(gsp)** - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT
In this paper we introduce a new class of sets called (gsp)**-closed sets in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce two new spaces namely, T_{gsp}** space, a_{T_{gsp}}** space. Further, (gsp)**-continuous, (gsp)**-irresolute mappings are also introduced and investigated.

Key words: (gsp)**closed set, (gsp)**-continuous map, (gsp)**-irresolute map, T_{gsp}**, a_{T_{gsp}}**-spaces.

1. INTRODUCTION

2. PRILIMINARIES
Throughout this paper (X,τ), (Y,σ) represent non-empty topological spaces of which no separation axioms are assumed unless otherwise stated. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure and the interior of A respectively. The class of all closed subsets of a space (X,τ) is denoted by C(X,τ). The smallest semi-closed (resp.pre-closed and α-closed) set containing a subset A of (X,τ) is called the semi-closure (resp.pre-closure and α-closure) of A and is denoted by scl(A)(resp.pcl(A) and αcl(A)).

Definition 2.1: A subset A of a topological space (X,τ) is called
1. a pre-open set [14] if A ⊆ int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
2. a semi-open set [12] if A ⊆ cl(int(A)) and a semi-closed set if cl(int(A)) ⊆ A.
3. a semi-preopen set [1] if A ⊆ cl(int(cl(A))) and a semi-preclosed set [1] if int(cl(int(A))) ⊆ A.
4. an α-open set [16] if A ⊆ cl(int(A)) and an α-closed set [16] if cl(int(cl(A)) ⊆ A.
5. a regular-open set [14] if int(cl(A) = A and regular-closed set [14] if A = int(cl(A)).

Definition 2.2: A subset A of a topological space (X,τ) is called
1. a generalized closed set (briefly g-closed) [1] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
2. generalized semi-closed set (briefly gs-closed) [3] if scl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
3. an α-generalized closed set (briefly gα-closed) [19] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
4. a generalized semi pre-closed set (briefly gsp-closed) [9] if sp cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
5. a regular generalized closed set (briefly rg-closed) [17] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X,τ).
(6) a generalized pre-closed set (briefly gp-closed) [13] if \( p \text{cl} (A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X,\tau)\).
(7) a generalized pre-regular-closed set (briefly gpr-closed)[10] if \( p \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular open in \((X,\tau)\).
(8) a g* -closed set \([18]\) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is g-open in \((X,\tau)\).
(9) a wg-closed set \([16]\) if \( \text{cl}(\text{int}(A)) \) whenever \( A \subseteq U \) and \( U \) is open in \((X,\tau)\).
(10) a (gsp)*-closed set \([20]\) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is gsp-open in \((X,\tau)\).

Definition 2.3: A function \( f : (X,\tau) \rightarrow (Y,\sigma) \) is called
(1) g-continuous \([4]\) if \( f^{-1}(V) \) is a g-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(2) \( \alpha g \)-continuous \([10]\) if \( f^{-1}(V) \) is an \( \alpha g \)-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(3) gs-closed \([7]\) if \( f^{-1}(V) \) is a gs-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(4) gsp-closed \([9]\) if \( f^{-1}(V) \) is a gsp-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(5) rg-continuous \([17]\) if \( f^{-1}(V) \) is a rg-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(6) gp-continuous \([2]\) if \( f^{-1}(V) \) is a gp-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(7) gpr-continuous \([10]\) if \( f^{-1}(V) \) is a gpr-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(8) gc*-closed \([18]\) if \( f^{-1}(V) \) is a gc*-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(9) gsp-closed \([16]\) if \( f^{-1}(V) \) is a gsp-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).
(10) \( (gsp)^* \)-closed \([20]\) if \( f^{-1}(V) \) is an \( (gsp)^* \)-closed set of \((X,\tau)\) for every closed set \( V \) of \((Y,\sigma)\).

3. Basic properties of \((gsp)^*\)-closed sets

We introduce the following definition.

Definition 3.1: A subset \( A \) of \((X,\tau)\) is said to be \((gsp)^*\)-closed if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((gsp)^*\)-open in \( X \).

Proposition 3.2: Every closed set is \((gsp)^*\)-closed.

Proof: Let \( A \) be closed set, Then \( \text{cl}(A) = A \).

Let \( A \subseteq U \) and \( U \) be \((gsp)^*\)-open.

Then \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((gsp)^*\)-open.

Therefore \( A \) is \((gsp)^*\)-closed.

Proposition 3.3: Every \((gsp)^*\)-closed set is gs-closed.

Proof: Let \( A \) be a \((gsp)^*\)-closed set. Let \( A \subseteq U \) and \( U \) be open. Then \( \text{cl}(A) \subseteq U \) since \( U \) is \((gsp)^*\)-open and \( A \) is \((gsp)^*\)-closed. \( \text{sc}(A) \subseteq \text{cl}(A) \subseteq U \). Hence \( A \) is gs-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.4: Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a\}, \{a, b\}\} \). Then \( A = \{b\} \) is gs-closed but not \((gsp)^*\)-closed in \((X,\tau)\).

Proposition 3.5: Every \((gsp)^*\)-closed set is \( \alpha g \)-closed, but not conversely.

Proof: Let \( A \) be a \((gsp)^*\)-closed set. \( \text{cl}(A) \subseteq U \) since \( U \) is \((gsp)^*\)-open and \( A \) is \((gsp)^*\)-closed. \( \alpha \text{cl}(A) \subseteq \text{cl}(A) \subseteq U \). Hence \( A \) is \( \alpha g \)-closed.

Example 3.6: Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a\}, \{a, b\}\} \). Then \( A = \{b\} \) is \( \alpha g \)-closed but not \((gsp)^*\)-closed in \((X,\tau)\).
Proposition 3.7: Every (gsp)**-closed set is gsp-closed, but not conversely.

**Proof:** Let A be a (gsp)**-closed set. Let $A \subseteq U$ and $U$ be open. Then $\text{cl}(A) \subseteq U$ since $U$ is (gsp)*-open and $A$ is (gsp)**-closed. $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence $A$ is gsp-closed.

Example 3.8: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is gsp-closed but not (gsp)**-closed in $(X, \tau)$.

Proposition 3.9: Every (gsp)**-closed set is rg-closed.

**Proof:** Let $A$ be a (gsp)**-closed set. Let $A \subseteq U$ and $U$ be regular open. Then $A \subseteq U$ and $U$ is (gsp)*-open and $\text{cl}(A) \subseteq U$, since $A$ is (gsp)**-closed. Hence $A$ is rg-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.10: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{a\}$ is rg-closed but not (gsp)**-closed in $(X, \tau)$.

Proposition 3.11: Every (gsp)**-closed set is gp-closed, but not conversely.

**Proof:** Let $A$ be a (gsp)**-closed set. Let $A \subseteq U$ and $U$ be open. Then $\text{cl}(A) \subseteq U$ since $U$ is (gsp)*-open and $A$ is (gsp)**-closed. $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence $A$ is gp-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.12: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is gp-closed but not (gsp)**-closed in $(X, \tau)$.

Proposition 3.13: Every (gsp)**-closed set is gpr-closed, but not conversely.

**Proof:** Let $A$ be a (gsp)**-closed set. Let $A \subseteq U$ and $U$ be regular open. Then $A \subseteq U$ and $U$ is (gsp)*-open and $\text{cl}(A) \subseteq U$, since $A$ is (gsp)**-closed. $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence $A$ is gpr-closed.

Example 3.14: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{a\}$ is gpr-closed but not (gsp)**-closed in $(X, \tau)$.

Proposition 3.15: Every (gsp)**-closed set is wg-closed, but not conversely.

**Proof:** Let $A$ be a (gsp)**-closed set. Let $A \subseteq U$ and $U$ be open. Then $U$ is (gsp)*-open and $\text{cl}(A) \subseteq U$, since $A$ is (gsp)**-closed. $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$. Hence $A$ is wg-closed.

Example 3.16: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $A = \{b\}$ is wg-closed but not (gsp)**-closed in $(X, \tau)$.

Proposition 3.17: If $A$ and $B$ are (gsp)**-closed sets then $A \cup B$ is also (gsp)**-closed.

**Proof:** follows from the fact that $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$.

Proposition 3.18: If $A$ is (gsp)**-closed set of $(X, \tau)$ such that $A \subseteq B \subseteq \text{cl}(A)$, then $B$ is also a (gsp)**-closed set of $(X, \tau)$.

**Proof:** Let $U$ be the (gsp)*-open set of $(X, \tau)$ such that $B \subseteq U$. Then $A \subseteq U$ where $U$ is (gsp)*-open. Since $A$ is (gsp)**-closed, $\text{cl}(A) \subseteq U$. Hence $B$ is (gsp)**-closed.

Proposition 3.19: If $A$ is (gsp)**-closed set of $(X, \tau)$, then $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set.

**Proof:** Let $F$ be (gsp)*-closed set of $(X, \tau)$ such that $F \subseteq \text{cl}(A) \setminus A$. Then $A \subseteq X \setminus F$. Since $A$ is (gsp)**-closed $\text{cl}(A) \subseteq X \setminus F$. This implies $F \subseteq X \setminus \text{cl}(A)$. Hence $F \subseteq (X \setminus \text{cl}(A)) \cap (\text{cl}(A) \setminus A) = \emptyset$. Hence $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set.

Proposition 3.20: If $A$ is both (gsp)*-open and (gsp)**-closed then $A$ is closed.
The above results can be represented in the following figure.

![Diagram showing relationships between various types of closed sets in topological spaces](image)

Where A → B represents A implies B and B need not imply A.

4. (gsp)**-continuous and (gsp)**-irresolute maps

We introduce the following definitions:

**Definition 4.1:** A function \( f: (X, \tau) \to (Y, \sigma) \) is called (gsp)**-continuous if \( f^{-1}(V) \) is a (gsp)**-closed set in \( (X, \tau) \) for every closed set \( V \) of \( (Y, \sigma) \).

**Definition 4.2:** A function \( f: (X, \tau) \to (Y, \sigma) \) is called (gsp)**-irresolute if \( f^{-1}(V) \) is a (gsp)**-closed set in \( (X, \tau) \) for every (gsp)**-closed set \( V \) of \( (Y, \sigma) \).

**Theorem 4.3:** Every continuous map is (gsp)**-continuous.

**Proof:** Let \( f: (X, \tau) \to (Y, \sigma) \) be a continuous map. Let \( F \) be a closed set in \( (Y, \sigma) \). Since \( f \) is continuous, \( f^{-1}(F) \) is closed in \( (X, \tau) \). Therefore \( f \) is (gsp)**-continuous.

**Theorem 4.4:** Every (gsp)**-continuous map is (1) gs-continuous (2) \( \alpha g \)-continuous (3) gsp-continuous (4) rg-continuous (5) gp-continuous (6) gpr-continuous and (7) wg-continuous but not conversely.

**Proof:** Let \( f: (X, \tau) \to (Y, \sigma) \) be (gsp)**-continuous and let \( F \) be a closed set of \( (Y, \sigma) \). Since \( f \) is (gsp)**-continuous, \( f^{-1}(F) \) is (gsp)**-closed in \( (X, \tau) \), then \( f^{-1}(F) \) is gs- closed, \( \alpha g \)-closed, gsp- closed, gp- closed, gpr- closed and wg- closed. Hence \( f \) is gs-continuous, \( \alpha g \)-continuous, gsp-continuous, rg-continuous, gp-continuous, gpr-continuous and wg-continuous.

**Example 4.5:** Let \( X = \{a, b, c\} = Y, \tau = \{\varnothing, X, \{a\}, \{a, b\}\}, \sigma = \{\varnothing, Y, \{a, c\}\} \) let \( f: (X, \tau) \to (Y, \sigma) \) be the identity map. The closed sets of \( Y \) are \( \varnothing, Y, \{a, b\} \). \( f^{-1}(\{a, b\}) = \{a, b\} \) is gpr- closed but not (gsp)**-closed and hence \( f \) is gpr-continuous but not (gsp)**-continuous.

**Example 4.6:** Let \( X = \{a, b, c\} = Y, \tau = \{\varnothing, X, \{a\}, \{a, b\}\}, \sigma = \{\varnothing, Y, \{b\}\} \) let \( f: (X, \tau) \to (Y, \sigma) \) be defined as \( f(a) = c, f(b) = a, f(c) = b \). \( f^{-1}(\{a, c\}) = \{a, b\} \) is gpr- closed in \( (X, \tau) \), but not (gsp)**-closed in \( (X, \tau) \). Hence \( f \) is gpr-continuous but not (gsp)**-continuous.

**Example 4.7:** Let \( X = \{a, b, c\} = Y, \tau = \{\varnothing, X, \{a\}, \{a, b\}\}, \sigma = \{\varnothing, Y, \{c\}\} \) let \( f: (X, \tau) \to (Y, \sigma) \) be the identity map. The closed sets of \( Y \) are \( \varnothing, Y, \{a, b\} \). \( f^{-1}(\{a, b\}) = \{a, b\} \) is rg- closed but not (gsp)**-closed and hence \( f \) is rg-continuous but not (gsp)**-continuous.

**Theorem 4.8:** Every (gsp)**-irresolute is (gsp)**-continuous.

**Proof:** Let \( f: (X, \tau) \to (Y, \sigma) \) be a (gsp)**-irresolute. Let \( V \) be a closed set of \( (Y, \sigma) \). Then \( V \) is (gsp)**-closed and \( f^{-1}(V) \) is (gsp)**-closed since \( f \) is a (gsp)**-irresolute. Hence \( f \) is (gsp)**-continuous.
Theorem 4.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $(gsp)^{**}$-irresolute then $f$ is

1. gs-continuous
2. $\alpha g$-continuous
3. gsp-continuous
4. rg-continuous
5. gp-continuous
6. gpr-continuous and
7. wg-continuous but not conversely.

Proof: Since every $(gsp)^{**}$-irresolute is $(gsp)^{**}$-continuous, $f$ is $(gsp)^{**}$-continuous. Then by theorem 4.4 the result follows.

Example 4.10: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a, f(b) = c, f(c) = b$. $f^{-1}(\{c\}) = \{b\}$ is gs- closed in $(X, \tau)$ and hence $f$ is gs-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{b\})$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.11: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a, f(b) = c, f(c) = b$. $f^{-1}(\{c\}) = \{b\}$ is gp- closed in $(X, \tau)$ and hence $f$ is gp-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{b\})$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.12: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = b, f(b) = c, f(c) = a$. $f^{-1}(\{c\}) = \{b\}$ is rg- closed in $(X, \tau)$ and hence $f$ is rg-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{c\}) = \{b\}$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.13: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. $f^{-1}(\{c\}) = \{c\}$ is gsp- closed in $(X, \tau)$ and hence $f$ is gsp-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{c\}) = \{c\}$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.14: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = a, f(b) = c, f(c) = b$. $f^{-1}(\{c\}) = \{b\}$ is wg- closed in $(X, \tau)$ and hence $f$ is wg-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{c\}) = \{b\}$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.15: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = b, f(b) = c, f(c) = a$. $f^{-1}(\{c\}) = \{b\}$ is $\alpha g$-closed in $(X, \tau)$ and hence $f$ is $\alpha g$-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{c\}) = \{b\}$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

Example 4.16: Let $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a, b\}, \sigma = \{\varphi, Y, \{a, b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a) = b, f(b) = c, f(c) = a$. $f^{-1}(\{c\}) = \{b\}$ is gpr- closed in $(X, \tau)$ and hence $f$ is gpr-continuous. $(gsp)^{**}$-closed sets of $(Y, \sigma)$ are $\varphi, Y, \{c\}, \{a, c\}$ and $\{b, c\}$. $f^{-1}(\{c\}) = \{b\}$ is not $(gsp)^{**}$-closed set in $(X, \tau)$. Hence $f$ is not a $(gsp)^{**}$-irresolute.

The above results can be represented in the following figure.

Where $A \rightarrow B$ represents $A$ implies $B$ and $B$ need not imply $A.$
5. APPLICATIONS OF (gsp)**-CLOSED SETS

Definition 5.1: A space \((X,\tau)\) is called a \(T^{gsp}\)-space if every (gsp)**-closed set is closed.

Definition 5.2: A space \((X,\tau)\) is called a \(aT^{gsp}\)-space if every \(\alpha g\)-closed set is (gsp)**-closed.

Theorem 5.3: Every \(T_{0}\)-space is \(T^{gsp}\)-space but not conversely.

Proof: Let \((X,\tau)\) be a \(T_{0}\)-space. Let \(A\) be a (gsp)**-closed set. Since every (gsp)**-closed set is gs-closed and hence \(A\) is gs-closed. Since \((X,\tau)\) is a \(T_{0}\)-space, \(A\) is closed. Hence \((X,\tau)\) is \(aT^{gsp}\)-space.

Example 5.4: Let \(X = \{a, b, c\}\) and \(\tau = \{(\varphi, X,\{a\},\{b\},\{a, b\}\}\}. \((X,\tau)\) is a \(T^{gsp}\)-space \(A = \{a\}\) is gs-closed, but it is not closed, and hence it is not a \(T_{0}\)-space. Hence a \(T^{gsp}\)-space need not be a \(T_{0}\)-space.

Theorem 5.5: Every \(T_{b}\)-space is a \(aT^{gsp}\)-space.

Proof: Let \((X,\tau)\) be a \(T_{b}\)-space. Let \(A\) be \(\alpha g\) -closed. Then \(A\) is gs-closed. Since the space is \(T_{b}\)-space, \(A\) is closed and hence \(A\) is (gsp)**-closed. Therefore the space \((X,\tau)\) is a \(aT^{gsp}\)-space.

Example 5.6: Let \(X = \{a, b, c\}\) and \(\tau = \{(\varphi, X,\{a\},\{b\},\{a, b\}\}\}. Here (gsp)**-closed sets are \((\varphi, X,\{a\},\{b\},\{a, c\}\})\) and \(\alpha g\)-closed sets are \((\varphi, X,\{a\},\{b\},\{a, c\}\})\) and the gs-closed sets are \((\varphi, X,\{a\},\{b\},\{c\}\})\). Since every \(\alpha g\)-closed set is (gsp)**-closed the space \((X,\tau)\) is a \(aT^{gsp}\)-space. \(A = \{a\}\) is gs-closed but it is not closed, and hence it is not a \(T_{0}\)-space.

Theorem 5.7: Let \(f: (X,\tau) \to (Y,\sigma)\) be (gsp)**-continuous map and let \((X,\tau)\) be \(aT^{gsp}\)-space then \(f\) is continuous.

Proof: Let \(f: (X,\tau) \to (Y,\sigma)\) be (gsp)**-continuous map. Let \(F\) be a closed set of \((Y,\sigma)\). Since \(f\) is (gsp)**-continuous, \(f^{-1}(F)\) is (gsp)**-closed set in \((X,\tau)\). Since \((X,\tau)\) is a \(T^{gsp}\)-space, \(f^{-1}(F)\) is closed in \((X,\tau)\). Therefore \(f\) is continuous.

Theorem 5.8: Let \(f: (X,\tau) \to (Y,\sigma)\) be \(\alpha g\) -continuous map where \((X,\tau)\) is a \(aT^{gsp}\)-space. Then \(f\) is (gsp)**-continuous.

Proof: Let \(f: (X,\tau) \to (Y,\sigma)\) be an \(\alpha g\) -continuous map. Let \(F\) be a closed set of \((Y,\sigma)\). Since \(f\) is \(\alpha g\)-continuous, \(f^{-1}(F)\) is \(\alpha g\)-closed set in \((X,\tau)\). Since \((X,\tau)\) is a \(aT^{gsp}\)-space, \(f^{-1}(F)\) is (gsp)**-closed in \((X,\tau)\). Therefore \(f\) is (gsp)**-continuous.

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