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GEOMETRIC MEAN LABELING OF SOME NEW DISCONNECTED GRAPHS

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ABSTRACT

***A** Graph G = (V, E) with p vertices and q edges is said to be a Geometric mean graph if is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 is such a way that when each edge e=uv is labeled with $f(e=uv) = \left[\sqrt{f(u)f(v)}\right]$ (or) $\left[\sqrt{f(u)f(v)}\right]$ then the resulting edge labels are distinct. In this case f is called Geometric mean labeling of G. In this paper we prove that some disconnected graphs are Geometric Mean graphs.*

Key Words: *Graph, Geometric Mean labeling, Path, Cycle, Comb, Triangular Snake, Quadrilateral Snake.*

1. INTRODUCTION

The graph considered here are simple, finite, connected and undirected graph. Let V(G) denote the vertex set and E(G) denote the edge set of G. For a detailed survey of graph labeling we refer to Gallain [1]. For all other standard terminology and notations we follow Harary [2]. S. Somasundaram and P. Vidyarani introduced the concept of Geometric Mean labeling of graphs in [3] and studied their behavior in [4], [5], [6] and [7]. In this paper we investigate the Geometric mean labeling behavior of some disconnected graphs. The following definitions are useful for our present study.

Definition1.1: A Graph G = (V, E) with p vertices and q edges is said to be a Geometric mean graph if is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 is such a way that when each edge e = uv is labeled with $f(e=uv) = \left[\sqrt{f(u)f(v)}\right]$ (or) $\left[\sqrt{f(u)f(v)}\right]$, then the resulting edge labels are distinct. In this case f is called *Geometric mean labeling* of G.

Definition1.2: The *union* of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 U G_2$ with vertex set $V = V_1 U V_2$ and the edge set $E = E_1 U E_2$.

Definition1.3: The *Corona* of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition1.4: A *Triangular Snake* T_n, is obtained from a path $u_1, u_2, \ldots u_n$ by joining u_i and u_{i+1} to a new vertex $v_i, 1 \le i \le n-1$.

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Definition1.5: A *Double Triangular Snake* $D(T_n)$ consists of two Triangular Snakes that have a common path.

Definition1.6: A *Quadrilateral Snake* Q_n is obtained from a path $u_1, u_2, ...u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and $w_i, 1 \le i \le n-1$.

Definition1.7: A *Double Quadrilateral Snake* $D(Q_n)$ consists of two Quadrilateral Snakes that have a common path.

Theorem1.8: Triangular Snake T_n is a Geometric Mean graph.

Theorem1.9: Double Triangular Snake $D(T_n)$ is a Geometric Mean graph.

Theorem1.10: Quadrilateral Snake Q_n is a Geometric Mean graph.

Theorem1.11: Double Quadrilateral Snake $D(Q_n)$ is a Geometric Mean graph.

2. MAIN RESULTS

Theorem2.1: $G_m \cup T_n$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let $v_1v_2...v_n$ be the path P_n. Let T_n be the triangular snake obtained from the path P_n by joining v_i and v_{i+1} to new vertex $w_i, 1 \le i \le n-1$. Let $G = C_m \cup T_n$.
Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = i, 1 \le i \le m$

$f(v_i) = m + 3i - 2, 1 \le i \le n$

$f(w_i) = m + 3i - 1, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.2: Geometric Mean labeling of $C_7 \cup T_6$ is given below.

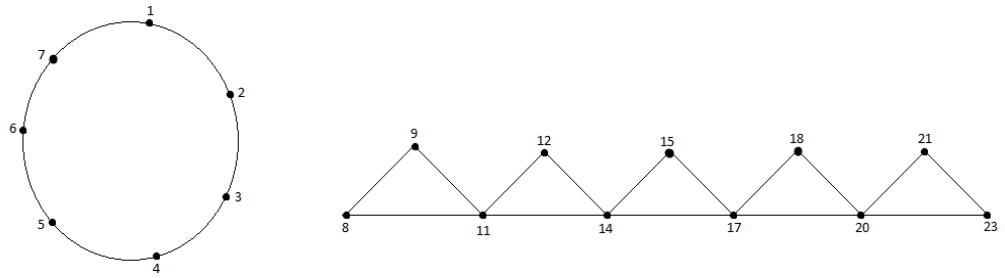

Figure-1

Theorem2.3: $(C_m \odot K_1) \cup T_n$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let v_i be the vertex of K_1 which is attached to the vertex $u_i, 1 \le i \le m$ of the cycle C_m. Let $w_1w_2...w_n$ be the path P_n Let T_n be the triangular snake obtained from P_n by joining w_i and w_{i+1} to a new vertex $x_i, 1 \le i \le n-1$. Let $G = (C_m \odot K_1) \cup T_n$.
Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = 2i - 1, 1 \le i \le 2$

$f(u_i) = 2i, 3 \le i \le m$

$f(v_i) = 2i, 1 \le i \le 2$

$f(v_i) = 2i-1, 3 \le i \le m$

$f(w_i) = 2m+3i-2, 1 \le i \le n$

$f(x_i) = 2m+3i-1, 1 \le i \le n-2$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.4: Geometric Mean labeling of $(C_7 \odot K_1) \cup T_5$ is given below.

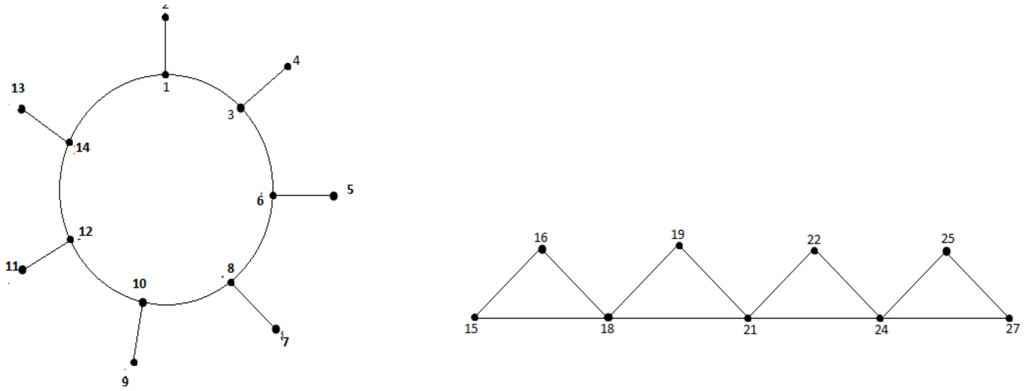

Figure-2

Theorem2.5: $C_m \odot D(T_n)$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let $v_1v_2...v_n$ be the path P_n. The double triangular snake $D(T_n)$ is obtained from the path P_n by joining v_i and v_{i+1} to two new vertices x_i and $y_i, 1 \le i \le n-1$. Let $G = C_m \odot D(T_n)$.

Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = i, 1 \le i \le m$

$f(v_i) = m+5i-4, 1 \le i \le n$

$f(x_i) = m+5i-3, 1 \le i \le n-1$

$f(y_i) = m+5i-2, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.6: Geometric Mean labeling of $C_7 \odot D(T_5)$ is given below.

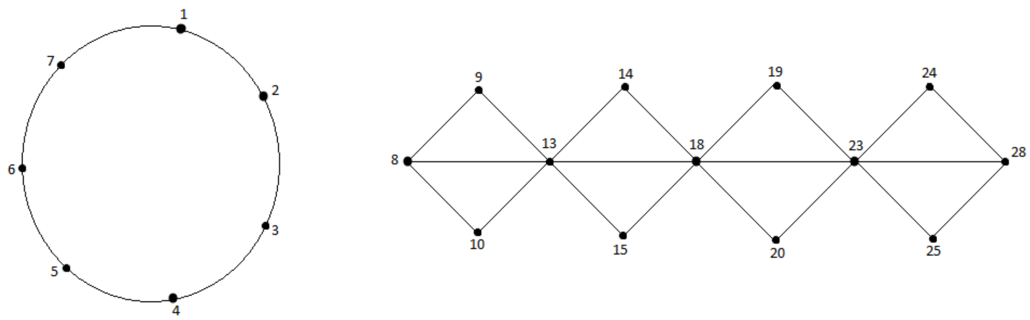

Figure-3

Theorem2.7: $(C_m \odot K_1) \cup (D(T_n))$ is a Geometric Mean graph.

Proof: Let the cycle C_m be $u_1u_2...u_mu_1$. Let v_i be the vertex of K_1 which is attached to the vertex $u_i, 1 \le i \le m$ of the cycle C_m. Let $w_1w_2...w_n$ be the path P_n The double triangular snake $D(T_n)$ is obtained by joining w_i and w_{i+1} to two new vertices x_i and $y_i, 1 \le i \le n-1$. Let $G = (C_m \odot K_1) \cup (D(T_n))$.

Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = 2i-1, 1 \le i \le 2$

$f(u_i) = 2i, 3 \le i \le m$

$f(v_i) = 2i, 1 \le i \le 2$

$f(v_i) = 2i-1, 3 \le i \le m$

$f(w_i) = 2m+5i-4, 1 \le i \le n$

$f(x_i) = 2m+5i-3, 1 \le i \le n-1$

$f(y_i) = 2m+5i-2, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.8: Geometric Mean labeling of $(C_6 \odot K_1) \cup (D(T_5))$ is given below.

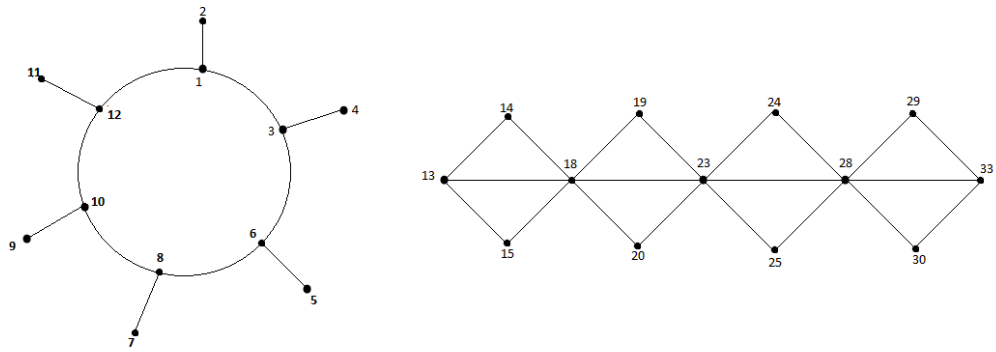

Figure-4

Theorem2.9: $C_m \cup Q_n$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let $v_1v_2...v_n$ be the path P_n. Let Q_n be the Quadrilateral snake obtained by joining v_i and v_{i+1} to two new vertices x_i and $y_i, 1 \le i \le n-1$ respectively and then joining x_i and y_i. Let $G = C_m \cup Q_n$.

Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = i, 1 \le i \le m$

$f(v_i) = m+4i-3, 1 \le i \le n$

$f(x_i) = m+4i-2, 1 \le i \le n-1$

$f(y_i) = m+4i-1, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.10: The labeling pattern of $C_7 \cup Q_5$ is given below.

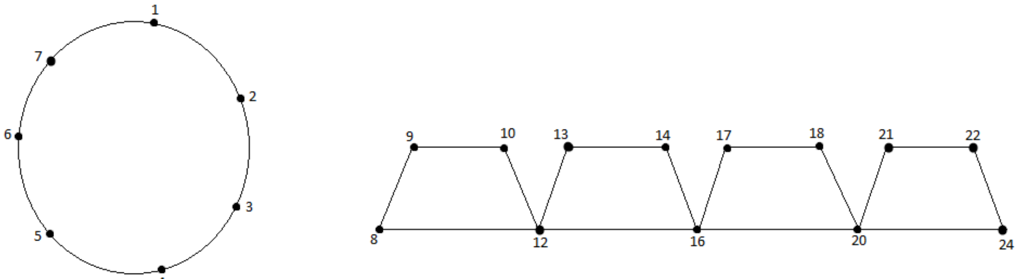

Figure-5

Theorem2.11: $(C_m \odot K_1) \cup (Q_n)$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let v_i be the vertex of K_1 which is attached to the vertex $u_i, 1 \le i \le m$ of the cycle C_m. Let $w_1w_2...w_n$ be the path P_n. Let x_i and $y_i, 1 \le i \le n-1$ be the vertices which are joined to w_i and w_{i+1} respectively. Join x_i and y_i. Let $G = (C_m \odot K_1) \cup (Q_n)$.

Define a function $f : V(G) \to \{1, 2, ...q+1\}$ by

$f(u_i) = 2i-1, 1 \le i \le 2$

$f(u_i) = 2i, 3 \le i \le m$

$f(v_i) = 2i, 1 \le i \le 2$

$f(v_i) = 2i-1, 3 \le i \le m$

$f(w_i) = 2m+4i-3, 1 \le i \le n$

$f(x_i) = 2m+4i-2, 1 \le i \le n-1$

$f(y_i) = 2m+4i-1, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G..

Example2.12: The labeling pattern of $(C_7 \odot K_1) \cup (Q_5)$ is given below.

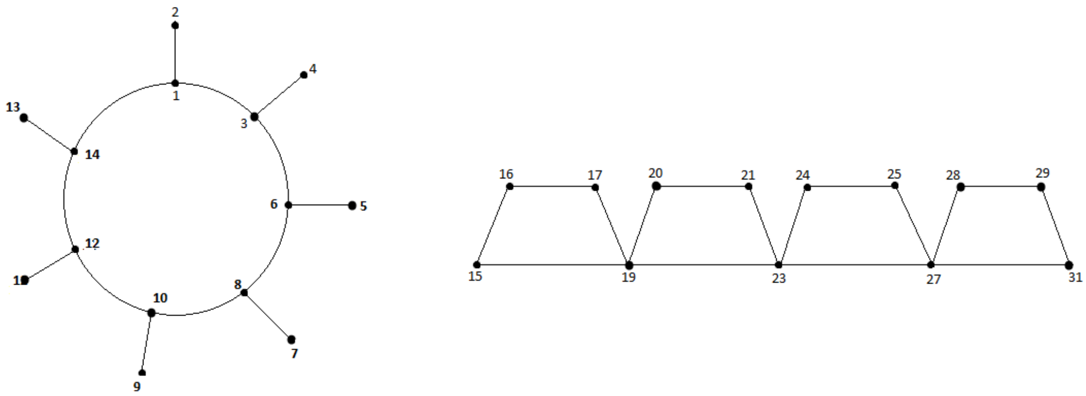

Figure-6

Theorem2.13: $C_m \odot D(Q_n)$ is a Geometric Mean graph.

Proof: Let $u_1u_2...u_mu_1$ be the cycle C_m. Let $v_i, x_i, y_i, x_i', y_i'$ be the vertices of $D(Q_n)$.
Let $G = C_m \odot D(Q_n)$.

Define a function $f : V(G) \to \{1, 2, \ldots q+1\}$ by

$f(u_i) = i, 1 \le i \le m$

$f(v_i) = m + 7i - 6, 1 \le i \le n$

$f(x_i) = m + 7i - 5, 1 \le i \le n-1$

$f(y_i) = m + 7i - 2, 1 \le i \le n-1$

$f(x_i^{'}) = m + 7i - 4, 1 \le i \le n-1$

$f(y_i^{'}) = m + 7i - 1, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric mean labeling of G.

Example2.14: The labeling pattern of $C_7 \odot D(Q_5)$ is given below.

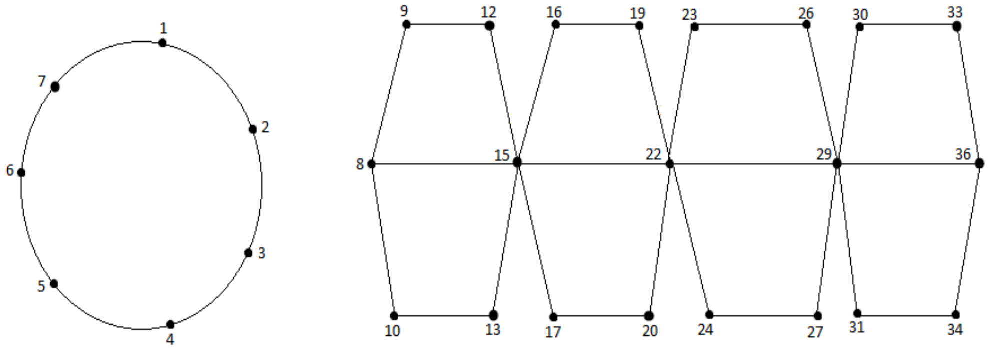

Figure-7

Theorem2.15: $(C_m \odot K_1) \cup D(Q_n)$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \ldots u_m u_1$ be the cycle C_m. Let v_i be the vertex of K_1 which is attached to the vertex $u_i, 1 \le i \le m$ of the cycle C_m. Let $w_i, x_i, y_i, x_i^{'}, y_i^{'}$ be the vertices of $D(Q_n)$. Let $G = (C_m \odot K_1) \cup D(Q_n)$.

Define a function $f : V(G) \to \{1, 2, \ldots q+1\}$ by

$f(u_i) = 2i - 1, 1 \le i \le 2$

$f(u_i) = 2i, 3 \le i \le m$

$f(v_i) = 2i, 1 \le i \le 2$

$f(v_i) = 2i - 1, 3 \le i \le m$

$f(w_i) = 2m + 7i - 6, 1 \le i \le n$

$f(x_i) = 2m + 7i - 5, 1 \le i \le n-1$

$f(y_i) = 2m + 7i - 2, 1 \le i \le n-1$

$f(x_i^{'}) = 2m + 7i - 4, 1 \le i \le n-1$

$f(y_i^{'}) = 2m + 7i - 1, 1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Geometric Mean labeling of G.

Example2.16: The labeling pattern of $\left(C_6 \odot K_1\right) \cup D\left(Q_5\right)$ is given below.

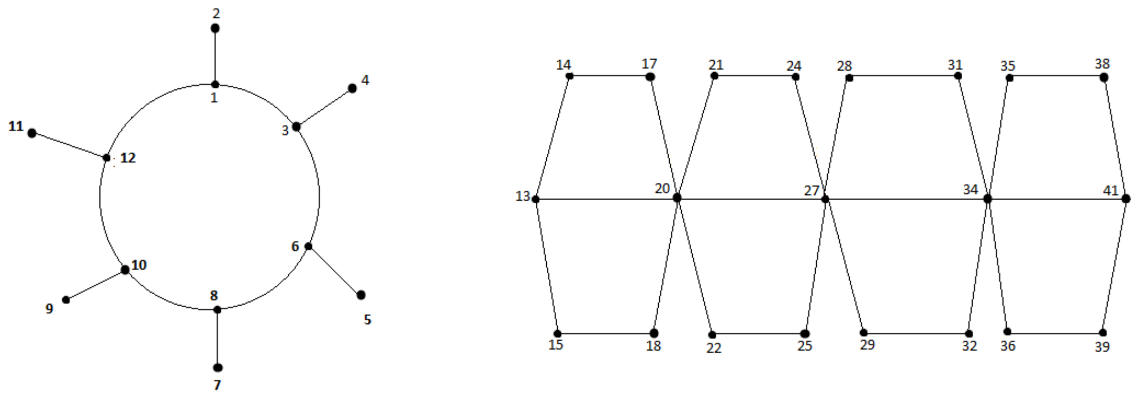

Figure-8

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