

(gsp)** - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

*In this paper we introduce a new class of sets called (gsp)**-closed sets in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce two new spaces namely, T_{gsp}^{**} space, ${}_aT_{gsp}^{**}$ space. Further, (gsp)**-continuous, (gsp)**-irresolute mappings are also introduced and investigated.*

Key words: (gsp)**-closed set, (gsp)**-continuous map, (gsp)**-irresolute map, T_{gsp}^{**} , ${}_aT_{gsp}^{**}$ -spaces.

1. INTRODUCTION

Levine [11] introduced the class of g-closed sets in 1970. Arya and Tour [3] defined gs-closed sets in 1990. Dontchev [9], Gnanambal [10] Palaniappan and Rao [17] introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively. Veerakumar [18] introduced g*-closed sets in 1991. Dontchev [8] introduced gsp-closed sets in 1995. Levine [11] Devi [6,8] introduced $T_{1/2}$ -spaces, T_b spaces and ${}_aT_b$ spaces respectively. PaulineMHelen[20] introduced (gsp)* sets. The purpose of this paper is to introduce the concepts of (gsp)**-closed set, (gsp)**-continuous map, (gsp)**-irresolute maps. T_{gsp}^{**} -space, ${}_aT_{gsp}^{**}$ -space are introduced and investigated.

2. PRILIMINARIES

Throughout this paper (X, τ) , (Y, σ) represent non-empty topological spaces of which no separation axioms are assumed unless otherwise stated. For a subset A of a space (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure and the interior of A respectively. The class of all closed subsets of a space (X, τ) is denoted by $C(X, \tau)$. The smallest semi-closed (resp. pre-closed and α -closed) set containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure and α -closure) of A and is denoted by $\text{scl}(A)$ (resp. $\text{pcl}(A)$ and $\alpha\text{cl}(A)$).

Definition 2.1: A subset A of a topological space (X, τ) is called

- (1) a pre-open set [14] if $A \subseteq \text{int}(\text{cl}(A))$ and a pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.
- (2) a semi-open set [12] if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
- (3) a semi-preopen set [1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and a semi-preclosed set [1] if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
- (4) an α -open set [16] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and an α -closed set [16] if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- (5) a regular-open set [14] if $\text{int}(\text{cl}(A)) = A$ and regular-closed set [14] if $A = \text{int}(\text{cl}(A))$.

Definition 2.2: A subset A of topological space (X, τ) is called

- (1) a generalized closed set (briefly g-closed) [1] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (2) generalized semi-closed set (briefly gs-closed) [3] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (3) an α -generalized closed set (briefly α g-closed) [19] if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (4) a generalized semi pre-closed set (briefly gsp-closed) [9] if $\text{sp cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (5) a regular generalized closed set (briefly rg-closed) [17] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .

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- (6) a generalized pre-closed set (briefly gp-closed) [13] if $p\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (7) a generalized pre regular-closed set (briefly gpr-closed)[10] if $p\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- (8) a g^* -closed set [18] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .
- (9) a wg -closed set [16] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (10) a $(gsp)^*$ -closed set [20] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp -open in (X, τ) .

Definition 2.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) g -continuous [4] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
- (2) αg -continuous [10] if $f^{-1}(V)$ is an αg -closed set of (X, τ) for every closed set V of (Y, σ) .
- (3) gs -continuous [7] if $f^{-1}(V)$ is a gs -closed set of (X, τ) for every closed set V of (Y, σ) .
- (4) gsp -continuous [9] if $f^{-1}(V)$ is a gsp -closed set of (X, τ) for every closed set V of (Y, σ) .
- (5) rg -continuous [17] if $f^{-1}(V)$ is a rg -closed set of (X, τ) for every closed set V of (Y, σ) .
- (6) gp -continuous [2] if $f^{-1}(V)$ is a gp -closed set of (X, τ) for every closed set V of (Y, σ) .
- (7) gpr -continuous [10] if $f^{-1}(V)$ is a gpr -closed set of (X, τ) for every closed set V of (Y, σ) .
- (8) g^* -continuous [18] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
- (9) wg -continuous [16] if $f^{-1}(V)$ is a wg -closed set of (X, τ) for every closed set V of (Y, σ) .
- (10) $(gsp)^*$ -continuous [20] if $f^{-1}(V)$ is an $(gsp)^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .

Definition 2.4: A topological space (X, τ) is said to be

- (1) a $T_{1/2}$ -space [11] if every g -closed set in it is closed.
- (2) a T_b space [6] if every gs -closed set in it is closed.
- (3) a $_{\alpha}T_b$ -space [8] if every αg -closed set in it is closed.
- (4) a $T_{1/2}^*$ -space [18] if every g^* -closed set in it is closed.
- (5) a T_{gsp}^* -space [20] if every $(gsp)^*$ -closed set is closed.
- (6) a gT_{gsp}^* -space [20] if every g -closed set is $(gsp)^*$ closed.

3. Basic properties of $(gsp)^*$ - closed sets

We introduce the following definition

Definition 3.1: A subset A of (X, τ) is said to be a $(gsp)^*$ -closed set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gsp)^*$ -open in X .

Proposition 3.2: Every closed set is $(gsp)^*$ -closed.

Proof: Let A be closed set, Then $\text{cl}(A) = A$.

Let $A \subseteq U$ and U be $(gsp)^*$ -open.

Then $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gsp)^*$ -open.

Therefore A is $(gsp)^*$ -closed.

Proposition 3.3: Every $(gsp)^*$ -closed set is gs -closed.

Proof: Let A be a $(gsp)^*$ -closed set. Let $A \subseteq U$ and U be open. Then $\text{cl}(A) \subseteq U$ since U is $(gsp)^*$ -open and A is $(gsp)^*$ -closed. $\text{scl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gs -closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.4: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is gs -closed but not $(gsp)^*$ -closed in (X, τ) .

Proposition 3.5: Every $(gsp)^*$ -closed set is αg -closed, but not conversely.

Proof: Let A be a $(gsp)^*$ -closed set. $\text{cl}(A) \subseteq U$ since U is $(gsp)^*$ -open and A is $(gsp)^*$ -closed. But $\alpha \text{cl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is αg -closed.

Example 3.6: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is αg -closed but not $(gsp)^*$ -closed in (X, τ) .

Proposition 3.7: Every (gsp)**-closed set is gsp-closed, but not conversely.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U be open. Then $\text{cl}(A) \subseteq U$ since U is (gsp)*-open and A is (gsp)**-closed. $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gsp-closed.

Example 3.8: Let $X = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is gsp-closed but not (gsp)**-closed in (X, τ) .

Proposition 3.9: Every (gsp)**-closed set is rg-closed.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U be regular open. Then $A \subseteq U$ and U is (gsp)*-open and $\text{cl}(A) \subseteq U$, since A is (gsp)**-closed. Hence A is rg-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.10: Let $X = \{a, b, c\}$, $\tau = \{\varphi, X, \{b\}, \{a, b\}\}$. Then $A = \{a\}$ is rg-closed but not (gsp)**-closed in (X, τ) .

Proposition 3.11: Every (gsp)**-closed set is gp-closed, but not conversely.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U be open. Then $\text{cl}(A) \subseteq U$ since U is (gsp)*-open and A is (gsp)**-closed. $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gp-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.12: Let $X = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is gp-closed but not (gsp)**-closed in (X, τ) .

Proposition 3.13: Every (gsp)**-closed set is gpr-closed, but not conversely.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U be regular open. Then $A \subseteq U$ and U is (gsp)*-open and $\text{cl}(A) \subseteq U$, since A is (gsp)**-closed. Then $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gpr-closed.

Example 3.14: Let $X = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then $A = \{a\}$ is gpr-closed but not (gsp)**-closed in (X, τ) .

Proposition 3.15: Every (gsp)**-closed set is wg-closed, but not conversely.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U be open. Then U is (gsp)*-open and $\text{cl}(A) \subseteq U$, since A is (gsp)**-closed. $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$. Hence A is wg-closed.

Example 3.16: Let $X = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is wg-closed but not (gsp)**-closed in (X, τ) .

Proposition 3.17: If A and B are (gsp)**-closed sets then $A \cup B$ is also (gsp)**-closed.

Proof: follows from the fact that $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$.

Proposition 3.18: If A is (gsp)**-closed set of (X, τ) such that $A \subseteq B \subseteq \text{cl}(A)$, then B is also a (gsp)**-closed set of (X, τ) .

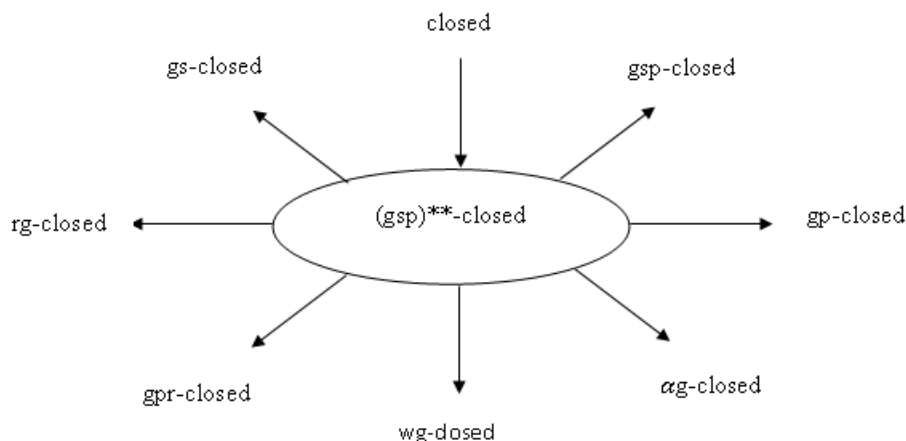
Proof: Let U be the (gsp)*-open set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$ where U is (gsp)*-open. Since A is (gsp)**-closed, $\text{cl}(A) \subseteq U$. Then $\text{cl}(B) \subseteq U$, Hence B is (gsp)**-closed.

Proposition 3.19: If A is (gsp)**-closed set of (X, τ) , then $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set.

Proof: Let F be (gsp)*-closed set of (X, τ) such that $F \subseteq \text{cl}(A) \setminus A$. Then $A \subseteq X \setminus F$. Since A is (gsp)**-closed $\text{cl}(A) \subseteq X \setminus F$. This implies $F \subseteq X \setminus \text{cl}(A)$. Hence $F \subseteq (X \setminus \text{cl}(A)) \cap (\text{cl}(A) \setminus A) = \emptyset$. Hence $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set.

Proposition 3.20: If A is both (gsp)*-open and (gsp)**-closed then A is closed.

The above results can be represented in the following figure.



Where $A \rightarrow B$ represents A implies B and B need not imply A .

4. (gsp)**-continuous and (gsp)**-irresolute maps

We introduce the following definitions:

Definition 4.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called (gsp)**-continuous if $f^{-1}(V)$ is a (gsp)**-closed set in (X, τ) for every closed set V of (Y, σ) .

Definition 4.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called (gsp)**-irresolute if $f^{-1}(V)$ is a (gsp)**-closed set in (X, τ) for every (gsp)**-closed set V of (Y, σ) .

Theorem 4.3: Every continuous map is (gsp)**-continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map. Let F be a closed set in (Y, σ) since f is continuous $f^{-1}(F)$ is closed in (X, τ) and hence $f^{-1}(F)$ is (gsp)**-closed. Therefore f is (gsp)**-continuous.

Theorem 4.4: Every (gsp)**-continuous map is (1) gs-continuous (2) α g-continuous (3) gsp-continuous (4) rg-continuous (5) gp-continuous (6) gpr-continuous and (7) wg-continuous but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be (gsp)**-continuous and let F be a closed set of (Y, σ) . Since f is (gsp)**-continuous $f^{-1}(F)$ is (gsp)**-closed in (X, τ) . Then $f^{-1}(F)$ is gs- closed, α g- closed, gsp- closed, rg- closed, gp- closed, gpr- closed and wg- closed. Hence f is gs-continuous, α g-continuous, gsp-continuous, rg-continuous, gp-continuous, gpr-continuous and wg-continuous.

Example 4.5: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, c\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. The closed sets of Y are $\emptyset, Y, \{b\}$. $f^{-1}(b) = b$ is not (gsp)**-closed in (X, τ) . Hence f is not (gsp)**-continuous. $f^{-1}(b) = b$ is gs- closed, α g- closed, gsp- closed, gp- closed and wg-closed. Hence f is gs-continuous, α g-continuous, gsp-continuous, gp-continuous and wg-continuous.

Example 4.6: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = c$, $f(b) = a$, $f(c) = b$. $f^{-1}\{a, c\} = \{a, b\}$ is gpr- closed in (X, τ) , but not (gsp)**-closed in (X, τ) . Hence f is gpr-continuous but not (gsp)**-continuous.

Example 4.7: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{c\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. The closed sets of Y are \emptyset, Y and $\{a, b\}$. $f^{-1}\{a, b\} = \{a, b\}$ is rg- closed but not (gsp)**-closed and hence f is rg-continuous but not (gsp)**- continuous.

Theorem 4.8: Every (gsp)**-irresolute is (gsp)**- continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a (gsp)**- irresolute. Let V be a closed set of (Y, σ) . Then V is (gsp)**-closed and $f^{-1}(V)$ is (gsp)**-closed since f is a (gsp)**-irresolute. Hence f is (gsp)**-continuous.

Theorem 4.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a (gsp)**-irresolute then f is

- (1) gs-continuous
- (2) α g-continuous
- (3) gsp- continuous
- (4) rg-continuous
- (5) gp-continuous
- (6) gpr-continuous and
- (7) wg-continuous but not conversely.

Proof: Since every (gsp)**-irresolute is (gsp)**- continuous, f is (gsp)**- continuous. Then by theorem 4.4 the result follows.

Example 4.10: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a$, $f(b) = c$, $f(c) = b$. $f^{-1}\{c\} = \{b\}$ is gs- closed in (X, τ) and hence f is gs- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

Example 4.11: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a$, $f(b) = c$, $f(c) = b$. $f^{-1}\{c\} = \{b\}$ is gp- closed in (X, τ) and hence f is gp- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

Example 4.12: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. $f^{-1}\{c\} = \{b\}$ is rg- closed in (X, τ) and hence f is rg- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

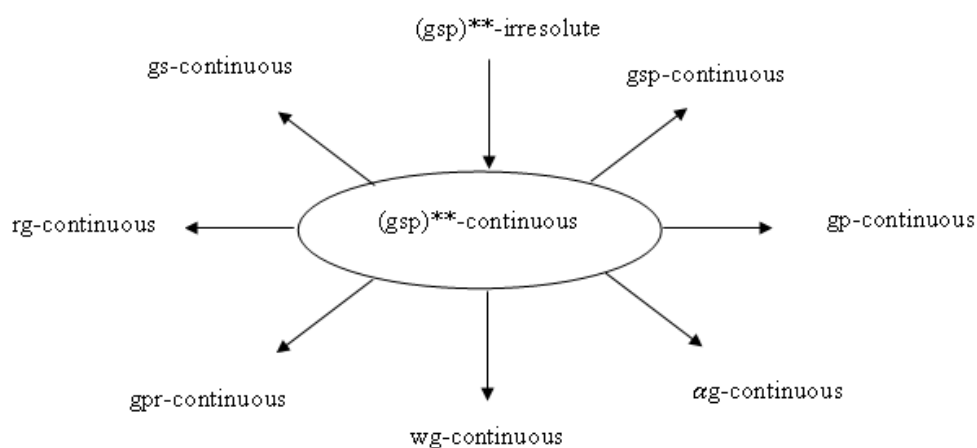
Example 4.13: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. $f^{-1}\{c\} = \{c\}$ is gsp- closed in (X, τ) and hence f is gsp- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{c\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

Example 4.14: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a$, $f(b) = c$, $f(c) = b$. $f^{-1}\{c\} = \{b\}$ is wg- closed in (X, τ) and hence f is wg- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

Example 4.15: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = c$, $f(c) = a$. $f^{-1}\{c\} = \{b\}$ is α g- closed in (X, τ) and hence f is α g- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

Example 4.16: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = c$, $f(c) = a$. $f^{-1}\{c\} = \{b\}$ is gpr- closed in (X, τ) and hence f is gpr- continuous. (gsp)**-closed sets of (Y, σ) are $\emptyset, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}\{c\} = \{b\}$ is not (gsp)**-closed set in (X, τ) . Hence f is not a (gsp)**-irresolute.

The above results can be represented in the following figure.



Where $A \rightarrow B$ represents A implies B and B need not imply A .

5. APPLICATIONS OF (gsp)**-CLOSED SETS

Definition 5.1: A space (X, τ) is called a T_{gsp}^{**} -space if every (gsp)**-closed set is closed.

Definition 5.2: A space (X, τ) is called a $_{\alpha}T_{gsp}^{**}$ -space if every αg -closed set is (gsp)**-closed.

Theorem 5.3: Every T_b -space is T_{gsp}^{**} -space but not conversely.

Proof: Let (X, τ) be a T_b -space. Let A be a (gsp)**-closed set. Since every (gsp)**-closed set is g_s -closed and hence A is g_s -closed. Since (X, τ) is a T_b -space, A is closed. Hence (X, τ) is a T_{gsp}^{**} -space.

Example 5.4: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. (X, τ) is a T_{gsp}^{**} -space $A = \{a\}$ is g_s -closed, but it is not closed, and hence it is not a T_b -space. Hence a T_{gsp}^{**} -space need not be a T_b -space.

Theorem 5.5: Every T_b -space is a $_{\alpha}T_{gsp}^{**}$ -space.

Proof: Let (X, τ) be a T_b -space. Let A be αg -closed. Then A is g_s -closed. Since the space is T_b -space, A is closed and hence A is (gsp)**-closed. Therefore the space (X, τ) is a $_{\alpha}T_{gsp}^{**}$ -space.

Example 5.6: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Here (gsp)**-closed sets are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$, αg -closed sets are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$ and the g_s -closed sets are $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Since every αg -closed set is (gsp)**-closed the space (X, τ) is a $_{\alpha}T_{gsp}^{**}$ -space. $A = \{a\}$ is g_s -closed, but it is not closed, and hence it is not a T_b -space.

Theorem 5.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be (gsp)**-continuous map and let (X, τ) be a T_{gsp}^{**} -space then f is continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be (gsp)**-continuous map. Let F be a closed set of (Y, σ) . Since f is (gsp)**-continuous, $f^{-1}(F)$ is (gsp)**-closed set in (X, τ) . Since (X, τ) is a T_{gsp}^{**} -space, $f^{-1}(F)$ is closed in (X, τ) . Therefore f is continuous.

Theorem 5.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be αg -continuous map where (X, τ) is a $_{\alpha}T_{gsp}^{**}$ -space. Then f is (gsp)**-continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a αg -continuous map. Let F be a closed set of (Y, σ) . Since f is αg -continuous, $f^{-1}(F)$ is αg -closed set in (X, τ) . Since (X, τ) is a $_{\alpha}T_{gsp}^{**}$ -space, $f^{-1}(F)$ is (gsp)**-closed in (X, τ) . Therefore f is (gsp)**-continuous.

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