption Π(x)∈J/I. Since J/I is an ideal of R/I, if $\Pi(y) \le \Pi(x)$ for some $\Pi(y) \in R/I$ then $\Pi(y) \in J/I$

$\Rightarrow y \le x$ and $y \in J$ (since Π is the natural homomorphism)

Thus, J is an ideal of R.

By Theorem (2.15), R/I is a regular quotient incline and by Lemma (3.1), J/I is an ideal of R/I.

Definition 3.4: For any $[x]_I, [y]_I \in R/I$, a relation ' ~ ' on R/I is defined by $[x]_I \sim [y]_I$ with respect to an ideal J/I of R/I if and only if there exist $[i_1]_I, [i_2]_I \in J/I$ such that $[x]_I + [i_1]_I = [y]_I + [i_2]_I$.
Trivially, the relation ' ~ ' is an equivalence relation on R/I.

Notation: For any $[x]_I \in R/I$, $[[x]_I]_{J/I} = \{[y]_I \in R/I \,/\, [x]_I \sim [y]_I$, with respect to J/I$\}$ denotes the equivalence class of $[x]_I$, with respect to an ideal J/I of R/I . In an incline R/I, for $[x]_I, [y]_I \in R/I$, $[x]_I \sim [y]_I$ if and only if $[[x]_I]_{J/I} = [[y]_I]_{J/I}$ and $[x]_I \nsim [y]_I$ if and only if $[[x]_I]_{J/I}$ and $[[y]_I]_{J/I}$ are disjoint.

Lemma 3.5: Let $[x]_{I\ be}$ any element of R/I. Then $[x]_I \in J/I$ if and only if $[[x]_I]_{J/I} = J/I$. In particular, $[[0]_I]_{J/I} = I$, where $[0]_I$ is the zero element of R/I.

Lemma 3.6: In a regular incline R/I, if J/I is an ideal of R/I, then the set of all equivalence classes, with respect to J/I, denoted by R/I ⁄ J/I is a regular incline under the operations,

(A.1) $[[x]_I]_{J/I} + [[y]_I]_{J/I} = [[x]_I + [y]_I]_{J/I}$.
(A.2) $[[x]_I]_{J/I} . [[y]_I]_{J/I} = [[x]_I . [y]_I]_{J/I}$, for any $[x]_I, [y]_I \in R/I$.

And $[[0]_I]_{J/I}$ is the zero element of R/I ⁄ J/I.

Lemma 3.7: If I and J are ideals of a regular incline R and $\Pi : R \to R/I$ is the natural homomorphism then $\Psi: R \to R/I ⁄ J/I$ is a surjective homomorphism and Ker $(\Psi) = J$.

Proof: Let $\Pi : R \to R/I$ be the natural homomorphism and Ψ be defined as

$$\Psi(x) = [\Pi(x)]_{J/I}, \text{ for all } x \in R.$$

To prove $\Psi: R \to R/I ⁄ J/I$ is a surjective homomorphism:

If $[x]_I \in R/I$, then $\Pi(x) = [x]_I$, for all $x \in R$ (by the Definition of Π).

Since Π is surjective, we have $[\Pi(x)]_{J/I} = \Psi(x)$.

Thus Ψ is surjective.

Now for $x, y \in R$, $\Psi(xy) = [\Pi(xy)]_{J/I}$
$= [\Pi(x).\Pi(y)]_{J/I}$ (by Definition 2.11)
$= [\Pi(x)]_{J/I} [\Pi(y)]_{J/I}$ (by Lemma 3.6)
$= \Psi(x).\Psi(y)$

$\Rightarrow \Psi(xy) = \Psi(x).\Psi(y)$

Also, $\Psi(x+y) = [\Pi(x+y)]_{J/I}$
$= [\Pi(x)+\Pi(y)]_{J/I}$ (by Definition 2.11)
$= [\Pi(x)]_{J/I}+[\Pi(y)]_{J/I}$ (by Lemma 3.6)
$= \Psi(x)+\Psi(y)$
$\Rightarrow \Psi(x+y) = \Psi(x)+\Psi(y)$
Thus, Ψ is homomorphism.

By Definition (2.13), we have $\text{Ker}(\Psi) = \{x \in R \mid \Psi(x) = [[0]_I]_{J/I} = J/I\}$ (by Lemma 3.5)

To prove $\text{Ker}(\Psi) = J$:

If $p \in J$ then by Lemma (3.3), $\Pi(p) \in J/I$.

Hence, $\Psi(p) = [\Pi(p)]_{J/I} = J/I$, the zero element of R/I ⁄ J/I (by Lemma 3.5)

Therefore, $J \subseteq \text{Ker}(\Psi)$.

On the other hand, if $q \in \text{Ker}(\Psi)$ then $\Psi(q) = J/I$ (3.1)

By Definition of Ψ, $\Psi(q) = [\Pi(q)]_{J/I}$ (3.2)

From equations (3.1) and (3.2) we have

$[\Pi(q)]_{J/I} = J/I$
$\Rightarrow \Pi(q) \in J/I$ (by Lemma 3.5)
$\Rightarrow q \in J$ (by Lemma 3.3)

Therefore, $\text{Ker}(\Psi) \subseteq J$
Thus, $\text{Ker}(\Psi) = J$

Theorem 3.8: Let R be a regular incline. If $\Pi : R \to R/J$ is the natural homomorphism and $\Psi: R \to R/I \,/\, J/I$ is a surjective homomorphism then there is a unique isomorphism $\Phi: R/J \to R/I \,/\, J/I$ such that $\Psi = \Phi \circ \prod$.

Proof: We define $\Phi: R/J \to R/I \,/\, J/I$ as $\Phi([x]_J) = \Psi(x)$, for all $x \in R$ (since $\Psi = \Phi \circ \prod$).

To prove Φ is well defined:

If $[x]_J = [y]_J$, for some $x, y \in R$

$\Rightarrow x \sim y$, with respect to J
$\Rightarrow x + j_1 = y + j_2$, for some $j_1, j_2 \in J$
$\Rightarrow \Psi(x+j_1) = \Psi(y+j_2)$
$\Rightarrow \Psi(x) + \Psi(j_1) = \Psi(y) + \Psi(j_2)$ (by Definition 2.11)
$\Rightarrow \Psi(x) + J/I = \Psi(y) + J/I$ (by Lemma 3.7)
$\Rightarrow \Psi(x) = \Psi(y)$ (since J/I is the zero element of $R/I \,/\, J/I$)

Therefore, $\Phi([x]_J) = \Psi(x) = \Psi(y) = \Phi([y]_J)$

Thus, Φ is well defined.

To prove Φ is homomorphism:

Let $[x]_J, [y]_J \in R/J$

Therefore, $\Phi([x]_J + [y]_J) = \Phi([x+y]_J)$ (by Theorem 2.15)
$= \Psi(x+y)$
$= \Psi(x) + \Psi(y)$ (by Definition 2.11)
$= \Phi([x]_J) + \Phi([y]_J)$

Also, $\Phi([x]_J . [y]_J) = \Phi([xy]_J)$ (by Theorem 2.15)
$= \Psi(xy)$
$= \Psi(x).\Psi(y)$ (by Definition 2.11)
$= \Phi([x]_J).\Phi([y]_J)$

Thus, Φ is homomorphism

To prove Φ is injective:

Suppose, $\Phi([x]_J) = \Phi([y]_J)$, for some $[x]_J, [y]_J \in R/J$

$\Rightarrow \Psi(x) = \Psi(y)$
$\Rightarrow [[x]_I]_{J/I} = [[y]_I]_{J/I}$
$\Rightarrow [x]_I \sim [y]_I$, with respect to J/I.
$\Rightarrow [x]_I + [j_1]_I = [x]_I + [j_2]_I$, for some $[j_1]_I, [j_2]_I \in J/I$
$\Rightarrow [x+j_1]_I = [y+j_2]_I$ (by Theorem 2.15)
$\Rightarrow x+j_1 \sim y+j_2$, with respect to I
$\Rightarrow x+j_1+i_1 = y+j_2+i_2$, for some $i_1, i_2 \in I$
$\Rightarrow x+j_3 = y+j_4$, where $j_3=j_1+i_1$, $j_4=j_2+i_2$ and since $I \subseteq J$ we have $j_3, j_4 \in J$
$\Rightarrow x \sim y$, with respect to J
$\Rightarrow [x]_J = [y]_J$

Thus, Φ is injective.

To prove Φ is surjective:

Since Ψ is surjective, for all $z = [[x]_I]_{J/I} \in R/I \,/\, J/I$ there is an $x \in R$ such that $\Psi(x) = z$. That is, $\Phi([x]_J) = z$.

Thus, Φ is surjective.

To prove Φ is unique:

Suppose that there exists an isomorphism $\Phi': R/J \to R/I \,/\, J/I$ such that $\Phi([x]_J) \neq \Phi'([x]_J)$.

Then $\Psi(x) = (\Phi'\circ\prod)(x) = \Phi'(\Pi(x)) = \Phi'([x]_J) \neq \Phi([x]_J) = \Phi(\Pi(x)) = (\Phi\circ\prod)(x) = \Psi(x)$.
$\Rightarrow \Psi(x) \neq \Psi(x)$.

This is a contradiction.

Thus, Φ is unique.

Theorem 3.9: Let R and L be Regular inclines and Π be a homomorphism from R into L. Then,

(i) The Kernel of Π is an ideal of R.
(ii) The image of Π is a subincline of L
(iii) There is a unique isomorphism Φ: L $\to$ R/ Ker(Π).

Proof: Let $\Pi : R \to L$ be homomorphism.

(i) Let Ker (Π) = $\{x\in R \,/\, \Pi(x) = 0_L$, the zero element of L$\}$ (by Definition 2.13)

Let x, y $\in$ Ker (Π)

$\Pi(x + y) = \Pi(x) + \Pi(y)$ (by Definition 2.11)
$= 0_L + 0_L$ (by Definition of Ker (Π))
$= 0_L$
$\Rightarrow x + y \in$ Ker (Π)

and, $\Pi(xy) = \Pi(x)\,\Pi(y)$ (by Definition 2.11)
$= 0_L \,.\, 0_L$ (by Definition of Ker (Π))
$= 0_L$
$\Rightarrow xy \in$ Ker(Π)

Therefore, Ker(Π) is a subincline of R.

Let x $\in$ Ker(Π) and y$\in$R such that $y \leq x$

$\Rightarrow y + x = x$ (by Definition 2.4)
$\Rightarrow \Pi(y + x) = \Pi(x)$
$\Rightarrow \Pi(y) + \Pi(x) = \Pi(x)$ (by Definition 2.11)
$\Rightarrow \Pi(y) + 0_L = 0_L$ (by Definition of Ker(Π))
$\Rightarrow \Pi(y) = 0_L$ (by Definition 2.2)
$\Rightarrow y \in$ Ker(Π)

Thus, Ker (Π) is an ideal of R.

(ii) The image of Π, $L' = \{\Pi(x) \,/\, x\in R\}$

Let $\Pi(x)$, $\Pi(y)$ be two arbitrary elements of L'.

Then $\Pi(x+y) = \Pi(x) + \Pi(y)$ (by Definition 2.11)
$\Rightarrow$ L' is closed under addition in L

Also, $\Pi(xy) = \Pi(x).\Pi(y)$ (by Definition 2.11)

$\Rightarrow$ L' is closed under multiplication in L.

Thus L' is a subincline of L.

(iii) Let $\Psi : R \to R/$ Ker(Π) be a surjective homomorphism and Ker (Ψ) = $\{x\in R \,/\, \Psi(x) = [0]_{Ker(\Pi)}\}$= Ker(Π).

Thus by Theorem 3.8, Φ is a unique isomorphism.

Theorem 3.10: Let R be a regular incline. Let S be a subincline of R and I be an ideal of R. Then,

(i) The sum S + I = $\{s+i \,/\, s\in S$ and $i\in I\}$ is a subincline of R.

(ii) The intersection $S \cap I$ is an ideal of S.
(iii) There is a unique isomorphism $\Phi : S/(S \cap I) \to (S + I)/I$.

Proof: Let S be a subincline of a regular incline R and I be an ideal of R.

(i) Let $s_1+i_1, s_2+i_2 \in S+I$ be two arbitrary elements of S+I.

$(s_1+i_1)+(s_2+i_2) = s_1+s_2+ i_1+i_2$
$= s_3+i_3$, where $s_3 = s_1+s_2$, $i_3 = i_1+i_2$
$\in S+I$ (by Definition 2.7 and 2.8)

Also, $(s_1+i_1)(s_2+i_2) = s_1s_2+s_1i_2+ i_1s_2+i_1i_2$
$= s_3+i_3$, where $s_3 = s_1s_2$, $i_3 = s_1i_2+ i_1s_2+i_1i_2$
$\in S+I$ (by Definition 2.7 and 2.8)

Thus S+I is a subincline of R.

(ii) Let $x, y \in S \cap I$ be two arbitrary elements of $S \cap I$.

Since $S \cap I \subseteq S$, by Definition 2.7, we have $x+y \in S \cap I$ and $xy \in S \cap I$.

Therefore, $S \cap I$ is a subincline of S.

Now let $x \in S \cap I$ and $y \in S$ such that $y \le x$, we have to show that $y \in S \cap I$.

We know that $x \in I$ and I is an ideal implies $y \in I$. (by Definition 2.8)

Since $y \in S$ it follows that, $y \in S \cap I$.

Thus $S \cap I$ is an ideal of S (by Definition 2.8)

(iii) Let $\Psi : S \to S+I/I$ be a surjective homomorphism and

$\mathrm{Ker}(\Psi) = \{x \in S \,/\, \Psi(x) = [0]_I = I\} = S \cap I$

Thus, by Theorem 3.8, Φ is a unique isomorphism.

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