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AN INVENTORY MODEL FOR DETERIORATING GOODS WITH TIME DEPENDENT QUADRATIC DEMAND AND TIME VARYING HOLDING COST WITH PARTIAL BACKLOGGING

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ABSTRACT

A deterministic inventory model is developed with time dependent demand and time varying holding cost and deterioration is considered to be time proportional. Shortages are allowed and demand is partially backlogged. The objective of the model is to develop an optimal policy that minimizes the total average cost. Numerical examples are used to illustrate the developed models. Sensitivity analysis of the optimal solution with respect to major parameters is carried out.

Key words: *Deterioration, quadratic demand, time varying holding cost.*

INTRODUCTION

Operation research's role in both, the public and the private sectors is increasing rapidly. Inventory modeling is an important part of Operation Research, which may be used in large numbers of problems. To make it applicable in real life situations researchers are busy in modifying the existing models on different parameters under various circumstances. First inventory model was given by Harris (1915). The model was based on constant demand without any deterioration function. However, in real life situation the demand may increase or decrease in the course of time. Harris work was generalized by Wilson (1934) who gave a formula to find economic order quantity. Ghare and Schrader (1963) developed a inventory model for an exponentially decaying demand. The basic theorem for time varying demand is given by Donald – son [1977] who established the classical no shortage inventory model with a linear trend in demand over a known and finite horizon. But this method is very difficult in computing. The difficulty of Donald son's approach has led to the development of heuristic methods. Silver [1978] derived simple heuristic procedures for Donald son's problems.

Dave and Patel [1981] first considered the inventory models for deteriorating items with time- varying demand. Wee HM, Law ST (2001) developed inventory models with deteriorating items taking into account the time value of money. Kumar N and Sharma A.K. (2001) developed an optimum ordering interval with known demand for items with variable rate of deterioration and shortages. Ouyang and cheng (2005) developed economic order quantity with exponential declining demand and partial backlogging. Sharma, M. M., Goel, V. C. and Yadav, R. K. (2009) developed inventory model for decaying items with ramp type demand, and partial backlogging. Abdul, I. & Murata, A. (2011). An inventory model for deteriorating items with varying demand pattern and unknown time horizon. Yadav and yadav (2013) work on volume flexibility in production model with weibull deterioration with partial backlogging and demand is cubic demand rate. Milu acharya and smrutirekha debata (2014) developed an inventory model for deteriorating goods with time dependent demand under partial backlogging.

In this paper we develop a deterministic inventory model is developed with time dependent demand and time varying holding cost and deterioration is considered to be time proportional. Shortages are allowed and demand is partially backlogged. The objective of the model is to develop an optimal policy that minimizes the total average cost.

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ASSUMPTIONS AND NOTATIONS

The basic assumptions are as given by

- The demand rate is time dependent and demand is quadratic.
- Shortages are allowed and partially backlogged.
- Lead time is zero.
- The replenishment rate is instantaneous.
- The deterioration rate is time proportional.
- C_2 is the shortage cost per unit per unit of time.
- Q_0 is the maximum inventory level during [0,T].
- S is the lost sale cost per unit.
- IS is the maximum inventory level during the shortage period.

The basic notations are as given by

- A is the fixed ordering cost per order.
- $\delta(t) = \delta t$ where δ *is the rate of deterioration.*
- $Q(t)$ is the level of inventory at time t ($0\leq t \geq T$).
- M is the unit cost of an item.
- Holding cost per unit time is time dependent and is assumed to be h(t) =h+αt where a > 0; h > 0.
- *The order* quantity of one cycle is q.
- γ is the backlogging rate; $0\leq \gamma \geq 1$.
- *At* time t_1, the inventory become zero and shortages starts occurs.

MATHEMATICAL MODEL

The rate of change of inventory during the stock period $[0, t_1]$ and shortage period $[t_1, t]$ is given by the differential equations:

$$\frac{dQ(t)}{dt} = -(a+bt+ct^2) - \delta(t).Q(t) \qquad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} = -\gamma(a+bt+ct^2) \qquad t_1 \leq t \leq T \tag{2}$$

The initial inventory level is Q_0 unit at time t=0; from t=0 to t=t_1,the inventory start reduces, owing to demand and deterioration, it reaches zero level at time t=t_1. at this time shortages started and is partially backlogged at the rate γ. At the end of the cycle, the inventory reaches a maximum shortage level as to clear the backlogged and again raises the inventory level to Q_0.

Thus boundary conditions are as follows:

$$Q_1(0) = Q_{0,} \qquad Q_1(t_1) = 0, \qquad Q_2(t_1) = 0.$$

The solutions of equations 1 and 2 with boundary conditions are as follows:

$$Q_1(t) = -e^{\delta\frac{t^2}{2}} \int_t^{t_1} (a+bt+ct^2).e^{\delta\frac{t^2}{2}}\,dt, \qquad 0 \leq t \leq t_1 \tag{3}$$

$$Q_2(t) = -\gamma[a(T-t_1) + \frac{b}{2}(T^2 - {t_1}^2) + \frac{c}{3}(T^3 - {t_1}^3)] \tag{4}$$

Using equation (3), we get the following

$$Q_0 = \int_0^{t_1} (a+bt+ct^2).e^{\delta\frac{t^2}{2}}\,dt \tag{5}$$

Inventory is available during the time interval (0,t_1). hence the holding cost for inventory is computed only for time [0, t_1].

Holding cost:

$$\begin{aligned}
\text{CH} &= \int_0^{t_1} h(t)\, Q_1(t) \\
&= \int_0^{t_1} (h+\alpha t)\, Q_1(t) \\
&= \int_0^{t_1} (h+\alpha t)\, e^{-\delta\frac{t^2}{2}} \int_t^{t_1} (a+bu+cu^2)\, e^{\delta\frac{u^2}{2}} du dt \\
&= (\text{ha}\,(\frac{{t_1}^2}{2} + \delta\frac{{t_1}^4}{12} + \delta^2\frac{{t_1}^6}{90}) + \text{hb}(\frac{{t_1}^3}{3} + \delta\frac{{t_1}^5}{15} + \delta^2\frac{{t_1}^7}{105}) + \text{hc}(\frac{{t_1}^4}{4} + \delta\frac{{t_1}^6}{9} + \delta^2\frac{{t_1}^8}{120}) + \alpha a(\frac{{t_1}^3}{6} + \delta\frac{{t_1}^5}{40} + \delta^2\frac{{t_1}^7}{336}) \\
&\quad + \alpha b(\frac{{t_1}^4}{8} + \delta\frac{{t_1}^6}{48} + \delta^2\frac{{t_1}^8}{384}) + \alpha c(\frac{{t_1}^5}{10} + \delta\frac{{t_1}^7}{84} + \delta^2\frac{{t_1}^7}{432}))
\end{aligned} \tag{6}$$

Shortages occur in the inventory in the time interval $[t_1, T]$.

The optimum level of shortage is present at t=T; therefore shortage cost is as follows

$$CS = C_2 \int_{t_1}^{T} -Q_2(t)\, dt$$

$$= [\gamma a C_2 (T-t_1)^2 + \frac{\gamma}{2} b C_2 (T-t_1)^2 (T+t_1) + \frac{\gamma c C_2}{3}(T-t_1)^2(T^2+{t_1}^2+Tt_1)] \qquad (7)$$

Lost sale cost

$$CLS = S\int_{t_1}^{T}(1-\gamma)(a+bt+ct^2)dt$$

$$= S\,(1-\gamma)[a(T-t_1)+\frac{b}{2}(T^2-{t_1}^2)+\frac{c}{3}(T^3-{t_1}^3)] \qquad (8)$$

$$CP = M\,(Q_0 + \int_{t_1}^{T}\gamma(a+bt+ct^2)dt$$

$$= MQ_0 + M\gamma a(T-t_1)+\frac{M\gamma b}{2}(T^2-{t_1}^2)+\frac{M\gamma c}{3}(T^3-{t_1}^3) \qquad (9)$$

Total cost is as follows:

TC = OC+CP+CH+CS+CLS

$$= [A + MQ_0 + M\gamma a(T-t_1)+\frac{M\gamma b}{2}(T^2-{t_1}^2)+\frac{M\gamma c}{3}(T^3-{t_1}^3) + (\,ha(\frac{{t_1}^2}{2}+\delta\frac{{t_1}^4}{12}+\delta^2\frac{{t_1}^6}{90}) + hb(\frac{{t_1}^3}{3}+\delta\frac{{t_1}^5}{15}$$
$$+\delta^2\frac{{t_1}^7}{105}) + hc(\frac{{t_1}^4}{4}+\delta\frac{{t_1}^6}{9}+\delta^2\frac{{t_1}^8}{120}) + \alpha a(\frac{{t_1}^3}{6}+\delta\frac{{t_1}^5}{40}+\delta^2\frac{{t_1}^7}{336}) + \alpha b(\frac{{t_1}^4}{8}+\delta\frac{{t_1}^6}{48}+\delta^2\frac{{t_1}^8}{384})$$
$$+\alpha c(\frac{{t_1}^5}{10}+\delta\frac{{t_1}^7}{84}+\delta^2\frac{{t_1}^7}{432})) + [\gamma a C_2(T-t_1)^2 + \frac{\gamma}{2}bC_2(T-t_1)^2(T+t_1)+\frac{\gamma c C_2}{3}(T-t_1)^2(T^2+{t_1}^2+Tt_1)]$$
$$+ S\,(1-\gamma)[a(T-t_1)+\frac{b}{2}(T^2-{t_1}^2)+\frac{c}{3}(T^3-{t_1}^3)]] \qquad (10)$$

Differentiating equations TC with respect to $t_1 and\ T$, we then get the following

$$\frac{\partial TC}{\partial t_1}\ and\ \frac{\partial TC}{\partial T}$$

To minimize the total cost we optimal value of T and t_1 can be obtained by solving the following differentiation equation:

$$\frac{\partial TC}{\partial t_1} = 0\ and\ \frac{\partial TC}{\partial T} = 0, \qquad (11)$$

Provided

$$\frac{\partial^2 TC}{\partial {t_1}^2}.\frac{\partial^2 TC}{\partial T^2} - (\frac{\partial^2 TC}{\partial T \partial t_1})^2 > 0\ and\ \frac{\partial^2 TC}{\partial {t_1}^2} > 0 \qquad (12)$$

By solving equation (12) the value of T and t_1 can be obtained and with the help of equation (11) we get the minimum inventory cost per unit of time.

SENSITIVITY ANALYSIS

Consider an inventory system with the following parameter in proper unit: A=2500, h=.01, M=.01, s=2.55, n=2, $\alpha = 10$, $\beta = 0.7$, a=7, b=28,c=28, theta=.65. The computer output by using maple mathematical software is t1=0.1578730591, T= 0.01098158558 and TC=2449.600. The variation in the parameter is as follows:

VARIATION IN PARAMETER α

α	T	t_1	TC
8	0.026809	0.17428	2499.58
9	0.0183	0.16544	2499.59
10	0.01099	0.1579	2499.59
11	0.00465	0.1513	2499.60

VARIATION IN PARAMETER β

β	T	t_1	TC
0.5	0.0010	0.2721	2499.068
0.6	0.0020	0.2143	2499.159
0.7	0.011	0.1579	2499.599
0.8	0.011	0.10111	2499.849

VARIATION IN PARAMETER M

M	T	t_1	TC
0.01	0.011	0.1579	2499.599
0.02	0.0056	0.16870	2499.604
0.03	0.003	0.16229	2499.609
0.04	0.001	0.156320	2499.609

VARIATION IN PARAMETER θ

θ	T	t_1	TC
0.55	0.01103065	0.1579	2499.5998
0.65	0.010981586	0.1579	2499.5999
0.75	0.010933	0.1578	2499.5999
0.85	0.0108837	0.1578	2499.6000

CONCLUDING REMARKS

In this paper, we developed a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. The deterioration factor taken into consideration in the present model, as almost all items undergo either direct spoilage (like fruits, vegetable etc) or physical decay (in case of radioactive substance etc.) in the course time, deterioration is natural feature in the inventory system. The model is very useful in the situation in which the demand rate is depending upon the time.

REFERENCES

1. Acharya, Milu., Debata Smrutirekha, 2014, “A inventory model for deteriorating with time dependent demand under partial backlogging”, International Journal Of research In Advent Technology, vol. 2, issue 1.
2. DR. Ravish kumar yadav, Amit kumar vasts (2014), A deteriorating inventory model for quadratic demand and constant holding cost with partial backlogging inflation, *IOSR, journal of mathematics pp 47-52.*
3. Giri B.C. and Chadhuri K.S. (1997), Heuristic models for deteriorating items with shortages and time-varying demand and costs, *International Journal of Systems Science, 28, 53-159.*
4. Goh M. (1994), EOQ models with general demand and holding cost functions, *European Journal of Operational Research, 73, 50-54.*
5. Hariga M.A. and Benkherouf L. (1994), Optimal and heuristic inventory replenishment models for deteriorating items with exponential time-varying demand, *European Journal of Operational Research, 79, 123-137.*
6. Kumar, N. and Sharma, A.K. 2001 “Optimum ordering interval with known demand for items with variable rate of deterioration and shortages, A.S.P vol. (111) 2, 95-100.
7. Manna, S.K. and Chaudhuri, K.S., 2006, "An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages," European Journal of Operational Research, 171, 557-566.
8. Nose, T., Ishii, H., Nishida, T., 1984, “Perishable inventory control with stochastic lead time and different selling prices,” European Journal of Operations Research, 18, 322-338.
9. Philip, G.C., 1974, “A generalized EOQ model for items with Weibull distribution deterioration,” American Institute of Industrial Engineers Transactions, 6, 159-162.
10. Roy, Ajanta, An inventory model for deteriorating items with price dependent demand and time varying holding cost. Advanced Modeling and Optimization, 10(2008), 25-37.
11. Ouyang,L.Y., Chang,C.T. and Teng, J.T, 2005 “An EOQ for deteriorating items under trade credits”, journal of the operational research society, 56, 719-726.
12. Wee, H. M., A deterministic lot-size inventory model for deterioration items with shortages and a declining market, Computers and Industrial Engineering, 22(1995), 345-356.

13. Yadav, R.K and Yadav, P. (2013), "volume flexibility in production model with cubic demand rate and weibull deterioration with partial backlogging", IOSR journal of mathematics 6(4), 29-39.

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