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A NEW FORM OF CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the notion of $I_{\pi gb}$-closed sets in ideal topological spaces and obtain their characterizations. Further, we discuss the continuity and irresoluteness via $I_{\pi gb}$-closed sets.

Keywords: *$I_{\pi gb}$ -closed, $I_{\pi gb}$ -open, $I_{\pi gb}$ -continuous, $I_{\pi gb}$ - $T_{1/2}$ space.*

Mathematics Subject Classification: *54A05, 54A10.*

I. INTRODUCTION

The notion of generalized open sets in a topological space called b-open sets was introduced by Andrijevic [3]. Jankovic and Hamlet [16] introduced the notion of I-open sets in topological spaces. The concept of ideals gained importance by the paper of Vaidyanathaswamy [29]. Navaneethakrishnan *et.al* [22, 23] has introduced regular g-closed sets and g-closed sets in ideal topological spaces. The class of b-open sets is contained in the class of semi-open and pre open sets. The class of generalized semi-closed, generalized semi-pre-open sets were discussed in [4, 7]. With advent of these notions, several research papers with interesting results came to existence [1, 3, 10, 11]

The aim of this paper is to study the notion of $I_{\pi gb}$ -closed sets and obtain their characterizations. In section 3, we study basic properties of $I_{\pi gb}$ -closed sets. In section 4, we characterize $I_{\pi gb}$ -open sets. Finally in section 5, $I_{\pi gb}$ -continuous and $I_{\pi gb}$ -irresolute functions are studied.

II. PRELIMINARIES

An ideal on a set X is a nonempty collection of subsets of X which satisfies
(i) $A \in I$ and $B \subseteq A$ implies $B \in I$,
(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. If I is an ideal on X, then (X,τ, I) is called an ideal topological space. For an ideal space (X,τ,I) and $A \subseteq X$, $A^*(I, \tau) = \{x \in X : U \cap A \notin I$ for every open set U containing x $\}$ is called the local function [18] of A with respect to I and τ .We simply write A* instead of A*(I, τ) in case there is no confusion. For every ideal topological space (X, τ, I),there exists a topology τ*(I), finer than τ generated by $\beta(I,\tau) = \{U - J : U \in \tau \text{ and } J \in I\}$. A subset A of an ideal topological space (X, τ, I) is said to be τ*-closed [16] or simply ∗-closed (resp.∗-perfect in itself [14]) if $A^* \subseteq A$ (resp.A=A*). A Kuratowski closure operator cl*( ) for a topology τ*(I,τ) called the ∗-topology defined by $cl^*(A) = A \cup A^*(X,\tau)$.

Throughout this paper (X,τ, I) and (Y,σ,I) represent topological spaces on which no separation axioms are assumed unless explicitly stated. The closure and interior of a subset A of a space (X,τ, I) will be denoted by cl(A) and int(A) .We can replace (X,τ,I) by X to avoid the chance of confusion.

Definition 2.1: A subset A of a space X is called
i) regular open set [24] if A= int(cl(A))
ii) b-open[2] or sp-open[6] or γ-open[5] if $A \subseteq cl(int(A)) \cup int(cl(A))$

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The complement of b-open (regular open) is called b-closed(regular closed).The finite union(intersection) of regular open set is called π-open(π-closed). If A is a subset of a space (X,τ,I) then the b-I-closure of A[20], denoted by $cl^*_b(A)$ is the smallest b-I-closed set containing A; the b-I-interior of A[20], denoted by $int_b I(A)$, is the largest b-I-open set contained in A.

The family of all b-open (resp. α-open, semi open, pre open, b-closed, pre closed) subsets of a space X is denoted by ($\alpha O(X)$,$SO(X)$,$PO(X)$,$bC(X)$,$PC(X)$).

Definition 2.2: A subset A of a space X is called

i) g-closed [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
ii) gb-closed [9, 13] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
iii) gp-closed [21] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
iv) πg-closed [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.
v) $\pi g\alpha$-closed [15] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.
vi) πgb-closed [28] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

The complement of g-closed (gb-closed, gp-closed, πg-closed, $\pi g\alpha$-closed, πgb-closed) is called g-open (gb-open, gp-open, πg-open, $\pi g\alpha$-open, πgb-open) respectively.

Definition 2.3: [15] A function $f: (X,\tau) \to (Y,\sigma)$ is π-open map if f(F) is **π-open map** in Y for every π-open in X.

Definition 2.4: [27] Let (X,τ) be a topological space, then a set $A \subseteq (X,\tau)$ is said to be **Q-set** if $int(cl(A)) = cl(int(A))$.

Definition 2.5: A subset A of an ideal space(X,τ, I) is said to be

i) I-open [19] if $A \subseteq int(A^*)$
ii) I_g-closed [8] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
i) $I_{\pi g}$-closed [26] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X,τ).

Definition 2.6: [12] A subset A of an ideal space(X,τ, I) is said t be

i) pre-I-closed if $cl^*(int(A)) \subseteq A$;
ii) semi-I-closed if $int(cl^*(A)) \subseteq A$;
iii) α-I-closed if $cl^*(int(cl^*(A))) \subseteq A$;
iv) b-I-closed if $cl^*(int(A) \cap int(cl^*(A)) \subseteq A$.

Definition 2.7: A function $f: (X,\tau, I) \to (Y,\sigma, I)$ is called

i) bI-irresolute [17] if for each bI-open set V in Y, $f^{-1}(V)$ is bI-open in X.
ii) bI-continuous [12] if for each open set V in Y, $f^{-1}(V)$ is bI-open in X.

III. $I_{\pi gb}$ -CLOSED SETS

Definition 3.1: A subset A of (X,τ,I) is called $I_{\pi gb}$-closed set if $bIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X,τ). By $I\pi GBC(X)$ we mean the family of all $I_{\pi gb}$ -closed subsets of the space(X,τ, I).

Example 3.2: Consider $X=\{a, b, c\}$,$\tau=\{X, \varphi,\{a\},\{b\}, \{a, b\}\}$ and $I=\{\varphi ,\{b\}\}$.Let $A=\{a, c\}$,then A is $I_{\pi gb}$-closed set.

Theorem 3.3:

1. Every closed set is $I_{\pi gb}$-closed set.
2. Every I-closed set is $I_{\pi gb}$-closed set.
3. Every g-closed is $I_{\pi gb}$-closed set.
4. Every πg-closed is $I_{\pi gb}$-closed set.
5. Every gb-closed is $I_{\pi gb}$-closed set.
6. Every gp-closed is $I_{\pi gb}$-closed set.
7. Every $\pi g\alpha$-closed is $I_{\pi gb}$-closed set.
8. Every πgb-closed set is $I_{\pi gb}$-closed set.
9. Every pI-closed set is $I_{\pi gb}$-closed set.
10. Every sI-closed set is $I_{\pi gb}$-closed set.
11. Every αI-closed set is $I_{\pi gb}$-closed set.
12. Every $*$-closed set is $I_{\pi gb}$-closed set.

Proof: Straight forward. Converse of the above need not be true as seen in the following examples.

Example 3.4: Consider X={a, b, c},τ={X, φ,{a},{b, c}} and I={φ ,{c}}. Let A={b},then A is $I_{\pi gb}$-closed set but not closed, I-closed..

Example 3.5: Let X={a, b, c},τ ={X, φ,{a},{c},{a, c}} and I={φ,{c}}. Let A={c}, then A is $I_{\pi gb}$-closed set but not g-closed, πg-closed set.

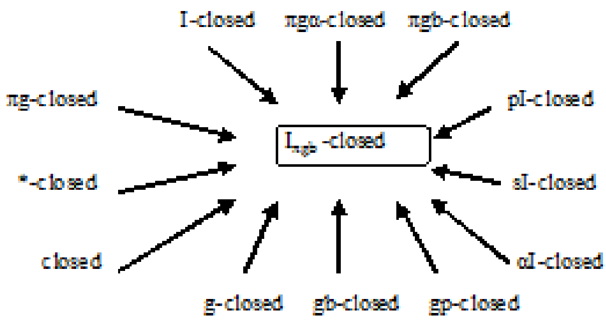
Example 3.6: Let X={a, b, c, d},τ ={X, φ,{a},{d},{a, d},{c, d},{a, c, d}} and I={φ, {d}}.Let A={a, c},then A is $I_{\pi gb}$-closed set but not gb-closed, gp-closed, πgα-closed and not πgb-closed set.

Example 3.7: Let X={a, b, c},τ ={X, φ,{a},{b},{a, b}} and I={φ,{b}}. Let A={a},then A is $I_{\pi gb}$-closed set but not pI-closed set.

Example 3.8: Consider X={a, b, c, d, e},τ ={X, φ,{a},{e},{a, e},{c, d},{a, c, d},{c, d, e},{a, c, d, e},{b, c, d, e}} and I={φ,{b},{e},{b, e}}.Let A={a, b, c, e},then A is $I_{\pi gb}$-closed set but not sI-closed set.

Example 3.9: Let X={a, b, c},τ ={X, φ,{a},{b, c}} and I={φ,{c}}.Let A={a, b},then A is $I_{\pi gb}$-closed set but not αI -closed set, ∗-closed.

Remark 3.10: The above discussions are shown in the figure below.



Theorem 3.11: If A is π-open and $I_{\pi gb}$-closed set, then A is bI-closed

Proof: Let A be π-open and $I_{\pi gb}$-closed set. Since A ⊆ A and A is π-open we have A is $I_{\pi gb}$-closed, bIcl(A) ⊆ A. Then A= bIcl(A).Hence A is bI-closed.

Theorem 3.12: If A is $I_{\pi gb}$-closed in (X,τ,I),then bIcl(A) – A does not contain any non empty π-closed set.

Proof: Let F be a non empty π-closed set such that F ⊆ bIcl(A) –A. Since A is $I_{\pi gb}$-closed, A ⊆ X – F where X – F is π-open implies bIcl(A) ⊆ X – F. Hence F ⊆ X – bIcl(A).Now F ⊆ bIcl(A) ∩ (X – bIcl(A)) implies F= φ which is a contradiction. Therefore bIcl(A) does not contain any non empty π-closed set.

Corollary 3.13: Let A be $I_{\pi gb}$ -closed in (X,τ,I).Then A is bI-closed if and only if bIcl(A) – A is π-closed.

Proof:
Necessity: Let A be bI-closed, then bIcl(A)=A. This implies bIcl(A) – A= ϕ which is π-closed.

Sufficiency: Assume bIcl(A)-A is π-closed. Then bIcl(A)-A=ϕ .Hence bIcl(A)=A implies A is bI-closed.

Remark 3.14: Finite Union of $I_{\pi gb}$-closed sets need not be $I_{\pi gb}$-closed.

Example 3.15: Consider X={a, b, c},τ={X, φ,{a},{c},{a, c}} and I={φ, {c}}. Let A={a},B={c}.Here A and B are $I_{\pi gb}$-closed set but A ∪ B = {a, c} is not $I_{\pi gb}$-closed set.

Remark 3.16: Finite Intersection of $I_{\pi gb}$-closed set need not be $I_{\pi gb}$-closed.

Example 3.17: Let X={a, b, c, d, e}, τ ={X, φ,{a},{e},{a, e},{c, d},{a, c, d},{c, d, e},{a, c, d, e},{b, c, d, e}} and I={φ,{b},{e},{b, e}}.Let A={b, c, e},B={a, c, d, e} are $I_{\pi gb}$ -closed set but A ∩ B={c,e} is not $I_{\pi gb}$-closed set.

Theorem 3.18: If A is $I_{\pi gb}$-closed and B is any set such that $A \subseteq B \subseteq bIcl(A)$,then B is $I_{\pi gb}$-closed set.

Proof: Let $B \subseteq U$ and U be π-open. Given $A \subseteq B$. Then $A \subseteq U$. Since A is $I_{\pi gb}$-closed, $A \subseteq U$ implies $bIcl(A) \subseteq U$. By assumption it follows that $bIcl(B) \subseteq bIcl(A) \subseteq U$. Hence B is $I_{\pi gb}$-closed.

IV. $I_{\pi gb}$ -OPEN SETS

Definition 4.1: A set $A \subseteq X$ is called **$I_{\pi gb}$-open** if its complement is $I_{\pi gb}$ -closed.

Remark 4.2: $bIcl(X - A) = X - bIint(A)$.

By IπGBO(X) we mean the family of all $I_{\pi gb}$-open subsets of the space(X, τ, I).

Theorem 4.3: A set $A \subseteq X$ is $I_{\pi gb}$-open if and only if $F \subseteq bI\text{-}int(A)$ whenever F is π-closed and $F \subseteq A$.

Proof:
Necessity: Let A be a $I_{\pi gb}$-open. Let F be a closed set and $F \subseteq A$, then $X - A \subseteq X - F$ where $X - F$ is π-open. By assumption, $bIcl(X - A) \subseteq X - F$. By remark 4.2, $X - bI\ int(A) \subseteq X - F$. Thus $F \subseteq bI\ int(A)$.

Sufficiency: Suppose F is π-closed and $F \subseteq A$ such that $F \subseteq bI\ int(A)$. Let $X - A \subseteq U$ where U is π-open. Then $X - U \subseteq A$ where $X - U$ is π-closed. By hypothesis, $X - U \subseteq bI\ int(A)$ implies $X - bI\ int(A) \subseteq U$ implies $bIcl(X - A) \subseteq U$. Thus X–A is $I_{\pi gb}$- closed and A is $I_{\pi gb}$-open.

Theorem 4.4: If $bI\ int(A) \subseteq B \subseteq A$ and A is $I_{\pi gb}$-open ,then B is $I_{\pi gb}$-open.

Proof: Let $bI\ int(A) \subseteq B \subseteq A$. Thus $X\text{-}A \subseteq X\text{-}B \subseteq bIcl(X\text{-}A)$.Since X-A is $I_{\pi gb}$-closed, By theorem 3.18, $(X\text{-}A) \subseteq (X\text{-}B) \subseteq bIcl(A)$ implies (X-B) is $I_{\pi gb}$-closed. Hence B is $I_{\pi gb}$-open.

Remark 4.5: For any $A \subseteq X$, $bI\ int(bIcl(A)\text{-}A) = \varphi$.

Example 4.6: Let X = {a, b, c, d}, τ = {φ, X, {a},{d},{a, d},{c, d},{a, c, d}},I={φ, {d}}. Let A = {a, d} be any subset of X. bIcl{a, d} –{a, d} ={b, c}.Then bI int [bIcl{a, d}-{a, d}]= bI int {b, c}= φ.

Theorem 4.7: If $A \subseteq X$ is $I_{\pi gb}$-closed, then bIcl(A)-A is $I_{\pi gb}$-open.

Proof: Let A be $I_{\pi gb}$-closed.Let F be π-closed. $F \subseteq bIcl(A)\text{-}A$ by theorem 3.12,F= φ. By remark 4.5, bI int(bIcl(A)-A)= φ. Thus $F \subseteq bI\ int(bIcl(A)\text{-}A)$.Thus bIcl(A)-A is $I_{\pi gb}$-open.

Corollary 4.8: Let A be a π-open, $I_{\pi gb}$-closed set. Then $A \cap F$ is $I_{\pi gb}$ -closed whenever $F \in bIcl(X)$.

Proof: Since A is $I_{\pi gb}$-closed and π-open, $bIcl(A) \subseteq A$ and thus A is bI-closed. Hence $A \cap F$ is bI-closed in X which implies $A \cap F$ is $I_{\pi gb}$-closed set in X.

Lemma 4.9[25]: Let $A \subseteq X$. If A is open or dense, then $\pi O(A, \tau/A) = V \cap A$ such that $V \in \pi O(X, \tau)$.

Theorem 4.10: Let $B \subseteq A \subseteq X$ where A is $I_{\pi gb}$-closed and π-open set. Then B is $I_{\pi gb}$- closed relative to A iff B is $I_{\pi gb}$-closed in X.

Proof: Let $B \subseteq A \subseteq X$, where A is $I_{\pi gb}$-closed and π -open set. Let B be $I_{\pi gb}$ -closed in A. Let $B \subseteq U$ where U is π –open in X. Since $B \subseteq A$, $B = B \cap A \subseteq U \cap A$, this implies $bIcl(B) = bIcl_A(B) \subseteq U \cap A \subseteq U$. Hence, B is $I_{\pi gb}$-closed in X.Let B be $I_{\pi gb}$- closed in X. Let $B \subseteq O$ where O is π -open in A. Then $O = U \cap A$ where U is π -open in X. This implies $B \subseteq O = U \cap A \subseteq U$. Since B is $I_{\pi gb}$-closed in X, $bIcl(B) \subseteq U$. Thus $bIcl_A(B) = A \cap bIcl(B) \subseteq U \cap A = O$. Hence, B is $I_{\pi gb}$ – closed relative to A.

Definition 4.11: A space (X,τ, I) is called an $I_{\pi gb}$-$T_{1/2}$ space if every $I_{\pi gb}$-closed is bI-closed.

Theorem 4.12:
i) $BIO(\tau) \subseteq I\pi GBO(\tau)$.
ii) A space (X,τ,I) is $I_{\pi gb}$- $T_{1/2}$ space if and only if $BIO(\tau) = I\pi GBO(\tau)$.

Proof: i) Let A be a bI-open, then X-A is bI-closed. So X-A is $I_{\pi gb}$ -closed. Thus A is $I_{\pi gb}$ -open. Hence $BIO(\tau) \subseteq I\pi GBO(\tau)$.

ii) **Necessity**: Let (X,τ,I) be $I_{\pi gb}$-$T_{1/2}$ space. Let $A \in I\pi GBO(\tau)$. Then X-A is $I_{\pi gb}$-closed. By hypothesis, X-A is bI-closed, thus $A \in BIO(\tau)$.Hence $BIO(\tau) = I\pi GBO(\tau)$. **Sufficiency**: Let $BIO(\tau) = I\pi GBO(\tau)$. Let A be $I_{\pi gb}$-closed. Then X-A is $I_{\pi gb}$-open. We have X-A $\in I\pi GBO(\tau)$ implies X-A $\in BIO(\tau)$. Hence A is bI-closed this implies (X,τ,I) is $I_{\pi gb}$ - $T_{1/2}$ space..

Theorem 4.13: For an ideal topological space (X,τ, I), the following are equivalent.

i) X is $I_{\pi gb}$ - $T_{1/2}$ space.
ii) Every singleton set is either π-closed or bI-open.

Proof:
(i) ⇒ (ii): Let X be a $I_{\pi gb}$ - $T_{1/2}$ space. Let $x \in X$ and assume that $\{x\}$ is not π-closed. Then clearly X-$\{x\}$ is trivially $I_{\pi gb}$-closed. Since X is $I_{\pi gb}$ - $T_{1/2}$ space, X-$\{x\}$ is bI-closed or $\{x\}$ is bI-open.

(ii) ⇒(i): Assume every singleton of X is either π-closed or bI-open. Let A be a $I_{\pi gb}$-closed set. Let $\{x\} \in bIcl(A)$.

Case-(i): Let $\{x\}$ be π-closed. Suppose$\{x\}$ does not belong to A, then $\{x\} \in bIcl(A)$-A by theorem 3.12, $\{x\} \in A$. Hence $bIcl(A) \subseteq A$.

Case-(ii): Let $\{x\}$ be bI-open. Since $\{x\} \in bIcl(A)$,we have $\{x\} \cap A \neq \phi$ implies$\{x\} \in A$. Therefore $bIcl(A) \subseteq A$ and A is bI-closed.

V. $I_{\pi gb}$ -CONTINUOUS and $I_{\pi gb}$ -IRRESOLUTE FUNCTIONS

Definition 5.1: A function f: $(X,\tau, I) \to (Y,\sigma)$ is called $I_{\pi gb}$-continuous if every $f^{-1}(V)$ is $I_{\pi gb}$-closed in (X,τ, I) for every closed set V of (Y,σ).

Definition 5.2: A function f: $(X,\tau, I) \to (Y,\sigma,I)$ is called $I_{\pi gb}$-irresolute if every $f^{-1}(V)$ is $I_{\pi gb}$-closed in (X,τ, I) for $I_{\pi gb}$-closed set V in (Y,σ,I).

Theorem 5.3: Every $I_{\pi gb}$–irresolute is $I_{\pi gb}$–continuous function.

Proof: Let f: $(X,\tau,I) \to (Y,\sigma,I)$ be $I_{\pi gb}$- continuous and V be $I_{\pi gb}$– closed in (Y,σ,I).But every $I_{\pi gb}$– closed sets need not be closed in (Y,σ, I).So there exists some sets which is not closed in (Y,σ).By definition, there exists some sets which are not $I_{\pi gb}$– closed in (X,τ,I) which implies f is not $I_{\pi gb}$–irresolute.

Example 5.4: Let $X=\{a, b, c\}$,$\tau =\{X, \varphi,\{a\},\{c\},\{a, c\}\}$,$\sigma =\{X, \varphi,\{a\},\{a, c\},\{a, b\}\}$ and $I=\{\varphi,\{c\}\}$. Let f: $(X,\tau,I) \to (X,\sigma,I)$ be the identity function, then f is $I_{\pi gb}$–continuous function but not $I_{\pi gb}$–irresolute.

Remark 5.5: Composition of two $I_{\pi gb}$– continuous need not be $I_{\pi gb}$–continuous.

Example 5.6: Let $X=\{a, b, c, d\}$,$\tau =\{X, \varphi,\{b\},\{c\},\{b, c\}\}$,$\sigma =\{X, \varphi,\{a, b, d\}\}$,$\eta=\{X, \varphi,\{a,d\}\}$ and $I=\{\varphi,\{b\}\}$.Define f: $(X,\tau,I) \to (X,\sigma)$ by f(a) = a, f(b) = c, f(c) = b, f(d) = d. Define g : $(X,\sigma,I) \to (X, \eta)$ by g(a) = d, g(b) = c, g(c) = b, g(d) = a. Then f and g are $I_{\pi gb}$– continuous but $g\circ f$ is not $I_{\pi gb}$– continuous.

Remark 5.7:

1. Every continuous function implies *-continuous function.
2. Every *-continuous function implies πgb-continuous function.
3. (3)Every πgb-continuous function implies $I_{\pi gb}$ - continuous function.

Definition 5.8: A function f : $(X,\tau,I) \to (Y,\sigma,I)$ is said to be pre-bI-closed if f(U) is bI-closed in Y for each bI-closed set in X.

Proposition 5.9: Let f : $(X,\tau,I) \to (Y,\sigma,I)$ be π-irresolute and pre-bI-closed map. Then f(A) is $I_{\pi gb}$–closed in Y for every $I_{\pi gb}$–closed set A of X.

Proof: Let A be $I_{\pi gb}$- closed in X. Let f(A) ⊆ V where V is π-open in Y. Then A ⊆ $f^{-1}(V)$ and A is $I_{\pi gb}$–closed in X implies bIcl(A) ⊆ $f^{-1}(V)$.Hence f(bIcl(A)) ⊆ V. Since f is pre-bI-closed, bIcl(f(A)) ⊆ bIcl(f(bIcl(A))) = f(bIcl(A)) ⊆ V. Hence f(A) is $I_{\pi gb}$– closed in Y.

Theorem 5.10: Let (X,τ, I) be a topological space if A ⊆ X is nowhere dense ,then A is $I_{\pi gb}$– closed.

Proof: Let A ⊆ U where U is π-open in X. Since A is nowhere dense, int(cl(A)) = φ.

Now bIcl(A) ⊆ cl(A) ⊆ int(cl(A)) = φ ⊆ U. Therefore A is $I_{\pi gb}$– closed in X.

Theorem 5.11: If an ideal topological space (X,τ, I) for each x ∈X,X\{x} is either $I_{\pi gb}$–closed or π-open in X.

Proof: Suppose X\{x} is not π-open, then X is the only π-open containing X\{x}.Hence bIcl(X\{x}) ⊆ X implies X\{x} is $I_{\pi gb}$–closed.

Definition 5.12: The intersection of all $I_{\pi gb}$-closed set containing A is called $I_{\pi gb}$–closure of A is denoted by $I_{\pi gb}$–cl(A).

Theorem 5.13: Let A⊆(X, τ, I) and x∈X. Then x ∈ $I_{\pi gb}$ –cl(A) if and only if V∩ A ≠ φ for every $I_{\pi gb}$– open V containing x.

Proof: Suppose x ∈ $I_{\pi gb}$–cl(A) and let V be an $I_{\pi gb}$ – open such that x ∈ V. Assume V ∩ A ≠ φ, then A ⊆ X\V implies $I_{\pi gb}$–cl(A) ⊆ X\V which implies x ∈ X\V ,thus V ∩ A ≠ φ for every $I_{\pi gb}$– open set V containing x. Conversely, suppose x ∉ $I_{\pi gb}$–cl(A) which implies x ∈X\$I_{\pi gb}$–cl(A) = V(say).Then V is $I_{\pi gb}$– open & x ∈ V. Also since A ⊆ $I_{\pi gb}$–cl(A),A ⊄ V implies V ∩ A = φ.Hence the proof.

Definition 5.14: An ideal topological space X is a $I_{\pi gb}$-space if every $I_{\pi gb}$-closed set is I-closed.

Theorem 5.15: If f: X → Y is π-open, bI-irresolute, pre bI-closed surjective function, if X is $I_{\pi gb}$ -$T_{1/2}$ space, then Y is $I_{\pi gb}$-$T_{1/2}$ space.

Proof: Let F be a $I_{\pi gb}$– closed set in Y. Let $f^{-1}(F)$ ⊆ U where U is π-open in X. Then F ⊆ f(U) and F is a $I_{\pi gb}$– closed in Y implies bIcl(F) ⊆ f(U).Since f is bI-irresolute, bIcl($f^{-1}(F)$) ⊆ bIcl(f^{-1}(bIcl(F))) = f^{-1}(bIcl(F)) ⊆ U. Therefore $f^{-1}(F)$ is $I_{\pi gb}$–closed in X.Since X is $I_{\pi gb}$-$T_{1/2}$ space,$f^{-1}(F)$ is bI-closed in X.Since f is pre-bI-closed, f($f^{-1}(F)$) = F is bI-closed in Y. Hence Y is $I_{\pi gb}$-$T_{1/2}$ space.

Proposition 5.16: Every $I_{\pi gb}$-space is $I_{\pi gb}$-$T_{1/2}$ space.

Proof: Let X be $I_{\pi gb}$-space, then every $I_{\pi gb}$-closed set is I-closed which implies (X,τ, I) is $I_{\pi gb}$- $T_{1/2}$ space.

Theorem 5.17: For an ideal topological space (X,τ, I),the following are equivalent.

(i) X is $I_{\pi gb}$ - $T_{1/2}$ space.
(ii) For every subset A ⊆ X,A is $I_{\pi gb}$ – open if and only if A is bI-open.

Proof:
(i) ⇒ (ii): Let A ⊆ X be $I_{\pi gb}$– open.Then (X-A) is $I_{\pi gb}$– closed and by (i),(X-A) is bI-closed implies A is bI-open.

Conversely, assume A is bI-open. Then (X-A) is bI-closed. As every bI-closed set is $I_{\pi gb}$– closed, (X-A) is $I_{\pi gb}$– closed implies A is $I_{\pi gb}$ – open.

(ii) ⇒ (i): Let A be $I_{\pi gb}$– closed set in X. Then (X-A) is $I_{\pi gb}$– open. Hence by (ii), (X-A) is bI-open implies A is bI-closed. Hence X is $I_{\pi gb}$- $T_{1/2}$ space.

Theorem 5.18: Let f: (X,τ,I) → (Y,σ,I) be a function.

(i) If f is $I_{\pi gb}$ -irresolute and X is $I_{\pi gb}$ - $T_{1/2}$ space, then f is bI-irresolute.
(ii) If f is $I_{\pi gb}$ -continuous and X is $I_{\pi gb}$ - $T_{1/2}$ space, then f is bI-continuous.

Proof:

(i) Let V be bI-closed in Y. Since f is $I_{\pi gb}$- irresolute, $f^{-1}(V)$ is $I_{\pi gb}$-closed in X. Since X is $I_{\pi gb}$- $T_{1/2}$ space, $f^{-1}(V)$ is bI-closed in X. Hence f is bI-irresolute.

(ii) Let V be bI-closed in Y. Since f is $I_{\pi gb}$-continuous, $f^{-1}(V)$ is $I_{\pi gb}$-closed in X. By assumption it is bI-closed in X. Hence f is bI-continuous.

Theorem 5.19: If the bijective f: $(X,\tau, I) \rightarrow (Y,\sigma,I)$ is bI-irresolute and π-open map, then f is $I_{\pi gb}$- irresolute.

Proof: Let V be $I_{\pi gb}$-closed in Y. Let $f^{-1}(V) \subseteq U$ where U is π-open in X. Hence $V \subseteq f(U)$ and f(U) is π-open implies $bIcl(V) \subseteq f(U)$. Since f is bI-irresolute, $f^{-1}(bIcl(V))$ is bI-closed.

Hence $bIcl(f^{-1}(V)) \subseteq bIcl(f^{-1}(bIclV)) = f^{-1}(bIcl(V)) \subseteq U$. Therefore f is $I_{\pi gb}$-irresolute.

Theorem 5.20: For a set $A \subseteq (X,\tau, I)$ if A is π-clopen, then A is Q-set, $I_{\pi gb}$–closed.

Proof: Let A be π-clopen, then A is both π-open and π-closed. Hence A is both open and closed. Therefore $cl(int(A)) = int(cl(A))$ which shows that A is Q-set. Also $bIcl(A) \subseteq cl(A) =A$, hence A is $I_{\pi gb}$–closed.

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