AN INVENTORY MODEL FOR DETERIORATING GOODS
WITH TIME DEPENDENT QUADRATIC DEMAND AND TIME VARYING HOLDING COST
WITH PARTIAL BACKLOGGING

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ABSTRACT

A deterministic inventory model is developed with time dependent demand and time varying holding cost and deterioration is considered to be time proportional. Shortages are allowed and demand is partially backlogged. The objective of the model is to develop an optimal policy that minimizes the total average cost. Numerical examples are used to illustrate the developed models. Sensitivity analysis of the optimal solution with respect to major parameters is carried out.

Key words: Deterioration, quadratic demand, time varying holding cost.

INTRODUCTION

Operation research’s role in both, the public and the private sectors is increasing rapidly. Inventory modeling is an important part of Operation Research, which may be used in large numbers of problems. To make it applicable in real life situations researchers are busy in modifying the existing models on different parameters under various circumstances. First inventory model was given by Harris (1915). The model was based on constant demand without any deterioration function. However, in real life situation the demand may increase or decrease in the course of time. Harris work was generalized by Wilson (1934) who gave a formula to find economic order quantity. Ghare and Schrader (1963) developed a inventory model for an exponentially decaying demand. The basic theorem for time varying demand is given by Donald – son [1977] who established the classical no shortage inventory model with a linear trend in demand over a known and finite horizon. But this method is very difficult in computing. The difficulty of Donald son’s approach has led to the development of heuristic methods. Silver [1978] derived simple heuristic procedures for Donald son’s problems.


In this paper we develop a deterministic inventory model is developed with time dependent demand and time varying holding cost and deterioration is considered to be time proportional. Shortages are allowed and demand is partially backlogged. The objective of the model is to develop an optimal policy that minimizes the total average cost.

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ASSUMPTIONS AND NOTATIONS

The basic assumptions are as given by

- The demand rate is time dependent and demand is quadratic.
- Shortages are allowed and partially backlogged.
- Lead time is zero.
- The replenishment rate is instantaneous.
- The deterioration rate is time proportional.
- $C_2$ is the shortage cost per unit per unit of time.
- $Q_0$ is the maximum inventory level during $[0,T]$.
- $S$ is the lost sale cost per unit.
- $IS$ is the maximum inventory level during the shortage period.

The basic notations are as given by

- $A$ is the fixed ordering cost per order.
- $M$ is the unit cost of an item.
- Holding cost per unit time is time dependent and is assumed to be $h(t)=h_0+t$ where $a>0$; $h>0$.
- The order quantity of one cycle is $q$.
- $\gamma$ is the backlogging rate; $0 \leq \gamma \leq 1$.
- At time $t_1$, the inventory become zero and shortages starts occurs.

MATHEMATICAL MODEL

The rate of change of inventory during the stock period $[0,t_1]$ and shortage period $[t_1,t]$ is given by the differential equations:

\[
\begin{align*}
\frac{dQ(t)}{dt} &= -(a + bt + ct^2) - \delta(t)Q(t) & 0 \leq t \leq t_1 \\
\frac{dQ(t)}{dt} &= -\gamma(a + bt + ct^2) & t_1 \leq t \leq T
\end{align*}
\]  

The solutions of equations 1 and 2 with boundary conditions are as follows:

\[
\begin{align*}
Q_1(t) &= -e^{\delta^2 t} \int_0^{t_1} (a + bt + ct^2) \cdot e^{\delta^2 t} \, dt, & 0 \leq t \leq t_1 \\
Q_2(t) &= -\gamma \left[ a(T - t_1) + \frac{b}{2} (T^2 - t_1^2) + \frac{c}{2} (T^3 - t_1^3) \right] & t_1 \leq t \leq T
\end{align*}
\]  

Using equation (3), we get the following

\[Q_0 = \int_0^{t_1} (a + bt + ct^2) \cdot e^{\delta^2 t} \, dt\]  

Inventory is available during the time interval $(0,t_1)$, hence the holding cost for inventory is computed only for time $[0,t_1]$.

Holding cost:

\[
CH = \int_0^{t_1} h(t) \, Q_1(t) \, dt = \int_0^{t_1} h(t) \, e^{-\delta^2 t} \int_0^{t_1} (a + bt + ct^2) \cdot e^{\delta^2 t} \, dt \, dt
\]  

\[
= \int_0^{t_1} \left[ (a + t^2_1 h_0 + \frac{b}{2} t^2_1 + \frac{c}{3} t^3_1 + \frac{d}{4} t^4_1 + \frac{e}{5} t^5_1 + \frac{f}{6} t^6_1 + \frac{g}{7} t^7_1) \right] \, dt
\]  

\[
+ \int_0^{t_1} \left[ (a + t^2_1 h_0 + \frac{b}{2} t^2_1 + \frac{c}{3} t^3_1 + \frac{d}{4} t^4_1 + \frac{e}{5} t^5_1 + \frac{f}{6} t^6_1 + \frac{g}{7} t^7_1) \right] \, dt
\]  

\[
= \left[ \frac{a t^2}{2} + \frac{b t^3}{3} + \frac{c t^4}{4} + \frac{d t^5}{5} + \frac{e t^6}{6} + \frac{f t^7}{7} \right]_0^{t_1} + \alpha a(\frac{t_1^3}{3} + \frac{t_1^4}{4} + \frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7})
\]  

\[
+ \beta b(\frac{t_1^4}{4} + \frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7}) + \gamma c(\frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7})
\]

\[
= \left[ \frac{a t^2}{2} + \frac{b t^3}{3} + \frac{c t^4}{4} + \frac{d t^5}{5} + \frac{e t^6}{6} + \frac{f t^7}{7} \right]_0^{t_1} + \alpha a(\frac{t_1^3}{3} + \frac{t_1^4}{4} + \frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7})
\]  

\[
+ \beta b(\frac{t_1^4}{4} + \frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7}) + \gamma c(\frac{t_1^5}{5} + \frac{t_1^6}{6} + \frac{t_1^7}{7})
\]  

\[
\]
Shortages occur in the inventory in the time interval $[t_1, T]$.

The optimum level of shortage is present at $t=T$; therefore shortage cost is as follows

$$CS = C_2 \int_{t_1}^{T} Q(t) \, dt$$

$$= [\gamma a C_2 (T - t_1)^2 + \frac{\xi}{2} b C_2 (T - t_1)^2 (T + t_1) + \frac{\rho C_2}{3} (T - t_1)^2 (T^2 + t_1^2 + T t_1)]$$

(7)

Lost sale cost

$$CLS = S \int_{t_1}^{T} (1 - \gamma) (a + bt + ct^2) \, dt$$

$$= S (1 - \gamma) [a(T - t_1) + \frac{b}{2} (T^2 - t_1^2) + \frac{\xi}{3} (T^3 - t_1^3)]$$

(8)

CP = M (Q_0 + \int_{t_1}^{T} y(a + bt + ct^2) \, dt)

$$= MQ_0 + Mya(T - t_1) + \frac{My^2}{2} (T^2 - t_1^2) + \frac{My^3}{3} (T^3 - t_1^3)$$

(9)

Total cost is as follows:

$$TC = OC + CP + CH + CS + CLS$$

Differentiating equations $TC$ with respect to $t_1$ and $T$, we then get the following

$$\frac{\partial TC}{\partial t_1} = 0 \text{ and } \frac{\partial TC}{\partial T} = 0,$$

(11)

Provided

$$\frac{\partial^2 TC}{\partial t_1^2} - (\frac{\partial^2 TC}{\partial T \partial t_1})^2 > 0 \text{ and } \frac{\partial^2 TC}{\partial t_1^2} > 0,$$

(12)

By solving equation (12) the value of $T$ and $t_1$ can be obtained and with the help of equation (11) we get the minimum inventory cost per unit of time.

**SENSITIVITY ANALYSIS**

Consider an inventory system with the following parameter in proper unit: $A=2500$, $h=.01$, $M=.01$, $s=2.55$, $n=2$, $a=10$, $b=.7$, $a=7$, $b=28$, $c=28$, $\theta=.65$. The computer output by using maple mathematical software is $t_1=0.1578730591$, $T=.01098158558$ and $TC=2449.600$. The variation in the parameter is as follows:

**VARIATION IN PARAMETER $\alpha$**

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<th>$\alpha$</th>
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<tr>
<td>8</td>
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VARIATION IN PARAMETER $M$

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VARIATION IN PARAMETER $\theta$

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CONCLUDING REMARKS

In this paper, we developed a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. The deterioration factor taken into consideration in the present model, as almost all items undergo either direct spoilage (like fruits, vegetable etc) or physical decay (in case of radioactive substance etc.) in the course time, deterioration is natural feature in the inventory system. The model is very useful in the situation in which the demand rate is depending upon the time.

REFERENCES


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