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EXTERNAL EQUITABLE DOMINATION IN FUZZY GRAPHS

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ABSTRACT

Let G be a fuzzy graph. A subset D of V is called externally fuzzy equitable dominating set of G if D is a dominating set of G and for every $v_1, v_2 \in V - D$ such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$. for some $u \in D$, $||N(v_1) \cap D| - |N(v_2) \cap D|| \leq 1$. An externally fuzzy equitable dominating set is also called complementary fuzzy dominating set. The minimum cardinality of a minimal externally fuzzy equitable dominating set of G is called externally fuzzy equitable domination number of G and is denoted by $\gamma^{(ee)f}(G)$. In this paper we introduce the concept of externally fuzzy equitable dominating set. Also we obtain some interesting result for this new parameter in external equitable domination in fuzzy graphs.

Key words: *External fuzzy equitable dominating set, minimal externally fuzzy equitable dominating set*

AMS Subject Classification : *05C72.*

1. INTRODUCTION

L.A.Zadeh (1965) introduced the concepts of a fuzzy subset of a set as a way for representing uncertainty. His idea have been applied to a wide range of scientific areas.

Fuzzy concepts is also introduced in Graph theory. Formally, a fuzzy graph $G = (V, \sigma, \mu)$ is a non empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V, σ is called the fuzzy vertex set of G and μ is called the fuzzy edge set of G. The concept of equitable domination [11] in graphs was introduced by Venkatrasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam. The notation of domination in fuzzy graphs [10] was developed by A. Somasundaram and S.Somasundaram. In this paper we introduce the concept of externally fuzzy equitable dominating set and obtain some interesting result for these new parameter in externally equitable domination in fuzzy graphs.

PRELIMINARIES

Definition 1.1: A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions σ:V→ [0,1] and $\mu: V \times V \to [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 1.2: Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define $\sigma_1$on $V_1$by $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1$on the collection $E_1$of two element subset of V_1 by $\mu_1(u, v) = \mu(u, v)$ for all $u, v \in V_1$. Then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $< V_1 >$.

Definition 1.3: The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $deg(u)$.

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Definition 1.4: Let G be a fuzzy graph. Let u and v be two vertices of G. A subset D of V is called a fuzzy equitable dominating set if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$ and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{ef}.

Definition 1.5: A vertex $u \in V$ is said to be degree equitable fuzzy graph with a vertex $v \in V$ if $|deg(u) - deg(v)| \leq 1$ and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$.

Definition 1.6: If D is an fuzzy equitable dominating set then any super set of D is a fuzzy equitable dominating set.

Definition 1.7:: Let G be a fuzzy graph. Let u and v be two vertices of a fuzzy connected dominating set D is to be a fuzzy equitable connected dominating set if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$ where $deg(u)$ denotes degree of vertex u and $deg(v)$ denotes the degree of vertex v and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of fuzzy equitable connected domination set is denoted by γ_c^{ef}.

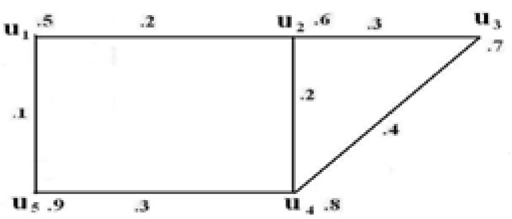
Definition 1.8: For every $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ also $deg(u) = deg(v) = r$ and in this case G is called regular fuzzy graph of degree r or a r-regular fuzzy graph.

Definition 1.9: A bipartite graph is a graph G whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge in G has one end vertex in V_1 and the other end vertex in V_2. (V_1, V_2) is called a bipartition of G.

Further if every vertex of V_1 is adjacent to every vertex of V_2, then G is called a complete bipartite graph. The complete bipartite graph with bipartition (V_1, V_2) such that $|V_1| = r$ and $|V_2| = s$ is denoted by $K_{r,s}$. $K_{1,r}$ is called a star.

2. EXTERNAL FUZZY EQUITABLE DOMINATION :

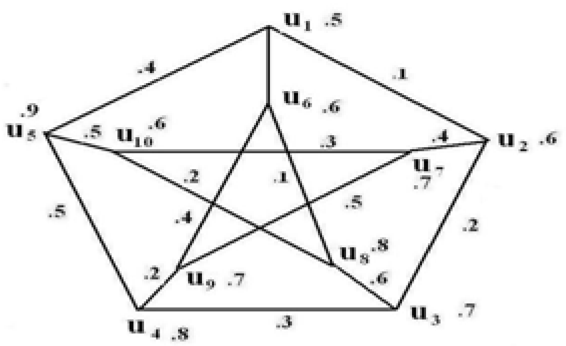
Definition 2.1: Let G be a fuzzy graph. A subset D of V is called externally fuzzy equitable dominating set of G if D is a dominating set of G and for every $v_1, v_2 \in V - D$ such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$ for some $u \in D$, $||N(v_1) \cap D| - |N(v_2) \cap D|| \leq 1$. An externally fuzzy equitable dominating set is also called complementary fuzzy dominating set. The minimum cardinality of a minimal externally fuzzy equitable dominating set of G is called externally fuzzy equitable domination number of G and is denoted by $\gamma^{(ee)f}(G)$.



$D = \{v_5, v_4\}$ is an externally fuzzy equitable dominating set and $\gamma^{(ee)f}(G) = 2$.

Theorem 2.1: An externally fuzzy equitable domination number of Petersen Graph (P) is four. i.e $\gamma^{(ee)f}(P) = 4$.

Proof:



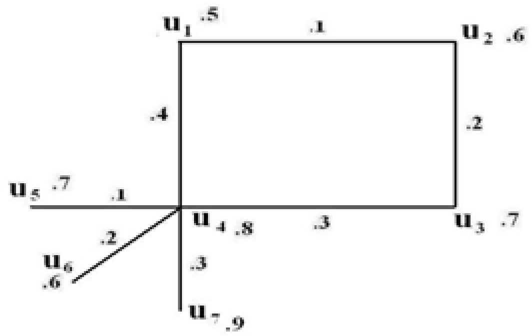
The minimum dominating set of the Petersen graphs are $S_1 = \{u_1, u_3, u_7\}$, $S_2 = \{u_1, u_4, u_{10}\}$, $S_3 = \{u_2, u_4, u_8\}$, $S_4 = \{u_2, u_5, u_6\}$, $S_5 = \{u_3, u_5, u_9\}$, $S_6 = \{u_6, u_7, u_4\}$, $S_7 = \{u_6, u_{10}, u_3\}$, $S_8 = \{u_7, u_8, u_5\}$, $S_9 = \{u_9, u_8, u_1\}$, $S_{10} = \{u_9, u_{10}, u_2\}$. None of these sets are externally fuzzy equitable. Therefore $\gamma^{(ee)f}(P) \geq 4$, But $\{u_1, u_6, u_8, u_9\}$ is an externally fuzzy equitable dominating set of P, $\gamma^{(ee)f}(P) \leq 4$ Therefore $\gamma^{(ee)f}(P) = 4$.

Theorem 2.2: Let G be a regular fuzzy graph. Then $\gamma^{f}(G) < \gamma^{(ee)f}(G)$.

Proof: Let G be a regular fuzzy graph. Then every vertex has the same degree say 3. $\gamma^{f}(G) = 3$ and $\gamma^{(ee)f}(G) = 4$. Therefore $\gamma^{f}(G) < \gamma^{(ee)f}(G)$.

Theorem 2.3: Let G be a fuzzy graph. Then the complement of a minimal externally fuzzy equitable dominating set need not be an externally fuzzy equitable dominating set.

Proof:



Let $D = \{u_2, u_4\}$ be a complement of a minimal externally fuzzy equitable dominating set $V - D = \{u_1, u_3, u_5, u_6, u_7\}$. This a dominating set but not externally fuzzy equitable dominating set.

Theorem 2.4: Let G be a connected fuzzy graph with at least three vertices. Then $\gamma^{(ee)f}(G) \leq n - 2$..

Proof: There exist two vertices in G say u_1, u_2 with equal degree. Let $S = V - \{u_1, u_2\}$. Since G has at least three vertices and G is connected. u_1 and u_2 are dominated by S. Therefore S is externally fuzzy equitable dominating set of G. Therefore $\gamma^{(ee)f}(G) \leq n - 2$..

Theorem 2.5: Let G be a connected regular fuzzy graph with at least four vertices, then $\gamma^{(ee)f}(G) \leq n - 3$ if $G \neq C_4$ and $\gamma^{(ee)f}(G) = n/2 = n - 2$ if $G = C_4$,

Proof: Suppose G is a connected regular fuzzy graph with at least four vertices.

Case (i): G contains a K_3 as a subgraph. Since $|V(G)| \geq 4$ and since G is regular, $D = V - \{u_1, u_2, u_3\}$ is an externally fuzzy equitable dominating set of G. Therefore $\gamma^{(ee)f}(G) \leq |D| = n - 3$.

Case (ii): G contains no K_3. G always contains a P_3. Let $\{u_1, u_2, u_3\}$ be the vertices of a P_3 with u_2 being adjacent with u_1 and u_3. If G is r-regular with $r \geq 3$. Then $V - \{u_1, u_2, u_3\}$ is an externally fuzzy equitable dominating set of G. Therefore $\gamma^{(ee)f}(G) \leq |D| = n - 3$.

Suppose G is 2-regular. Then G is a cycle say C_n. If $n \geq 5$ then $\gamma^{(ee)f}(G) \leq |D| = n - 3$. If $n = 4$, then $G = C_4$, $\gamma^{(ee)f}(G) = n/2 = n - 2$.

Theorem 2.6: Let G be a fuzzy graph. Then $\gamma^{(ee)f}(G) = n - 1$ if and only if $G = K_2 \cup \overline{K_{n-2}}$.

Proof: Let $G = K_2 \cup \overline{K_{n-2}}$ $(n \geq 2)$. Obviously $\gamma^{(ee)f}(G) = n - 1$. Conversely, suppose $\gamma^{(ee)f}(G) = n - 1$.If G is connected and has at least three vertices then $\gamma^{(ee)f}(G) \leq n - 2$.. Therefore G is either K_2 or a disconnected graph. Suppose $G \neq K_2$. Then G is disconnected with at least three vertices. Let $G_1, G_2, \ldots\ldots G_k$ be the components of G. Since $\gamma^{(ee)f}(G) = n - 1$ and since $\gamma^{(ee)f}(G) = \sum_{i=1}^{k} \gamma^{(ee)f}(G_i)$. None of the components can have three or more vertices. Therefore $|V(G_i)| \leq 2$ for every i, $1 \leq i \leq k$. If G_i and G_j $(i \neq j)$ have two vertices, then $\gamma^{(ee)f}(G) \leq n - 2$. If every G_i has exactly one vertex, then $\gamma^{(ee)f}(G) = n$. Therefore there exist exactly one G_i which has two vertices and all others have exactly one vertex. Therefore $G = K_2 \cup \overline{K_{n-2}}$ $(n \geq 2)$.

Theorem 2.7: Let G be a fuzzy graph with at least three vertices. Then $\gamma^{(ee)f}(G) = n - 2$. if and only if G is H or $H \cup \overline{K_{n-|H|}}$ where $H = C_4$ or P_4, P_3, K_4 or $2K_2$.

Proof: If G is any one of the fuzzy graph mentioned in the theorem, then $\gamma^{(ee)f}(G) = n - 2$.

Conversely suppose $\gamma^{(ee)f}(G) = n-2$. If G is connected and if G has at least 5 vertices, then $\gamma^{(ee)f}(G) \leq n-3$, a contradiction that either G is disconnected or G is connected and has at most 4 vertices.

If $|V(G)| = 4$, then $\gamma^{(ee)f}(G) = 2$ if and only if $G = P_4$ or C_4. If G is connected and $|V(G)| = 3$, $\gamma^{(ee)f}(G) = 2$ if and only if $G = P_3$ or K_3 .

Let G be disconnected. Let $G_1, G_2, \ldots\ldots G_k$ be the components of G $(k \geq 2)$. Suppose $|V(G_i)| = 4$ for some i. Since G_i is connected, $\gamma^{(ee)f}(G_i)$ cannot be 3. Also if $\gamma^{(ee)f}(G_i) = 1$ then $\gamma^{(ee)f}(G) = \sum_{i=1}^{k} \gamma^{(ee)f}(G_i) \leq n-3$, a contradiction. Therefore, $\gamma^{(ee)f}(G_i) = 2$. Therefore, $G = C_4 \cup \overline{K_{n-4}}$ or $G = P_4 \cup \overline{K_{n-4}}$.

If $|V(G_i)| = 3$, then $\gamma^{(ee)f}(G_i) = 1$. Therefore $\gamma^{(ee)f}(G) = \sum_{i=1}^{k} \gamma^{(ee)f}(G_i) \leq n-2$. Since $\gamma^{(ee)f}(G) = n-2$, we get $|V(G_j)| = 1$ for every $j \neq i$. Therefore, $G_3 = C_3 \cup \overline{K_{n-3}}$ or $G = P_3 \cup \overline{K_{n-3}}$.

Suppose $|V(G_i)| = 2$, then $\gamma^{(ee)f}(G_i) \leq n-1$. Since $\gamma^{(ee)f}(G) = n-2$ there exists a graph G_j such that G_j is connected and $\gamma^{(ee)f}(G_j) = |V(G_j)| - 1$. If H is connected and $|V(H)| \geq 3$ then $\gamma^{(ee)f}(H) \leq |V(H)| - 2$. There G_j has less than three vertices. If G_j is K_1 , then $\gamma^{(ee)f}(G_j) \neq |V(G_j)| - 1$. Therefore G_j is K_2 . $\gamma^{(ee)f}(G) = \sum_{i=1}^{k} \gamma^{(ee)f}(G_i) \leq n-2$. But $\gamma^{(ee)f}(G) = n-2$. Therefore $G_r = K_1$, for every r, $r \neq i, j$. Therefore $G = 2K_2 \cup \overline{K_{n-4}}$.

Theorem 2.8: $\gamma^{(ee)f}(G) = n-m$ if and only if G contains exactly $\gamma^{(ee)f}$ components, each of which is a star, $K_{1,n}$, $n \geq 0$

Proof: Suppose $G = K_{1,n_1} \cup K_{1,n_2} . \cup \ldots . \cup K_{1,n_r}$, then

$$\gamma^{(ee)f}(G) = \sum_{i=1}^{r} \gamma^{(ee)f}(K_{1,n_i}) = r.$$

$$= (n_1+1) + (n_2+1) + \ldots\ldots\ldots\ldots\ldots + (n_r+1) - (n_1+n_2+,,,,,,,,+n_r) = |V(G)| - |E(G)|.$$

Conversely, suppose $\gamma^{(ee)f}(G) = n-m$. Suppose G has t components, $t \neq \gamma^{(ee)f}$. Since any externally fuzzy equitable dominating set should have at least one vertex from each of the components $t \leq \gamma^{(ee)f}$. If $t < \gamma^{(ee)f}$, then the number of edges in G is $m(G) \geq n(G) - t \geq n(G) - \gamma^{(ee)f}$, a contradiction. Therefore $t \geq \gamma^{(ee)f}$. Hence $t = \gamma^{(ee)f}$. Suppose $G_1, G_2, \ldots\ldots G_r$ are the externally fuzzy equitable components of G. Suppose G_i is not a star. Since G_i is connected and not a star it follows that $diam(G_i) = 1$ or ≥ 3 or G_i has a cycle with $\gamma(G_i) = 1$. If $diam(G_i) = 1$ then G_i is complete and $n(G_i) < m(G_i)$. Therefore $n - m = \sum_{i=1}^{t} |n(G_i) - m(G_i)|$. since $n(G_i) - m(G_i)$ is negative there exists some j such that $n(G_j) - m(G_j) \geq 2$, contradiction. Therefore G_i is not complete for any i. Suppose G_i has diameter ≥ 3 since $t = \gamma^{(ee)f}$ $\gamma^{(ee)f}(G_i) = 1$ for every i, $1 \leq i \leq t$. Since G_i has diameter ≥ 3, $\gamma^{(ee)f}(G_i) > 1$, a contradiction. Suppose G_i has a cycle with $\gamma(G_i) = 1$. Then $m(G_i) \geq n(G_i)$. Therefore $n(G_i) - m(G_i) \leq 0$,

$$m(G) = m(G_j) + \sum_{j=1, j\neq i}^{t} |m(G_i)| \geq n(G_i) + \sum_{i=1, j\neq i}^{t} |n(G_i) - 1| = n(G) - (t-1).$$

Therefore $\gamma^{(ee)f} > t - 1 \geq n(G) - m(G)$, a contradiction. Therefore each G_i is a star.

3. CONCLUSION

In this paper we define new parameter called external equitable domination in fuzzy graphs. We can extend this concept to outdegree fuzzy equitable domination and study the characteristics of this new parameter.

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