ANTI $\Omega$-FUZZY SUBBIGROUP OF A BIGROUP

T. JUSTIN PRABU*
Department of Mathematics,
Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.

K. ARJUNAN
Department of Mathematics,
H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti $\Omega$-fuzzy subbigroup of a bigroup.

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INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [10], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [2] and Gyu Ihn Chae, Young Sik Park and Chul Hwan Park [3] have introduced and defined a new algebraic structure called $\Omega$-bifuzzy subsemigroup. After that A. Solairaju, R. Nagarajan [7, 8, 9] and K. Arjunan, Selvak Kumaraen [6] extend the theory to many algebraic structure. N. Palaniappan & K. Arjunan [4] defined a new algebraic structure called anti fuzzy ideal. In this paper, we introduce the some theorems in anti $\Omega$-fuzzy subbigroup of a bigroup.

1. PRELIMINARIES

1.1 Definition: A set $(G, +, \cdot)$ with two binary operations $+$ and $\cdot$ is called a bigroup if there exist two proper subsets $G_1$ and $G_2$ of $G$ such that

(i) $G = G_1 \cup G_2$
(ii) $(G_1, +)$ is a group
(iii) $(G_2, \cdot)$ is a group.

1.2 Definition: Let $X$ be a non–empty set. A fuzzy subset $A$ of $X$ is a function $A: X \rightarrow [0, 1]$.

1.3 Definition: Let $G = (G_1 \cup G_2, +, \cdot)$ be a bigroup. Then a fuzzy set $A$ of $G$ is said to be a fuzzy subbigroup of $G$ if there exist two fuzzy subsets $A_1$ of $G_1$ and $A_2$ of $G_2$ such that

(i) $A = A_1 \cup A_2$
(ii) $A_1$ is a fuzzy subgroup of $(G_1, +)$
(iii) $A_2$ is a fuzzy subgroup of $(G_2, \cdot)$.

1.4 Definition: Let $G = (G_1 \cup G_2, +, \cdot)$ be a bigroup and $\Omega$ be a nonempty set. The fuzzy subset $A: G \times \Omega \rightarrow [0, 1]$ of $G$ is said to be a $\Omega$-fuzzy subbigroup of $G$ if there exist two fuzzy subsets $A_1$ of $G_1$, $A_2$ of $G_2$, $A_3$ of $G_3$ such that

(i) $A = A_1 \cup A_2$
(ii) $A_1$ is a $\Omega$-fuzzy subgroup of $(G_1, +)$
(iii) $A_2$ is a $\Omega$-fuzzy subgroup of $(G_2, \cdot)$.

Corresponding Author: T. Justin Prabu*, Department of Mathematics, Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.
1.5 Definition: Let $G = (G_1 \cup G_2, +, \cdot)$ be a bigroup and $\Omega$ be a nonempty set. The fuzzy subset $A_G: G \times \Omega \rightarrow [0, 1]$ of $G$ is said to be a anti $\Omega$-fuzzy subbigroup of $G$ if there exist two fuzzy subsets $A_1: G_1 \times \Omega \rightarrow [0, 1]$ of $G_1$ and $A_2: G_2 \times \Omega \rightarrow [0, 1]$ of $G_2$ such that

(i) $A = A_1 \cup A_2$
(ii) $A_1$ is an anti $\Omega$-fuzzy subgroup of $(G_1, +)$
(iii) $A_2$ is an anti $\Omega$-fuzzy subgroup of $(G_2, \cdot)$.

2. PROPERTIES

2.1 Theorem: If $A = M \cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E \cup F$, then $\mu_M(x, q) = \mu_N(x, q)$ for all $x, e$ in $E$ and $q$ in $\Omega$.

Proof: Let $x, e$ in $E$ and $x, e'$ in $F$ and $q$ in $\Omega$.

Let $x, y$ and $e'$ in $F$ and $q$ in $\Omega$.

Therefore $\mu_M(x, y, q) = \mu_N(x, y, q)$ for all $x, e'$ in $F$ and $q$ in $\Omega$.

2.2 Theorem: If $A = M \cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $\mu_M(x, y, q) = \mu_N(e, q)$ gives $\mu_M(x, q) = \mu_N(y, q)$ for all $x, y$ and $e$ in $E$ and $q$ in $\Omega$.
(ii) $\mu_M(x, y, q) = \mu_N(e', q)$ gives $\mu_M(x, q) = \mu_N(y, q)$ for all $x, y$ and $e'$ in $F$ and $q$ in $\Omega$.

Proof:

(i) Let $x, y$ and $e$ in $E$ and $q$ in $\Omega$. Then $\mu_M(x, y, q) = \mu_N(x, y, q)$ is $\mu_M(x, y, q)$.

{[...]}
Proof:
(i) Let $x$ and $y$ belong to $E$ and $q$ in $\Omega$. Then $\mu_B(x, q) = \mu_B(x+y, q) \leq \max \{ \mu_B(x, q), \mu_B(y, q) \} = \max \{ \mu_B(x, q), \mu_B(y, q) \}$. Therefore $\mu_B(x, q) = \mu_B(y, q)$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$.

(ii) Let $x$ and $y$ belong to $F$ and $q$ in $\Omega$. Then $\mu_B(x, q) = \mu_B(x+y, q) \leq \max \{ \mu_B(x, q), \mu_B(y, q) \} = \max \{ \mu_B(x, q), \mu_B(y, q) \}$. Therefore $\mu_B(x, q) = \mu_B(y, q)$ for all $x$ and $y$ in $F$ and $q$ in $\Omega$.

2.6 Theorem: If $A = M \cup N$ is an anti $\Omega$-fuzzy subgroup of a bigroup $G = E \cup F$, then
(i) $\mu_A(x+y, q) = \max \{ \mu_A(x, q), \mu_A(y, q) \}$ for each $x$ and $y$ in $E$ and $q$ in $\Omega$ with $\mu_A(x, q) \neq \mu_A(y, q)$.
(ii) $\mu_A(xy, q) = \max \{ \mu_A(x, q), \mu_A(y, q) \}$ for each $x$ and $y$ in $F$ and $q$ in $\Omega$ with $\mu_A(x, q) \neq \mu_A(y, q)$.

Proof:
(i) Let $x$ and $y$ belong to $E$ and $q$ in $\Omega$. Assume that $\mu_B(x, q) < \mu_B(y, q)$, then $\mu_b(y, q) = \mu_B(-x+y, q) \leq \max \{ \mu_B(x, q), \mu_B(-x+y, q) \} = \max \{ \mu_B(x, q), \mu_B(-x+y, q) \}$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$.

(ii) Let $x$ and $y$ belong to $F$ and $q$ in $\Omega$. Assume that $\mu_B(x, q) < \mu_B(y, q)$, then $\mu_b(y, q) = \mu_B(x+y, q) \leq \max \{ \mu_B(x, q), \mu_B(x+y, q) \} = \max \{ \mu_B(x, q), \mu_B(x+y, q) \}$ for all $x$ and $y$ in $F$ and $q$ in $\Omega$.

2.7 Theorem: If $A = M \cup N$ and $B = O \cup P$ are two anti $\Omega$-fuzzy subbigroups of a bigroup $G = E \cup F$, then their union $A \cup B$ is an anti $\Omega$-fuzzy subgroup of $G$.

Proof: Let $A = M \cup N = \{ \{x, q), \mu_A(x, q) \} / x \in E$ and $q \in \Omega \}$ and $N = \{ \{x, q), \mu_N(x, q) \} / x \in F$ and $q \in \Omega \}$ and $B = O \cup P = \{ \{x, q), \mu_P(x, q) \} / x \in G$ and $q \in \Omega \}$ where $O = \{ \{x, q), \mu_O(x, q) \} / x \in F$ and $q \in \Omega \}$ and $P = \{ \{x, q), \mu_P(x, q) \} / x \in E$ and $q \in \Omega \}$. Let $C = A \cup B = R \cup S$ where $C = \{ \{x, q), \mu_C(x, q) \} / x \in G$ and $q \in \Omega \}$. Let $x$ and $y$ belong to $E$ and $q$ in $\Omega$. Then $\mu_C(x+y, q) = \max \{ \mu_C(x+y, q), \mu_C(x, y) \} = \max \{ \mu_C(x+y, q), \mu_C(x, y) \}$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$.

2.8 Theorem: The union of a family of anti $\Omega$-fuzzy subbigroups of a bigroup $G$ is an anti $\Omega$-fuzzy subbigroup of $G$.

Proof: It is trivial.

2.9 Theorem: If $A = M \cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E \cup F$, then
(i) $\mu_A(x+y, q) = \mu_A(x, y, q)$ if and only if $\mu_A(x, q) = \mu_A(-y+x+y, q)$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$.
(ii) $\mu_A(xy, q) = \mu_A(y, x, q)$ if and only if $\mu_A(x, q) = \mu_A(y^2, x, q)$ for all $x$ and $y$ in $F$ and $q$ in $\Omega$.

Proof:
(i) Let $x$ and $y$ be in $E$ and $q$ in $\Omega$. Assume that $\mu_A(x+y, q) = \mu_A(x, y, q)$, then $\mu_A(y, x, q) = \mu_A(x, y, q)$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$. Conversely assume that $\mu_A(x, q) = \mu_A(-y+x+y, q)$, then $\mu_A(x+y, q) = \mu_A(x, y, q)$ for all $x$ and $y$ in $E$ and $q$ in $\Omega$.

(ii) Let $x$ and $y$ be in $F$ and $q$ in $\Omega$. Assume that $\mu_A(x+y, q) = \mu_A(y, x, q)$, then $\mu_A(x^2, q) = \mu_A(x, x, q)$ for all $x$ and $y$ in $F$ and $q$ in $\Omega$. Conversely, assume that $\mu_A(x, q) = \mu_A(-y^2+x+y, q)$, then $\mu_A(x+y, q) = \mu_A(x, x, q)$ for all $x$ and $y$ in $F$ and $q$ in $\Omega$.

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