b-H-OPEN SETS AND DECOMPOSITION OF CONTINUITY VIA HEREDITARY CLASSES

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ABSTRACT

In this paper, we introduce and study the notions of b-H-open sets and (bH, λ)-continuity in hereditary generalized topological spaces. We also find the decomposition of (µ, λ)-continuity and (σH, λ)-continuity.

Keywords: generalized topology, Hereditary classes and b-H-open.

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1. INTRODUCTION AND PRELIMINARIES

A family µ of subsets of X is called a generalized topology (GT)[1], if ∅ ∈ µ and µ is closed under arbitrary union. The generalized topology µ is said to be strong [7], if X ∈ µ. A hereditary class H of X is a non-empty collection of subsets of X such that A ⊆ B, B ∈ H implies A ∈ H [2]. If µ is a GT on X and A ⊆ X, x ∈ X then x ∈ Anµ[2] iff x ∈ M ∈ µ ⇒ M ∩ A /∈ H. A function f: (X, µ, H) → (Y, λ) is called (µ, λ)-continuous [1] (resp. (αH, λ)-continuous [9], (πH, λ)-continuous [9], (σH, λ)-continuous [9], (δH, λ)-continuous [8]) if the inverse image of each λ-open set in Y is µ-open (resp. α-H-open, π-H-open, σ-H-open, δ-H-open).

Definition 1.1: A subset A of a hereditary generalized topological space (X, µ, H) is said to be

(a) α - H -open [2] if A ⊆ iµ (cµ (iµ (A))),
(b) β - H -open [2] if A ⊆ cµ (iµ (cµ (A))),
(c) σ - H -open [2] if A ⊆ cµ (iµ (A)),
(d) π - H -open [2] if A ⊆ iµ (cµ (A)),
(e) δ - H -open [2] if iµ (cµ (A)) ⊆ cµ (iµ (A)),
(f) strong β - H -open [2] if A ⊆ cµ (iµ (cµ (A))),
(g) S - H -set [10] if iµ (A) = cµ (iµ (A)),
(h) t - H -set [10] if iµ (cµ (A)) = iµ (A),
(i) BH -set [10] if A = U ∩ V, where U ∈ µ and V is t - H -set.

Lemma 1.2: [6] Let (X, µ, H) be a hereditary generalized topological space and A, B be subsets of X. Then the following holds:

(a) If A ⊆ B, then A'⊆ B'.
(b) If G ∈ µ, then G ∩ A'⊆ (G ∩ A)'.
(c) A' = cµ (A') ⊆ cµ (A).

2. b-H-OPEN SETS

Definition 2.1: A subset A of a hereditary generalized topological space (X, µ, H) is said to be b - H-open, if A ⊆ iµ (cµ (A)) U cµ (iµ (A)).

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Proposition 2.2: Let $(X, \mu, H)$ be a hereditary generalized topological space. Then the following holds:

(a) Every $\sigma$ - $H$-open set is $b$ - $H$-open.
(b) Every $\pi$ - $H$-open set is $b$ - $H$-open.
(c) Every $b$ - $H$-open set is strong $\beta$ - $H$-open.
(d) Every $b$ - $H$-open set is $\beta$ - $H$-open.

Proof:

(a) Let $A$ be $\sigma$ - $H$-open. Then $A \subseteq c_\mu^\ast (i_\mu (A)) \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A))$. Hence $A$ is $b$ - $H$-open.

(b) Let $A$ be $\pi$ - $H$-open. Then $A \subseteq i_\mu (c_\mu^\ast (A)) \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A))$. Hence $A$ is $b$ - $H$-open.

(c) Let $A$ be $b$-open.

Then $A \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A)) \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A)) = c_\mu^\ast (i_\mu (A)) \cup c_\mu^\ast (i_\mu (A)) = c_\mu^\ast (i_\mu (A))$. Hence $A$ is strong $\beta$ - $H$-open.

(d) Let $A$ be $b$ - $H$-open.

Then $A \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A)) \subseteq c_\mu^\ast (i_\mu (A)) \cup c_\mu^\ast (i_\mu (A)) = c_\mu^\ast (i_\mu (A)) \cup c_\mu^\ast (i_\mu (A)) = c_\mu^\ast (i_\mu (A))$. Hence $A$ is $\beta$ - $H$-open.

Remark 2.3: The following examples show that the converse of Proposition 2.2 need not be true.

Example 2.4: Let $X = \{a, b, c, d\}$, $\mu = \emptyset$, $\{b, c\}, \{b, c, d\}$, and $H = \emptyset$. Then

1. $A = \{a\}$ is $b$ - $H$-open set but not $\sigma$ - $H$-open.
2. $A = \{a, b\}$ is $b$ - $H$-open set but not $\pi$ - $H$-open.
3. $A = \{a, c\}$ is $b$ - $H$-open but not $b$ - $H$-open.

Theorem 2.5: Let $(X, \mu, H)$ be a hereditary generalized topological space and $A \subseteq X$. If $A$ is $b$ - $H$-open and $S$ - $H$-set, then $A$ is $\pi$ - $H$-open.

Proof: If $A$ is $S$ - $H$-set, then $i_\mu (A) = c_\mu^\ast (i_\mu (A))$. Since $A$ is $b$ - $H$-open, then $A \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A)) = i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A)) = i_\mu (A) \cup c_\mu^\ast (i_\mu (A)) = i_\mu (A) \sqcup i_\mu (A) = i_\mu (A) \sqcup i_\mu (A) = i_\mu (c_\mu^\ast (A))$. Hence $A$ is $\pi$ - $H$-open.

Theorem 2.6: Let $(X, \mu, H)$ be a hereditary generalized topological space and $A \subseteq X$. If $A$ is $b$ - $H$-open and $t$ - $H$-set, then $A$ is $\sigma$ - $H$-open.

Proof: If $A$ is $t$ - $H$-set, then $i_\mu (A) = c_\mu (i_\mu (A))$. Since $A$ is $b$ - $H$-open, then $A \subseteq i_\mu (c_\mu (A)) \cup c_\mu (i_\mu (A)) = i_\mu (c_\mu (A)) \cup c_\mu (i_\mu (A)) = i_\mu (A) \cup c_\mu (i_\mu (A)) = i_\mu (A) \cup (i_\mu (A))^* = i_\mu (A) \cup (i_\mu (A))^* = c_\mu^\ast (i_\mu (A))$. Hence $A$ is $\sigma$ - $H$-open.

Proposition 2.7: Let $(X, \mu, H)$ be a hereditary generalized topological space. Then the following are equivalent.

(a) Every $\beta$ - $H$-open set is $\sigma$ - $H$-open.
(b) Every $\beta$ - $H$-open set is $\sigma$ - $H$-open.
(c) Every $\pi$ - $H$-open set is $\sigma$ - $H$-open.

Proof: It follows from Proposition 2.2.

Proposition 2.8: Let $(X, \mu, H)$ be a hereditary generalized topological space. Then arbitrary union of $b$ - $H$-open sets is $b$ - $H$-open.

Proof: Let $U_a$ be $b$ - $H$-open for $a \in A$, we have $U_a \subseteq i_\mu (c_\mu^\ast (U_a)) \cup c_\mu^\ast (i_\mu (U_a))$. Then Lemma 1.2, we have $U_a \subseteq i_\mu (c_\mu^\ast (U_a)) \cup c_\mu^\ast (i_\mu (U_a)) = U_a \subseteq (U_a \cup i_\mu (U_a))^* \cup i_\mu (U_a \cup U_a)^* = U_a \cup U_a = U_a$. Hence $U_a$ is $b$ - $H$-open.

Proposition 2.9: Let $(X, \mu, H)$ be a hereditary generalized topological space and $A$, $B$ be subsets of $X$. If $A$ is $b$ - $H$-open and $B$ is $\mu$ - $H$-open, then $A \cap B$ is $b$ - $H$-open.

Proof: If $A$ is $b$ - $H$-open, then $A \subseteq i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A))$ and $A \cap B \subseteq (i_\mu (c_\mu^\ast (A)) \cup c_\mu^\ast (i_\mu (A))) \cap B = (i_\mu (c_\mu^\ast (A)) \cap B) \cup (c_\mu^\ast (i_\mu (A)) \cap B) = (i_\mu (A \cup A^*) \cap B) \cup ((i_\mu (A))^* \cap B) = (i_\mu (A \cup A^*) \cap B) \cup ((i_\mu (A))^* \cap B) = (i_\mu (A) \cap B) \cup ((i_\mu (A))^* \cap B) = (i_\mu (A) \cap B) \cup ((i_\mu (A))^* \cap B) = i_\mu (A \cap B) \cup (i_\mu (A))^* \cap B = i_\mu (A \cap B) \cup i_\mu (A \cap B) = i_\mu (A \cap B) \cup i_\mu (A \cap B) = i_\mu (A \cap B)$. Hence $A \cap B$ is $b$ - $H$-open.
Remark 2.10: The following examples show that the intersection of two \( b - H \)-open sets need not be \( b - H \)-open.

Example 2.11: Let \( X = \{a, b, c, d\} \), \( \mathcal{M} = \{\emptyset, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\} \) and \( H = \{\emptyset, \{a\}, \{b\}\} \).

Consider \( A = \{a, b\} \) and \( B = \{a, c, d\} \) are \( b - H \)-open sets, but \( A \cap B = \{b\} \) is not \( b - H \)-open set.

Proposition 2.12: Let \( (X, \mu, H) \) be a hereditary generalized topological space and \( A \subseteq X \). Then the following are equivalent:

(a) \( A \) is \( \sigma - H \)-open.
(b) \( A \) is both \( b - H \)-open and \( \delta - H \)-open.

Proof:

(a) \( \Rightarrow \) (b): If \( A \) is \( \sigma - H \)-open, then \( A \subseteq c_{\mu}(i_{\mu}(A)) \). Now \( i_{\mu}(c_{\mu}(A)) \subseteq c_{\mu}(i_{\mu}(A)) \subseteq c_{\mu}(i_{\mu}(A)) \). Hence \( A \) is \( \delta - H \)-open. Obviously \( A \) is \( b - H \)-open.

(b) \( \Rightarrow \) (a): If \( A \) is \( b - H \)-open and \( \delta - H \)-open, then \( A \subseteq i_{\mu}(c_{\mu}(A)) \cup c_{\mu}(i_{\mu}(A)) \) and \( i_{\mu}(c_{\mu}(A)) \subseteq c_{\mu}(i_{\mu}(A)) \), therefore \( A \subseteq c_{\mu}(i_{\mu}(A)) \). Hence \( A \) is \( \sigma - H \)-open.

Remark 2.13: The following example shows that the notions \( b - H \)-open and \( \delta - H \)-open are independent.

Example 2.14: Let \( X = \{a, b, c, d\} \), \( \mathcal{M} = \{\emptyset, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\} \) and \( H = \{\emptyset, \{a\}, \{b\}\} \). Then

(a) \( A = \{c\} \) is \( b - H \)-open but not \( \delta - H \)-open.
(b) \( A = \{a\} \) is \( \delta - H \)-open but not \( b - H \)-open.

Proposition 2.15: Let \( (X, \mu, H) \) be a hereditary generalized topological space and \( x \in X \). Then \( \{x\} \) is \( \mu \)-open if and only if \( \{x\} \) is \( \sigma - H \)-open.

Proof:

Let \( \{x\} \) be a \( \mu \)-open. Then \( \{x\} = i_{\mu}(\{x\}) \subseteq c_{\mu}(i_{\mu}(\{x\})) \). Hence \( \{x\} \) is \( \sigma - H \)-open. Conversely, assume that \( \{x\} \) is \( \sigma - H \)-open. Then \( \{x\} \subseteq c_{\mu}(i_{\mu}(\{x\})) \). Now \( i_{\mu}(\{x\}) \) is either \( \{x\} \) or \( \emptyset \). We have \( c_{\mu}(\emptyset) = \emptyset \), but \( \{x\} \subseteq c_{\mu}(i_{\mu}(\{x\})) \), so \( i_{\mu}(\{x\}) \neq \emptyset \). Hence \( i_{\mu}(\{x\}) = \{x\} \). Thus \( \{x\} \) is \( \mu \)-open.

Lemma 2.16: Let \( (X, \mu, H) \) be a hereditary generalized topological space, \( A \subseteq X \) and \( U \in \mu \). If \( A \cap U = \emptyset \), then \( c_{\mu}(A) \cap U = \emptyset \).

Proposition 2.17: Let \( (X, \mu, H) \) be a hereditary generalized topological space and \( x \in X \). Then the following are equivalent:

(a) \( \{x\} \) is \( \pi - H \)-open.
(b) \( \{x\} \) is \( b - H \)-open.
(c) \( \{x\} \) is strong \( \beta - H \)-open.

Proof:

(a) \( \Rightarrow \) (b) and (b) \( \Rightarrow \) (c) follows from Proposition 2.2.

(c) \( \Rightarrow \) (a): Assume that \( \{x\} \) is strong \( \beta - H \)-open and \( \{x\} \) is not \( \pi - H \)-open. Then \( \{x\} \not\subseteq i_{\mu}(c_{\mu}(\{x\})) \), that is, \( \{x\} \cap i_{\mu}(c_{\mu}(\{x\})) = \emptyset \). We have \( i_{\mu}(c_{\mu}(\{x\})) \) is \( \mu \)-open, it follows from Lemma 2.15, \( c_{\mu}(\{x\}) \cap i_{\mu}(c_{\mu}(\{x\})) = \emptyset \) and thus \( i_{\mu}(c_{\mu}(\{x\})) = \emptyset \). Therefore \( c_{\mu}(i_{\mu}(c_{\mu}(\{x\}))) = \emptyset \). But \( \{x\} \) is strong \( \beta - H \)-open, a contradiction. Hence \( \{x\} \) is \( \pi - H \)-open.

Proposition 2.18: Let \( (X, \mu, H) \) be a hereditary generalized topological space and \( x \in X \). Then the following are equivalent:

(a) \( \{x\} \) is \( \pi - H \)-open.
(b) \( \{x\} \) is \( b - H \)-open.
(c) \( \{x\} \) is strong \( \beta - H \)-open.

Proof:

(a) \( \Rightarrow \) (b) and (b) \( \Rightarrow \) (c) follows from Proposition 2.2.

(c) \( \Rightarrow \) (a): Assume that \( \{x\} \) is strong \( \beta - H \)-open and \( \{x\} \) is not \( \pi - H \)-open. Then \( \{x\} \not\subseteq i_{\mu}(c_{\mu}(\{x\})) \), that is, \( \{x\} \cap i_{\mu}(c_{\mu}(\{x\})) = \emptyset \). We have \( i_{\mu}(c_{\mu}(\{x\})) \) is \( \mu \)-open, it follows from Lemma 2.15, \( c_{\mu}(\{x\}) \cap i_{\mu}(c_{\mu}(\{x\})) = \emptyset \) and thus \( i_{\mu}(c_{\mu}(\{x\})) = \emptyset \). Therefore \( c_{\mu}(i_{\mu}(c_{\mu}(\{x\}))) = \emptyset \). But \( \{x\} \) is strong \( \beta - H \)-open, a contradiction. Hence \( \{x\} \) is \( \pi - H \)-open.
Proposition 2.18: Let \((X, \mu, H)\) be a hereditary generalized topological space and \(A \subseteq X\) such that \((i_\mu(A^*)) \subseteq i_\mu(A^*)\). Then the following are equivalent:

(a) \(A \subseteq i_\mu(A^*)\).

(b) \(A\) is \(b\)-\(H\)-open and \(A \subseteq A^*\).

Proof:

(a) \(\Rightarrow\) (b): If \(A \subseteq i_\mu(A^*) \subseteq A^*\). Since \(A \subseteq i_\mu(A^*) \subseteq i_\mu(A^*) \cup i_\mu(A) \subseteq i_\mu(A \cup A) = i_\mu(c_\mu(A^*)) \subseteq i_\mu(c_\mu(A)) \cup c_\mu(i_\mu(A))\). Then \(A\) is \(b\)-\(H\)-open.

(b) \(\Rightarrow\) (a): If \(A\) is a \(b\)-\(H\)-open and \(A \subseteq A^*\), then \(A \subseteq i_\mu(c_\mu(A)) \subseteq i_\mu(c_\mu(A^*)) \cup i_\mu(A) = i_\mu(A \cup A^*) \cup (i_\mu(A) \cup (i_\mu(A))^*) \subseteq (i_\mu(A)^* \cup i_\mu(A)) \cup (i_\mu(A))^* = i_\mu(A^*) \cup (i_\mu(A))^* = i_\mu(A^*)\).

Definition 2.19: A subset \(A\) of a hereditary generalized topological space \((X, \mu, H)\) is said to be \(b\)-\(H\)-closed if its complement is \(b\)-\(H\)-open.

Theorem 2.20: Let \((X, \mu, H)\) be a hereditary generalized topological space and \(A \subseteq X\). If \(A\) is \(b\)-\(H\)-closed, then \(i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A)) \subseteq A\).

Proof: If \(A\) is \(b\)-\(H\)-closed, then \(X - A\) is \(b\)-\(H\)-open. We have \(X - A \subseteq c_\mu(i_\mu(X - A)) \cup i_\mu(c_\mu(X - A)) \subseteq c_\mu(i_\mu(A)) \cup i_\mu(c_\mu(A)) = (X - (i_\mu(c_\mu(A)))) \cup (X - (c_\mu(i_\mu(A)))) \subseteq (X - (i_\mu(c_\mu(A)))) \cup (X - (c_\mu(i_\mu(A)))) = X - (i_\mu(c_\mu(A)) \cup c_\mu(i_\mu(A)))\). Hence \(i_\mu(c_\mu(A)) \cup c_\mu(i_\mu(A)) \subseteq A\).

Remark 2.21: The following example shows that the converse of theorem 2.20 need not be true.

Example 2.22: Let \(X = \{a, b, c, d\}, \mu = \{\emptyset, \{a\}, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}\) and \(H = \{\emptyset, \{a\}, \{b\}\}\). Let \(A = \{c\}\), then \(i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A)) \subseteq A\) but \(A\) is not \(b\)-\(H\)-closed.

Corollary 2.23: Let \((X, \mu, H)\) be a hereditary generalized topological space and \(A \subseteq X\) such that \(X - i_\mu(c_\mu(A)) = c_\mu(i_\mu(X - A))\) and \(X - c_\mu(i_\mu(A)) = i_\mu(c_\mu(X - A))\). Then \(A\) is \(b\)-\(H\)-closed if and only if \(i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A)) \subseteq A\).

Proof: By theorem 2.20, if \(A\) is \(b\)-\(H\)-closed, then \(i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A)) \subseteq A\). Conversely, if \(i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A)) \subseteq A\), then \(X - A \subseteq X - (i_\mu(c_\mu(A)) \cap c_\mu(i_\mu(A))) \subseteq (X - i_\mu(c_\mu(A))) \cup (X - c_\mu(i_\mu(A))) = i_\mu(c_\mu(X - A)) \cup i_\mu(c_\mu(X - A)) = c_\mu(i_\mu(X - A)) \cup i_\mu(c_\mu(X - A)) = c_\mu(i_\mu(X - A)) \cup i_\mu(c_\mu(X - A))\). Therefore, \(X - A\) is \(b\)-\(H\)-open and hence \(A\) is \(b\)-\(H\)-closed.

Definition 2.24: A subset \(A\) of a hereditary generalized topological space \((X, \mu, H)\) is said to be strong \(B_{H}\)-set if \(A = U \cap V\), where \(U \in \mu\) and \(V\) is a \(t\)-\(H\)-set and \(i_\mu(c_\mu(V)) = c_\mu(i_\mu(V))\).

Proposition 2.25: Let \((X, \mu, H)\) be a hereditary generalized topological space and \(A \subseteq X\). If \(A\) is a strong \(B_{H}\)-set, then \(A\) is \(B_{H}\)-set.

Proof: Obvious.

Remark 2.26: The following example shows that the converse of proposition 2.25 need not be true.

Example 2.27: Let \(X = \{a, b, c, d\}, \mu = \{\emptyset, \{a\}, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}\) and \(H = \{\emptyset, \{a\}, \{b\}\}\). \(A = \{b\}\) is \(B_{H}\)-set but not strong \(B_{H}\)-set.

Proposition 2.28: Let \((X, \mu, H)\) be a strong hereditary generalized topological space and \(A \subseteq X\). Then the following are equivalent:

(a) \(A\) is \(\mu\)-open.

(b) \(A\) is \(b\)-\(H\)-open and a strong \(B_{H}\)-set.

Proof:

(a) \(\Rightarrow\) (b): Clearly every \(\mu\)-open set is \(b\)-\(H\)-open. Now every \(\mu\)-open set is strong \(B_{H}\)-set, because \(X\) is \(t\)-\(H\)-set and \(i_\mu(c_\mu(X)) = c_\mu(i_\mu(X))\).

(b) \(\Rightarrow\) (a): If \(A\) is \(b\)-\(H\)-open and strong \(B_{H}\)-set, then \(A \subseteq i_\mu(c_\mu(A)) \cup c_\mu(i_\mu(A)) = i_\mu(c_\mu(U \cap V)) \cup c_\mu(i_\mu(U \cap V))\), where \(U\) is \(\mu\)-open and \(V\) is \(t\)-\(H\)-set and \(i_\mu(c_\mu(V)) = c_\mu(i_\mu(V))\). Hence \(A \subseteq (i_\mu(c_\mu(U)) \cap i_\mu(c_\mu(V))) \cup (c_\mu(i_\mu(U)) \cap c_\mu(i_\mu(V))) \subseteq U \cap i_\mu(c_\mu(V)) \cup U \cap c_\mu(i_\mu(V)) = U \cap i_\mu(c_\mu(V)) = U \cap i_\mu(V) = i_\mu(U \cap V) = i_\mu(A)\). Hence \(A\) is \(\mu\)-open.

Remark 2.29: The following examples show that the notions of \(b\)-\(H\)-open and strong \(B_{H}\)-set are independent.
Example 2.30: Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$ and $H = \{\emptyset, \{a\}, \{b\}\}$.
1. $A = \{d\}$ is strong $B_H$-set but not $b$-H-open.
2. $A = \{a, b\}$ is $b$-H-open but not strong $B_H$-set.

3. DECOMPOSITION OF $(\mu, \lambda)$-CONTINUITY AND $(\sigma_{\mathcal{H}}, \lambda)$-CONTINUITY

Definition 3.1: A function $f: (X, \mu, H) \to (Y, \lambda)$ is called $(b_{\mathcal{H}}, \lambda)$-Continuous function, if the inverse image of each $\lambda$-open set in $Y$ is $b$-H-open in $X$.

Proposition 3.2: If a function $f: (X, \mu, H) \to (Y, \lambda)$ is either $(\sigma_{\mathcal{H}}, \lambda)$-continuous or $(\pi_{\mathcal{H}}, \lambda)$-continuous, then $f$ is $(b_{\mathcal{H}}, \lambda)$-continuous.

Proof: Obvious.

Remark 3.3: The following example shows that the converse of Proposition 3.2 need not be true.

Example 3.4: Let $X = Y = \{a, b, c, d\}$, $\mu = \lambda = \{\emptyset, \{a\}, \{c\}, \{b, d\}, \{a, c\}, \{a, b, c\}, X\}$ and $H = \{\emptyset, \{a\}, \{b\}\}$. Define $f: (X, \mu, H) \to (Y, \lambda)$ by $f(a) = a$, $f(b) = b$. Then $f$ is $(b_{\mathcal{H}}, \lambda)$-continuous but not strong $B_H$-continuous. In Example 3.8 $f$ is strong $B_H$-continuous but not $(b_{\mathcal{H}}, \lambda)$-continuous.

Theorem 3.6: If a function $f: (X, \mu, H) \to (Y, \lambda)$ is strong $B_H$-continuous, then $f$ is $B_H$-continuous.

Proof: It follows from Proposition 2.25.

Remark 3.7: The following example shows that the converse of Theorem 3.6 need not be true.

Example 3.8: Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, c\}\}$ and $H = \{\emptyset, \{a\}, \{b\}\}$. Also, let $Y = X$ and $\lambda = \{\emptyset, \{a\}, \{a, c\}\}$. Define $f: (X, \mu, H) \to (Y, \lambda)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then $f$ is strong $B_H$-continuous but not $B_H$-continuous.

Proposition 3.9: Let $(X, \mu, H)$ be a strong hereditary generalized topological space. For a function $f: (X, \mu, H) \to (Y, \lambda)$, the following are equivalent:
(a) $f$ is $(\mu, \lambda)$-continuous,
(b) $f$ is $(b_{\mathcal{H}}, \lambda)$-continuous and strong $B_H$-continuous.

Proof: This is an immediate consequence from Proposition 2.28.

Remark 3.10: The following examples show that the notions of $(b_{\mathcal{H}}, \lambda)$-continuous and $B_H$-continuous are independent.

Example 3.11: Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, X\}$ and $H = \{\emptyset, \{d\}\}$. Also, let $Y = \{a, b, c\}$ and $\lambda = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f: (X, \mu, H) \to (Y, \lambda)$ such that $f(a) = f(b) = a$, $f(c) = d = f(d)$. Then $f$ is $(b_{\mathcal{H}}, \lambda)$-continuous but not strong $B_H$-continuous. In Example 3.8 $f$ is strong $B_H$-continuous but not $(b_{\mathcal{H}}, \lambda)$-continuous.

Proposition 3.12: Let $(X, \mu, H)$ be a hereditary generalized topological space. For a function $f: (X, \mu, H) \to (Y, \lambda)$, the following are equivalent:
(a) $f$ is $(\sigma_{\mathcal{H}}, \lambda)$-continuous.
(b) $f$ is $(b_{\mathcal{H}}, \lambda)$-continuous and $(\delta_{\mathcal{H}}, \lambda)$-continuous.

Proof: This is an immediate consequence from Proposition 2.12.

Remark 3.13: The notions of $(\delta_{\mathcal{H}}, \lambda)$-continuous and $(b_{\mathcal{H}}, \lambda)$-continuous are independent as shown in the following examples.

Example 3.14: Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{b, d\}, \{a, b, c\}, X\}$ and $H = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Also let $Y = X$ and $\lambda = \{\emptyset, \{a\}, \{c\}, \{b, d\}, \{a, c\}, \{a, b, c\}, X\}$. Define function $f: (X, \mu, H) \to (Y, \lambda)$ such that $f(a) = a$, $f(b) = f(c) = b$. Then $f$ is $(\delta_{\mathcal{H}}, \lambda)$-continuous but neither $(b_{\mathcal{H}}, \lambda)$-continuous nor $(\sigma_{\mathcal{H}}, \lambda)$-continuous.
Example 3.15: Let \( X = \{a, b, c, d\} \), \( \mu = \{\emptyset, \{c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, X\} \) and \( H = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, d\}\} \). Let \( Y = \{a, b, c\} \), \( \lambda = \{\emptyset, \{a, c\}, \{b, c\}, \{a, b, c\}\} \). Define the identity function \( f: X \rightarrow Y \) is \((b_H, \lambda)\)-continuous but it is neither \((\delta_H, \lambda)\)-continuous nor \((\sigma_H, \lambda)\)-continuous.

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