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# A STUDY ON THE DOMINATING TRANSVERSAL CHROMATIC NUMBER

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***ABSTRACT***

*Domination is a fast developing area in Graph Theory. The concept of domination leads to very interesting research work with good applications.*

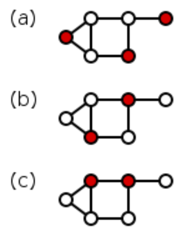
*In this paper three areas of Graph Theory namely Domination, Coloring and Transversal theory are combined judiciously to define a new domination parameter. A result on std-set is given which can be used to verify whether the given std-set is minimal or not. Also, $\gamma_{st}$ i.e., the dominating transversal chromatic number for different graphs such as Petersen graph, Wheel, Herschel graph and Grotszch graph are given.*

***Key words:*** *Dominating colour transversal set, $\chi$-partition, Herschel graph, Grotszch graph, Petersen Graph and Wheel.*

## BASIC DEFINITIONS AND RESULTS

**Definition 1.1:** Let G = (V, E) be a graph. A non-empty set D ⊆ V is called a dominating set of G if every vertex in V−D is adjacent to some vertex of D. Let $\hat{C}$ be the collection of all dominating sets of G. The member of $\hat{C}$ whose cardinality is minimum is called a -set and its cardinality is defined to be the domination number of G and it is denoted by $\gamma$ .

**Example:** In the following graph, The Red Vertices denote the Dominating sets.




**Definition 1.2:** A partition of the vertex set V into a minimum number of disjoint equivalence classes of independent sets, called colour classes, is said to be a $\chi$ -partition of G and this minimum number is denoted by $\chi$.

**Definition 1.3:** A dominating set is called a dominating colour transversal set (std-set) if it has a non-empty intersection with every colour class of some $\chi$ –partition of G. This set is called an std-set because it is transversal of at least one (single) $\chi$ -partition of G.

**Definition 1.4:** An std-set with minimum cardinality is called a $\gamma_{st}$ -set and its cardinality is denoted by $\gamma_{st}$.

**Definition 1.5:** Let $\prod$ be a $\chi$ -partition of a graph G. A set D ⊆ V is said to be a transversal of $\prod$ if D intersects every colour class of $\prod$.

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**Definition 1.6:** *A dominating set* D ⊆ V *is called a* dominating colour transversal set *(std-set) of a graph* G *if* D *is a transversal of at least one* $\chi$ *-partition of* G. We call this set an std-set because it is a dominating colour transversal of at least one (single) $\chi$ -partition. *An std-set is minimal if none of its proper subsets is an std-set.*

**Definition 1.7:** *Let* $\hat{C}$ ={D ⊆ V | D *is an std-set of* G}. $\hat{C}$ *is non empty as* $v \in \hat{C}$. Let $D^1 \in \hat{C}$ *whose cardinality is minimum.* $D^1$ is called a $\gamma_{st}$ *-set and its cardinality is called the* dominating colour transversal number *denoted by* $\gamma_{st}$.

**Result1.8:** *For any graph* G, $1 \leq \gamma \leq \gamma_{st}$

**Result1.9:** For any Bipartite graph G, $\gamma_{st}$ is either $\gamma$ or $\gamma_{+1}$ since by adding if necessary a single vertex to a $\gamma_{-set}$, we get a $\gamma_{st}$-set for G.

**Theorem1.10:** For any std-set D is minimal⟺for every u∈D, any of the following holds:

(a) u is an isolate of D
(b) There exist a vertex v∈V-D∋N(v)∩D={u}.
(c) For every $\chi$ *–partition*, ∏={$V_1$,$V_2$,$V_3$,…,$V_k$}, There exist one $V_i$ ∋ $V_i$∩D={u}or ∅.

**Proof:** Let D be an std-set.

Suppose D is minimal.

Then D-{u} is not an std-set for every u∈D.

⇒ either D-{u} is not a dominating set or not a transversal of every $\chi$ *–partition* of G.

**Case-1:** Suppose D-{u} is not a dominating set.

Then there exists a vertex v∈ $(V-D) \cup \{u\}$ that is not adjacent to any vertex of D-{u}.

If u = v, then u is an isolate of D.

If u ≠v, then v is adjacent to u but not to any other vertex of D.

Hence N (v) ∩D= {u}.

**Case-2:** Suppose D-{u} is not a traversel for every $\chi$ *–partition* of {$V_1$, $V_2$, $V_3$,…,$V_k$}.

$\Rightarrow D-\{u\} \cap V_i = \emptyset,$ for some i.

i.e., $V_i \cap D = \{u\} or\ \emptyset$ for some i

Hence (c) is satisfied.

Conversely, assume any one of the three conditions.

We prove that D is a minimal std-set.

Suppose D is not a minimal std-set.

Then D is an std-set but not minimal.

⇒ D and D-{u} are std-sets for some u∈D.

Let {$V_1$, $V_2$, $V_3$,…,$V_k$}be a $\chi$ *–partition* of V for which D-{u} and D are transversals.

Then D-{u}∩ $V_i \neq \emptyset$ and D ∩ $V_i \neq \emptyset$ for every i.

$\Rightarrow$ D ∩ $V_i \neq$ {u}or ∅, contradicting condition (c)

**Result 1.11:** For any graph G, $\gamma \leq \gamma_g \leq \gamma_{st.}$

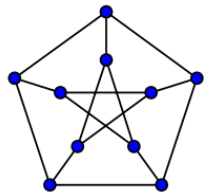
**Result1.12:** If G has K-components $G_1$, $G_{2,\ \ldots,}$ $G_k$ such that $\chi(G_1) \geq \chi(G_i)$, for i=1,2,….,k, then
$\gamma_{st(G)} \leq \gamma_{st}(G_1) + \sum_{i=2}^{k} \gamma(G_i)$.

**THE DOMINATING COLOUR TRANSVERSAL NUMBER FOR SOMESTANDARD GRAPHS**

**1. PETERSEN GRAPH**

In the mathematical field of graph theory, the **Petersen graph** is an undirected graph with 10 vertices and 15 edges. It is a small graph that serves as a useful example and counterexample for many problems in graph theory. The Petersen graph is named for Julius Petersen, who in 1898 constructed it to be the smallest bridgeless cubic graph with no three-edge-coloring.

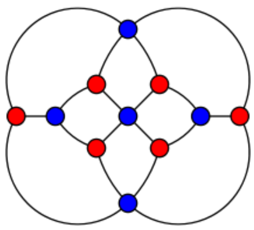
*For the Petersen graph* P, $\gamma_{st} = 4$.



**2. HERSCHEL GRAPH**

In graph theory, a branch of mathematics, the **Herschel graph** is a bipartite undirected graph with 11 vertices and 18 edges, the smallest non-Hamiltonian polyhedral graph. It is named after British astronomer Alexander Stewart Herschel, who wrote an early paper concerning William Rowan Hamilton's icosian game: the Herschel graph describes the smallest convex polyhedron for which this game has no solution. However, Herschel's paper described solutions for the icosian game only on the graphs of the regular tetrahedron and regular icosahedron; it did not describe the Herschel graph.

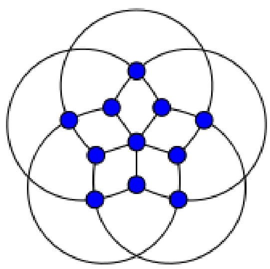
The Herschel graph H as shown below is bipartite since it contains no odd cycles. Hence $\chi(H) = 2$ and $\gamma_{st}=3$.



**3. GROTZSCH GRAPH**

In the mathematical field of graph theory, the **Grötzsch graph** is a triangle-free graph with 11 vertices, 20 edges. It is named after German mathematician Herbert Grötzsch.

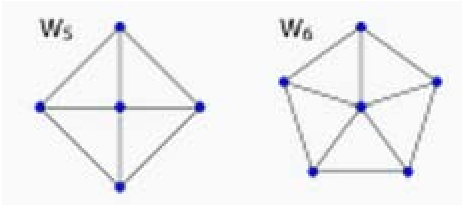
G4 is a triangle free 4-chromatic graph we can easily verify that $\gamma_{st} = 4$.



**4. WHEEL GRAPH**

In the mathematical discipline of graph theory, a **wheel graph** $W_n$ is a graph with $n$ vertices ($n \geq 4$), formed by connecting a single vertex to all vertices of an ($n$-1)-cycle. The numerical notation for wheels is used inconsistently in the literature: some authors instead use $n$ to refer to the length of the cycle, so that their $W_n$ is the graph we denote $W_{n+1}$.[1] A wheel graph can also be defined as the 1-skeleton of an ($n$-1)-gonal pyramid.

$$\gamma_{st}(W_p) = \begin{cases} 3, & \text{if p is odd} \\ 4, & \text{if p is even} \end{cases}$$




## APPLICATIONS

1. Dominating sets are useful in routing problems. How many internet routers do you need so that every computer in your business has internet access?
2. It is also useful in scheduling problems. Suppose that some people are attending a conference. Each person writes down a list of events they want to attend. What is the smallest number of time slots needed so that everyone can attend all sessions that they want to attend (assuming there are enough rooms)? Make a graph whose vertices are the different events, with an edge between two vertices $x$ and $y$ if there is at least one person who wants to attend events $x$ and $y$. The smallest number of events is the chromatic number of the graph.
3. Suppose you have hazardous chemicals that cannot be stored together. To find the minimum number of storage units you need to store all your chemicals?
4. suppose you have a set of volunteers and certain volunteers who won't work with each other. How do you group them to minimize the number of groups

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