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GENERAL PRODUCTION AND SALES BY MARKOVIAN MANPOWER AND MACHINE SYSTEM

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ABSTRACT

A production and sale system is considered. During the operation time a machine produces random number of products. After operation time, sale time starts, and it has one among two distinct distributions depending on the magnitude of production time is within or exceeding a random threshold magnitude. Two models are treated. In model 1, the machine operation time has exponential distribution; repair, recruitment the production and sale times have general distribution. In model 2, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. Joint transforms of the variables, their means and Co-variances with numerical results are presented.

***Mathematics Subject Classification**: 91B70.*

***Keywords**: Storage system, Production and Sale, Repair and Recruitment, Joint transform.*

1. INTRODUCTION

In manufacturing models to get the return on investment and to pay minimum interest, it is natural that when the production time is more, the sale time is made short so as to cut cost. It has been noticed that when the units produced are more, financial supports for the customers are provided to clear products early. These are widely felt in perishable commodity sectors where many banking institutions provide required finance for the purchase.

Storage systems of (s, S) type was studied by Arrow, Karlin and Scrat [1]. Such systems with random lead times and unit demand were treated by Danial and Ramanarayanan [2]. Models with bulk demands were analyzed by Ramanarayanan and Jacob [9]. Murthy and Ramanarayanan [5, 6, 7, 8] considered several (s, S) inventory systems. Kun-Shan Wu, Ouyang and Liang-Yuh [3] studied (Q, r, L) inventory model with defective items. Usha, Nithyapriya and Ramanarayanan [10] considered storage systems with random sales time depending on production. General Manpower and Machine system with Markovian production were analyzed by Harikumar [4]. In this paper, two models are treated. In model 1, the machine operation time is exponential, repair, recruitment, production and sale times have general distributions. Sales are done one by one. In model 2, when the operation time more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one.

The joint transforms, the means and covariance of production time and sale time with the numerical examples are presented.

2.1 MODEL-1

1. The inter production times of products by a manpower and machine system are independent and identically distributed (i.i.d) random variables with Cdf F(x) and pdf f(x).
2. The sales time of products are independent with Cdf H(z) and pdf h(z). The sale time starts when production is stopped.
3. The machine producing products has operation time which is exponential with parameter λ.
4. Inter-departure time of employees attending the machine is exponential with parameter μ. The manpower system collapses with probability p when an employee leaves and with probability q it survives.

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5. The production stops when both machine and manpower are in failed state. When either manpower or machine is alone in failed state, the failed one is hired till both became unavailable.
6. The repair time distribution of the machine is general with Cdf R(y) and pdf r(y)and recruitment time of each employee is general with Cdf $R_1(y)$ and pdf $r_1(y)$. All recruitments and repair are done one by one.

ANALYSIS

To study the above model 1, the joint probability density function of three variables $(X, \hat{R}, \hat{S})$ namely the operation time of manpower and machine system, sum of recruitment and repair times, sales time of products is written as follows.

$$f(x,y,z) = \sum_{i=1}^{\infty}(\lambda e^{-\lambda x} pq^{i-1}\int_0^x e^{-\mu u}\frac{(\mu u)^{i-1}}{\underline{|i-1}}\mu du + (1-e^{-\lambda x})e^{-\mu x}\frac{(\mu x)^{i-1}}{\underline{|i-1}}q^{i-1}p\mu)r(y)©r_{1i}(y) \\ [\sum_{k=0}^{\infty}(F_k(x)-F_{k+1}(x))h_k(z)] \tag{1}$$

Here © indicates convolution of functions and suffix k indicates so many folds Stieltjes convolution of Cdf or convolutions of pdf as the case may be. The first term of equation (1) inside the bracket is the part of the pdf that the machine fails when the manpower system is in collapsed state and the second term is the part of the pdf that the manpower collapses when the machine is already in failed state. The repair and the recruitment are considered in the convolution ©. The square bracket presents that the products produced during the operation time are sold one by one.

We now find the triple Laplace transform as follows for the joint pdf

$$f^*(\xi,\eta,\varepsilon) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x-\eta y-\varepsilon z} f(x,y,z)dxdydz \tag{2}$$

This gives on simplification

$$f^*(\xi,\eta,\varepsilon) \\ = p\mu r^*(\eta)r_1^*(\eta)\left\{\begin{array}{l} \dfrac{\lambda}{\mu(1-qr_1^*(\eta))}\left[\begin{array}{l}\dfrac{1}{(\xi+\lambda)}\dfrac{(1-f^*(\xi+\lambda))}{(1-h^*(\varepsilon)f^*(\xi+\lambda))} \\ -\dfrac{1}{[\xi+\lambda+\mu(1-qr_1^*(\eta))]}\dfrac{(1-f^*(\xi+\lambda+\mu(1-qr_1^*(\eta))))}{(1-h^*(\varepsilon)f^*(\xi+\lambda+\mu(1-qr_1^*(\eta))))}\end{array}\right] \\ +\dfrac{1}{[\xi+\mu(1-qr_1^*(\eta))]}\dfrac{(1-f^*(\xi+\mu(1-qr_1^*(\eta))))}{(1-h^*(\varepsilon)f^*(\xi+\mu(1-qr_1^*(\eta))))} \\ -\dfrac{1}{[\xi+\lambda+\mu(1-qr_1^*(\eta))]}\dfrac{(1-f^*(\xi+\lambda+\mu(1-qr_1^*(\eta))))}{(1-h^*(\varepsilon)f^*(\xi+\lambda+\mu(1-qr_1^*(\eta))))} \end{array}\right\} \tag{3}$$

Using differentiation (3) we can find

$$E(X) = -\frac{\partial}{\partial\xi}f^*(\xi,\eta,\varepsilon)\,|\,\xi=\eta=0 = \frac{1}{\lambda}+\frac{1}{\mu p}-\frac{1}{\lambda+\mu p} \tag{4}$$

After simplification it may be obtained that

$$E(\hat{R}) = -\frac{\partial}{\partial\eta}f^*(\xi,\eta,\varepsilon)\,a\;\; \xi,\eta,\varepsilon=0 = E(R)+\frac{E(R_1)}{p} \tag{5}$$

Similarly, we get

$$E(\hat{S}) = -\frac{\partial}{\partial\varepsilon}f^*(\xi,\eta,\varepsilon)\,at\,\xi=\eta=\varepsilon=0$$

and

$$E(\hat{S}) = E(H)\left[\frac{f^*(\lambda)}{1-f^*(\lambda)} + \frac{f^*(\mu p)}{1-f^*(\mu p)} - \frac{f^*(\lambda+\mu p)}{1-f^*(\lambda+\mu p)}\right] \tag{6}$$

The Laplace transforms of the pdf of X and Ŝ is given by

$$\begin{aligned} f^*(\xi,0,\varepsilon) = {} & \frac{\lambda}{(\xi+\lambda)}\frac{(1-f^*(\xi+\lambda))}{(1-h^*(\varepsilon)f^*(\xi+\lambda))} - \frac{\lambda+\mu p}{(\xi+\lambda+\mu p)}\frac{(1-f^*(\xi+\lambda+\mu p))}{(1-h^*(\varepsilon)f^*(\xi+\lambda+p\mu))} \\ & + \frac{p\mu}{(\xi+p\mu)}\frac{(1-f^*(\xi+\mu p))}{(1-h^*(\varepsilon)f^*(\xi+\mu p))} \end{aligned} \tag{7}$$

The product moment of X and Ŝ is

$$E(X\hat{S}) = \frac{\partial^2}{\partial\xi\partial\eta} f^*(\xi,0,\varepsilon) \mid \xi = 0 = \varepsilon$$

and is given by after simplification

$$E(X\hat{S}) = E(H)\left[\begin{aligned} & \frac{1}{\lambda}\frac{f^*(\lambda)}{(1-f^*(\lambda))} - \frac{f^{*'}(\lambda)}{(1-f^*(\lambda))^2} - \frac{1}{\lambda+p\mu}\frac{f^*(\lambda+\mu p)}{1-f^*(\lambda+\mu p)} \\ & + \frac{f^{*'}(\lambda+\mu p)}{(1-f^*(\lambda+\mu p))^2} + \frac{1}{p\mu}\frac{f^*(\mu p)}{(1-f^*(\mu p))} - \frac{f^{*'}(\mu p)}{(1-f^*(\mu p))^2} \end{aligned}\right] \tag{8}$$

Since, $Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S}),$ equation (8), (4) and (6) may be used for writing $Cov(X,\hat{S})$

2.2 MODEL-2

In this model we treat the previous model with all assumptions (1), (3), (4), (5) and (6) except (2) given for sales.

Assumptions for sales

2.1 When the operation time X is more than a threshold time U, the sales are done all together. It is assigned to an agent whose sales time distribution function is G_1 (z) with pdf g_1 (z).
2.2 When the operation time is less than the threshold time U, the sales are done one by one with cdf G_2 (z) with pdf g_2(z).
2.3 The threshold U has exponential distribution with parameter α.

ANALYSIS

Using the arguments given for model 1, we note that the joint pdf of $(X,\hat{R},\hat{S})$ (operation time, repair-recruitment time, sales time) as follows.

$$\begin{aligned} f(x,y,z) = {} & \sum_{i=1}^{\infty}\left(\lambda e^{-\lambda x} pq^{i-1}\int_0^x e^{-\mu u}\frac{(\mu u)^{i-1}}{\underline{|i-1}}\mu du + (1-e^{-\lambda x})e^{-\mu x}\frac{(\mu x)^{i-1}}{\underline{|i-1}}q^{i-1}p\mu\right)r(y)©r_{1i}(y) \\ & \left[\sum_{k=0}^{\infty}(F_k(x)-F_{k+1}(x))((1-e^{-\alpha x})g_1(z)+e^{-\alpha x}g_{2,k}(z))\right] \end{aligned} \tag{9}$$

We use the same arguments given for model 1 for all terms except the last square bracket where the sales time pdf is g_1 (z) when (X>U) the operation time is greater than the threshold and when the operation time is less than the threshold (X<U) the k products are sold one by one with sale time pdf g_2 (z) where suffix ‘k’ indicates the k-fold convolution.

Using the previous arguments the triple Laplace transform can be seen as

$$f^*(\xi,\eta,\varepsilon)=p\mu r^*(\eta)r_1^*(\eta)g_1^*(\varepsilon)\left\{\begin{array}{l}\frac{\lambda}{\mu(1-qr_1^*(\eta))}\left[\frac{1}{(\xi+\lambda)}-\frac{1}{\xi+\lambda+\mu(1-r_1^*(\eta)q)}\right]\\+\frac{1}{\xi+\mu(1-qr_1^*(\eta))}-\frac{1}{\xi+\lambda+\mu(1-qr_1^*(\eta))}\\-\frac{\lambda}{\mu(1-qr_1^*(\eta))}\left[\frac{1}{\xi+\alpha+\lambda}-\frac{1}{\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))}\right]\\-\frac{1}{\xi+\alpha+\mu(1-qr_1^*(\eta))}+\frac{1}{\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))}\end{array}\right\}$$

$$+p\mu r^*(\eta)r_1^*(\eta)\left\{\begin{array}{l}\frac{\lambda}{\mu(1-r_1^*(\eta)q)}\left[\begin{array}{l}\frac{1}{(\xi+\alpha+\lambda)}\frac{(1-f^*(\xi+\alpha+\lambda))}{)(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\lambda))}\\-\frac{1}{(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta)))}\\\times\frac{(1-f^*(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))))}\end{array}\right]\\\frac{1}{(\xi+\alpha+\mu(1-qr_1^*(\eta)))}\frac{(1-f^*(\xi+\alpha+\mu(1-qr_1^*(\eta))))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\mu(1-qr_1^*(\eta))))}\\-\frac{1}{(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta)))}\frac{(1-f^*(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\lambda+\mu(1-qr_1^*(\eta))))}\end{array}\right\} \quad (10)$$

Since there is only change in sales pattern, E(X) and $E(\hat{R})$ are same as that of the results obtained in model 1.

$$E(\hat{S})=-\frac{\partial}{\partial\varepsilon}f^*(\xi,\eta,\varepsilon)\,|\,\xi=\eta=\varepsilon=0.$$

This gives

$$E(\hat{S})=E(H_1)\left[1-\frac{\lambda\mu p(2\alpha+\mu p+\lambda)}{(\alpha+\lambda)(\alpha+\mu p)(\alpha+\lambda+\mu p)}\right]$$
$$+E(H_2)\left[\begin{array}{l}\frac{\lambda}{(\alpha+\lambda)}\frac{f^*(\alpha+\lambda)}{(1-f^*(\alpha+\lambda))}+\frac{\mu p}{(\alpha+\mu p)}\frac{f^*(\alpha+\mu p)}{(1-f^*(\alpha+\mu p))}\\-\frac{(\lambda+\mu p)}{(\alpha+\lambda+\mu p)}\frac{f^*(\alpha+\lambda+\mu p)}{(1-f^*(\alpha+\lambda+\mu p))}\end{array}\right] \quad (11)$$

The equation (10) gives the Laplace transform of pdf of X and Ŝ as follows.

$$f^*(\xi,0,\varepsilon)=g_1^*(\varepsilon)\{\frac{\lambda}{(\xi+\lambda)}-\frac{\lambda}{(\xi+\lambda+\mu p)}+\frac{p\mu}{(\xi+p\mu)}-\frac{p\mu}{(\xi+\lambda+p\mu)}$$
$$-\frac{\lambda}{(\xi+\alpha+\lambda)}+\frac{\lambda}{(\xi+\alpha+\lambda+\mu p)}-\frac{p\mu}{(\xi+\alpha+p\mu)}+\frac{p\mu}{(\xi+\lambda+\alpha+p\mu)}\}$$
$$+\frac{\lambda}{(\xi+\alpha+\lambda)}\frac{(1-f^*(\xi+\alpha+\lambda))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\lambda))}$$
$$-\frac{\lambda+\mu p}{(\xi+\alpha+\lambda+\mu p)}\frac{(1-f^*(\xi+\alpha+\lambda+\mu p))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\lambda+p\mu))}$$
$$+\frac{p\mu}{(\xi+\alpha+p\mu)}\frac{(1-f^*(\xi+\alpha+\mu p))}{(1-g_2^*(\varepsilon)f^*(\xi+\alpha+\mu p))} \quad (12)$$

The joint moment

$E(X\hat{S}) = \frac{\partial^2}{\partial\xi\partial\varepsilon} f^*(\xi,0,\varepsilon) \,|\, \xi = 0 = \varepsilon$, can be seen as

$$E(X\hat{S}) = E(H_1)\left\{\begin{array}{l} \frac{1}{\lambda} - \frac{\lambda}{(\lambda+\mu p)^2} + \frac{1}{\mu p} - \frac{\mu p}{(\lambda+\mu p)^2} - \frac{\lambda}{(\alpha+\lambda)^2} \\ + \frac{\lambda}{(\alpha+\lambda+\mu p)^2} - \frac{\mu p}{(\alpha+\mu p)^2} + \frac{\mu p}{(\lambda+\alpha+\mu p)^2} \end{array}\right\}$$

$$+E(H_2)\left\{\begin{array}{l} \frac{\lambda}{(\alpha+\lambda)^2}\frac{f^*(\alpha+\lambda)}{(1-f^*(\alpha+\lambda))} - \frac{\lambda}{(\alpha+\lambda)}\frac{f^{*'}(\alpha+\lambda)}{(1-f^*(\alpha+\lambda))^2} \\ -\frac{(\lambda+\mu p)}{(\alpha+\lambda+\mu p)^2}\frac{f^*(\alpha+\lambda+\mu p)}{(1-f^*(\alpha+\lambda+\mu p))} + \frac{(\lambda+\mu p)}{(\alpha+\lambda+\mu p)}\frac{f^{*'}(\alpha+\lambda+\mu p)}{(1-f^*(\alpha+\lambda+\mu p))^2} \\ + \frac{\mu p}{(\alpha+\mu p)^2}\frac{f^*(\alpha+\mu p)}{(1-f^*(\alpha+\mu p))} - \frac{\mu p}{(\alpha+\mu p)}\frac{f^{*'}(\alpha+\mu p)}{(1-f^*(\alpha+\mu p))^2} \end{array}\right\} \quad (13)$$

Now,

$Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S})$ can be written using equations (13), (11) and (4).

3. NUMERICAL EXAMPLES FOR MODEL 1 & 2

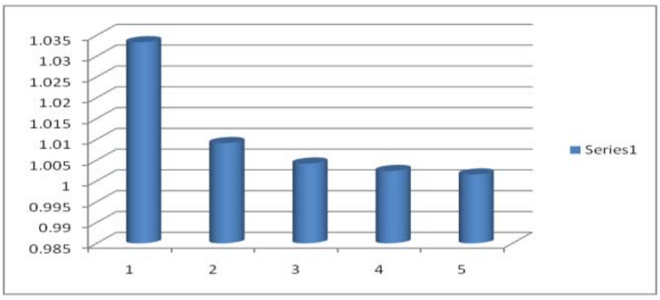
The usefulness of the results obtained is presented by numerical examples. The two models 1 and 2 are considered together, since there is only change in sales pattern E(X) and $E(\hat{R})$ are same for models 1 and 2.

3.1 Numerical values for model 1

Let α = 0.4, p=0.2, q=0.8, E(R)=10, $E(R_1)$ = 5, E(H)=20, μ=5, λ=5,10,15,20,25 and δ=1,2,3,4,5. Here 'f' is an exponential density function with parameter 'δ'.

The table and graph for E(X)

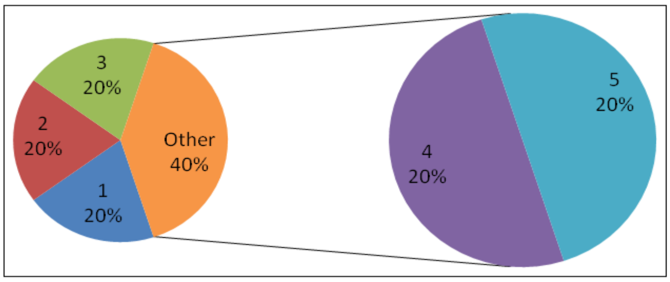
λ/μ	5	5	5	5	5
5	1.033333	1.009091	1.004167	1.002381	1.001538
10	1.033333	1.009091	1.004167	1.002381	1.001538
15	1.033333	1.009091	1.004167	1.002381	1.001538
20	1.033333	1.009091	1.004167	1.002381	1.001538
25	1.033333	1.009091	1.004167	1.002381	1.001538



From the table and graph it is observed that, when λ increases the expected operation time E(X) decreases.

The table and graph for $E(\hat{R})$

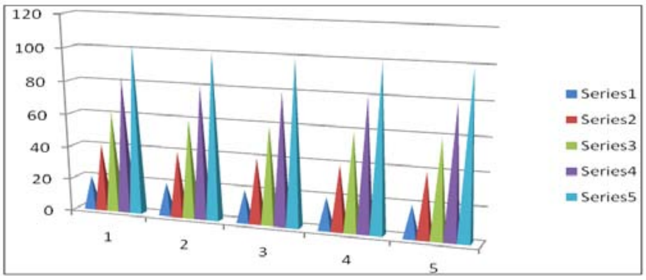
λ/μ	5	5	5	5	5
5	35	35	35	35	35
10	35	35	35	35	35
15	35	35	35	35	35
20	35	35	35	35	35
25	35	35	35	35	35



From the table and graph it is observed that, when λ increases the expected repair time $E(\hat{R})$ remains constant.

The table and graph for $E(\hat{S})$

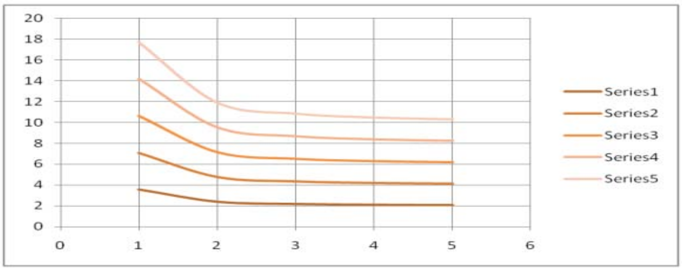
λ/δ	1	2	3	4	5
5	20.66667	20.18182	20.08333	20.04762	20.03077
10	41.33333	40.36364	40.16667	40.09524	40.06154
15	62	60.54545	60.25	60.14286	60.09231
20	82.66667	80.72727	80.33333	80.19048	80.12308
25	103.3333	100.9091	100.4167	100.2381	100.1538



From the table and graph it is observed that, when λ increases the expected sales time $E(\hat{S})$ increases and when δ increases the expected sales time $E(\hat{S})$ decreases.

The table and graph for $E(X,\hat{S})$

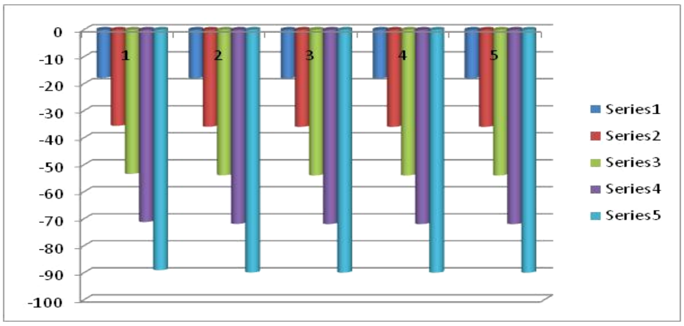
λ/δ	1	2	3	4	5
5	3.544444	2.383471	2.169965	2.095465	2.061041
10	7.088889	4.766942	4.339931	4.19093	4.122083
15	10.63333	7.150413	6.509896	6.286395	6.183124
20	14.17778	9.533884	8.679861	8.381859	8.244166
25	17.72222	11.91736	10.84983	10.47732	10.30521



From the table and graph it is observed that, when λ increases the expected product moment of X and $\hat{S}$ $E(X,\hat{S})$ increases and when δ increases $E(X,\hat{S})$ decreases.

The table and graph for $Cov(X,\hat{S})$

λ/δ	1	2	3	4	5
5	-17.8111	-17.9818	-17.997	-17.9999	-18.0005
10	-35.6222	-35.9636	-35.9941	-35.9998	-36.0011
15	-53.4333	-53.9455	-53.9911	-53.9997	-54.0016
20	-71.2444	-71.9273	-71.9882	-71.9995	-72.0022
25	-89.0556	-89.9091	-89.9852	-89.9994	-90.0027



From the table and graph it is observed that when both λ and δ both increase, the $Cov(X,\hat{S})$ decrease.

3.2 Numerical values for model 2

Let $\alpha = 0.4$, p=0.2, q=0.8, E(R)=10, E(R_1) =5, E(H_1)= 5, E(H_2)=10, μ=5, λ=5,10,15,20,25 and δ=10,20,30,40,50.

Here 'f' is an exponential density function with parameter 'δ'

The table and graph for E(X)

λ/μ	5	5	5	5	5
5	1.033333	1.009091	1.004167	1.002381	1.001538
10	1.033333	1.009091	1.004167	1.002381	1.001538
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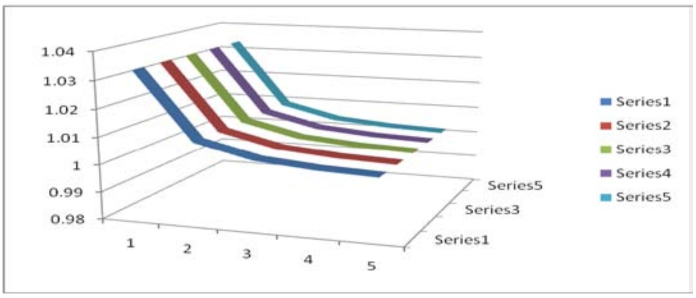
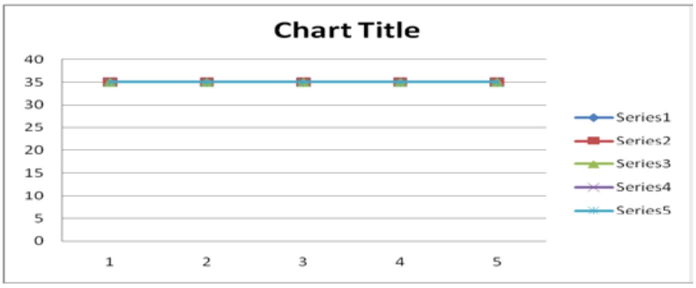
20	1.033333	1.009091	1.004167	1.002381	1.001538
25	1.033333	1.009091	1.004167	1.002381	1.001538



From the table and graph it is observed that, when λ increases the expected operation time E(X) decreases.

The table and graph for $E(\hat{R})$

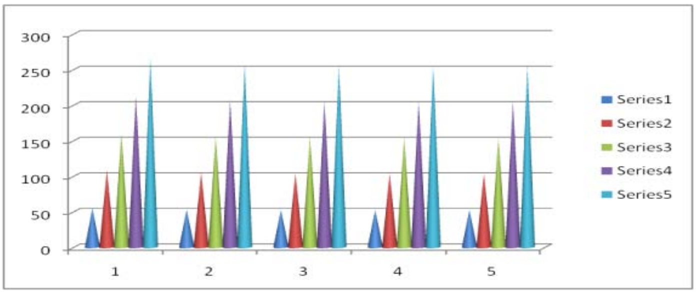
λ/δ	10	20	30	40	50
5	35	35	35	35	35
10	35	35	35	35	35
15	35	35	35	35	35
20	35	35	35	35	35
25	35	35	35	35	35



From the table and graph it is observed that, when λ increases the expected repair time $E(\hat{R})$ remains constant.

The table and graph for $E(\hat{S})$

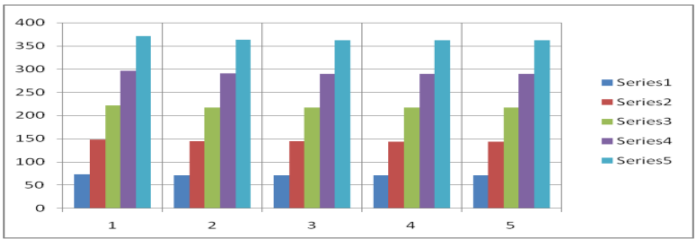
λ/δ	10	20	30	40	50
5	55.00519	53.24727	52.8329	52.67385	52.59648
10	108.5289	105.0491	104.2293	103.9146	103.7614
15	162.0427	156.8509	155.6257	155.1552	154.9263
20	215.5614	208.6528	207.0221	206.3959	206.0913
25	269.0802	260.4546	258.4186	257.6366	257.2562



From the table and graph it is observed that, when λ increases the expected sales time $E(\hat{S})$ increases and when δ increases the expected sales time $E(\hat{S})$ decreases.

The table and graph for $E(X,\hat{S})$

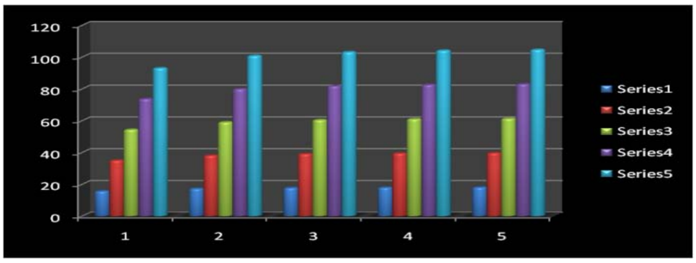
λ/δ	10	20	30	40	50
5	72.35006	70.83472	70.68825	70.57877	70.55825
10	147.0094	144.0141	143.6155	143.5077	143.4671
15	221.6687	217.1934	216.5977	216.4366	216.3759
20	296.328	290.3728	289.58	289.3655	289.2847
25	370.9873	363.5521	362.5622	362.2944	362.1935



From the table and graph it is observed that, when λ increases $E(X,\hat{S})$ increases and when δ increases $E(X,\hat{S})$ decreases.

The table and graph for $Cov(X,\hat{S})$

λ/δ	10	20	30	40	50
5	15.51135	17.10838	17.58021	17.77951	17.88085
10	34.86797	38.00998	38.95188	39.34571	39.54601
15	54.22458	58.91658	60.32956	60.91191	61.21117
20	73.58119	79.82318	81.69523	82.47811	82.87633
25	92.93781	100.7298	103.0669	104.0448	104.5415



From the table it is observed that when λ and δ increase, the $Cov(X,\hat{S})$ increases.

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