SUBDIVISION OF SUPER GEOMETRIC MEAN LABELING
FOR TRIANGULAR SNAKE GRAPHS

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(Received On: 29-07-15; Revised & Accepted On: 22-08-15)

ABSTRACT

Let \( f: V(G) \rightarrow \{1,2,...,p+q\} \) be an injective function. For a vertex labeling \( "f" \), the induced edge labeling \( f^*(e=uv) \) is defined by, \( f^*(e) = \sqrt{f(u)f(v)} \) or \( \sqrt{f(u)+f(v)} \). Then \( "f" \) is called a “Super Geometric mean labeling” if \( \{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, ..., p+q\} \). A graph which admits Super Geometric mean labeling is called “Super Geometric mean graph”.

In this paper we prove that \( S[A(T_n)], S[D(T_n)], S[A(D(T_n))] \), Subdivision of triple Triangular snake \( S[T(T_n)] \) and Subdivision of alternate triple Triangular snake graphs \( S[A(T(T_n))] \) are Super Geometric mean graphs.

Key Words: Graph, Geometric mean graph, Super Geometric mean graph, Triangular snake, Double Triangular snake and Triple Triangular snake.

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected graph \( G=(V,E) \) with \( p \) vertices and \( q \) edges. For a detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2].

The concept of “Geometric mean labeling” has been introduced by S.Somasundaram, R. Ponraj and P. Vidhyarani in [6].

In this paper we investigate Super Geometric mean labeling behavior of \( S[A(T_n)], S[D(T_n)], S[A(D(T_n))] \), Subdivision of triple Triangular snake \( S[T(T_n)] \) and Subdivision of alternate triple Triangular snake \( S[A(T(T_n))] \).

We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition: 1.1 A graph \( G=(V,E) \) with \( p \) vertices and \( q \) edges is called a “Geometric mean graph” if it is possible to label the vertices \( x \in V \) with distinct labels \( f(x) \) from \( 1,2,...,q+1 \) in such a way that when each edge \( e=uv \) is labeled with, \( f(e=uv) = \sqrt{f(u)f(v)} \) or \( \sqrt{f(u)+f(v)} \) then the edge labels are distinct. In this case, “\( f \) ” is called a “Geometric mean labeling” of \( G \).

Definition: 1.2 Let \( f: V(G) \rightarrow \{1,2,...,p+q\} \) be an injective function. For a vertex labeling \( "f" \), the induced edge labeling \( f^*(e=uv) \) is defined by, \( f^*(e) = \sqrt{f(u)f(v)} \) or \( \sqrt{f(u)+f(v)} \). Then “\( f" \) is called a “Super Geometric mean labeling” if \( \{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, ..., p+q\} \). A graph which admits Super Geometric mean labeling is called “Super Geometric mean graph”.

Corresponding Author: 3B. Shiny*
S. S. Sandhya, E. Ebin Raja Merly, B. Shiny* / Subdivision of Super Geometric Mean Labeling for Triangular Snake Graphs / IJMA- 6(8), August-2015.

**Definition:** 1.3 If \( e=uv \) is an edge of \( G \) and \( w \) is not a vertex of \( G \), then \( e \) is said to be subdivided when it is replaced by the edges \( uw \) and \( wv \). The graph obtained by subdividing each edge of a graph \( G \) is called the **Subdivision** of \( G \) and it is denoted by \( S(G) \).

For example,

\[
\begin{align*}
G: & \quad u \quad \longrightarrow \quad v \\
S(G): & \quad u \quad \longrightarrow \quad w \quad \longrightarrow \quad v
\end{align*}
\]

**Definition:** 1.4 A **Triangular snake** \( T_n \) is obtained from a path \( u_1u_2…u_n \) by joining \( u_i \) and \( u_{i+1} \) to a new vertex \( v_i \) for \( 1 \leq i \leq n-1 \). That is every edge of a path is replaced by a triangle \( C_3 \).

**Definition:** 1.5 An **Alternate Triangular snake** \( A(T_n) \) is obtained from a path \( u_1u_2…u_n \) by joining \( u_i \) and \( u_{i-1} \) (alternatively) to a new vertex \( v_i \). That is every alternate edge of a path is replaced by a triangle \( C_3 \).

**Definition:** 1.6 A **Double Triangular snake** \( D(T_n) \) consists of two Triangular snakes that have a common path.

**Definition:** 1.7 An **Alternate Double Triangular snake** \( A[D(T_n)] \) consists of two Alternate Triangular snakes that have a common path.

**Definition:** 1.8 A **Triple Triangular snake** \( T(T_n) \) consists of three Triangular snakes that have a common path.

**Definition:** 1.9 An **Alternate Triple Triangular snake** \( A[T(T_n)] \) consists of three Alternate Triangular snakes that have a common path.

**Theorem 1.10:** \( T_n, A(T_n), D(T_n) \) and \( A[D(T_n)] \) are Mean graphs.

**Theorem 1.11:** \( T_n, A(T_n), D(T_n) \) and \( A[D(T_n)] \) are Harmonic mean graphs.

**Theorem 1.12:** \( T_n, A(T_n), D(T_n), A[\{T(T_n)\}], T(T_n) \) and \( A[\{T(T_n)\}] \) are Geometric mean graphs.

**Theorem 1.13:** \( T_n, A(T_n), D(T_n), A[D(T_n)], T(T_n) \) and \( A[T(T_n)] \) are Super Geometric mean graphs.

2. **MAIN RESULTS**

**Theorem:** 2.1 Subdivision of Alternate Triangular snake \( S[A(T_n)] \) is a Super Geometric mean graph.

**Proof:** Let \( A(T_n) \) be an Alternate Triangular snake which is obtained from a path \( P_n=u_1u_2…u_n \) by joining \( u_i \) and \( u_{i+1} \) alternatively to a new vertex \( v_i \).

Let \( S[A(T_n)]=A(T_N) = G \) be a graph obtained by subdividing all the edges of \( A(T_n) \).

Here we consider the following cases.

**Case 1:** If \( T_n \) starts from \( u_1 \),

Let \( t_i, 1 \leq i \leq n-1 \) be the vertices which subdivide the edges \( u_iu_{i+1} \).

Let \( r_i \) be the vertices which subdivide the edges \( u_{2i-1}v_i \).

Let \( s_i \) be the vertices which subdivide the edges \( u_{2i}v_i \).

We have to consider two subcases.

**Subcase (1) (a):** If \( \text{‘}n\text{’} \) is odd, then

Define a function \( f: V[A(T_N)] \rightarrow \{1,2,…,p+q\} \) by,

\[
\begin{align*}
f(u_1) &= 8 \\
f(u_{2i-1}) &= 15i-14, \; 2 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil + 1 \\
f(u_{2i}) &= 15i-3, \; 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil \\
f(t_1) &= 10
\end{align*}
\]
The labeling pattern of $S[A(T_7)]$ is shown in the following figure.

\[ f(t_{2i-1}) = 15i-9, \ 2 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(t_{2i}) = 15i-1, \ 1 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(v_{i}) = 1 \]
\[ f(s_i) = 15i-5, \ 2 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(ri) = 15i-11, \ 1 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(s_i) = 15i-8, \ 2 \leq i \leq \left( \frac{n-1}{2} \right) \]

From the above labeling pattern, we get, \{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1, 2, \ldots, p+q\}

∴ In this case, “f” provides a Super Geometric mean labeling of $A(T_N)$

**Subcase (1) (b):** If ‘n’ is even, then

Define a function $f: V[A(T_N)] \to \{1, 2, \ldots, p+q\}$ by,
\[ f(u_{t_{2i-1}}) = 15i-14, \ 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(u_{t_{2i}}) = 15i-3, \ 1 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(t_{1}) = 10 \]
\[ f(t_{2i-1}) = 15i-9, \ 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(t_{2i}) = 15i-1, \ 1 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(v_{i}) = 1 \]
\[ f(s_i) = 15i-5, \ 2 \leq i \leq \left( \frac{n}{2} \right) \]
The labeling pattern of \( S[A(T_n)] \) is given below.

From the above labeling pattern, we get, \( \{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\} \)

In this case, \( A(T_n) \) is a Super Geometric mean graph.

**Case 2:** If \( T_n \) starts from \( u_3 \),

Let \( t_i, 1 \leq i \leq n-1 \) be the vertices which subdivide the edges \( u_iu_{i+1} \).

Let \( r_i \) and \( s_i \) be the vertices which subdivide the edges \( u_2v_i \) and \( u_{2i+1}v_i \) respectively.

Here we have to consider two subcases.

**Subcase (2) (a):** If \( n \) is odd, then

Define a function \( f: V[A(T_n)] \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
f(u_2i-1) &= 15i-14, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) + 1 \\
f(u_2i) &= 15i-10, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) \\
f(t_{2i-1}) &= 15i-12, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) \\
f(t_{2i}) &= 15i-5, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) \\
f(r_i) &= 15i-7, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) \\
f(s_i) &= 15i-1, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right) \\
f(v_i) &= 15i-4, \quad 1 \leq i \leq \left(\frac{n-1}{2}\right)
\end{align*}
\]

The labeling pattern of \( S[A(T_7)] \) is displayed below.
From the above labeling pattern, we get \( \{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\} \)

Hence \( A(T_n) \) admits a Super Geometric mean labeling.

**Subcase (2) (b):** If \( n \) is even, then

Define a function \( f: V[A(T_N)] \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
    f(u_{2i-1}) &= 15i - 14, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(u_{2i}) &= 15i - 10, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(t_{2i-1}) &= 15i - 12, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(t_{2i}) &= 15i - 5, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(r_i) &= 15i - 7, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(s_i) &= 15i - 1, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(v_i) &= 15i - 4, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
    f(x_i) &= 15i - 5, 1 \leq i \leq n-1 \\
    f(w_i) &= 15i - 6, 2 \leq i \leq n-1 \\
    f(y_i) &= 15i - 9, 2 \leq i \leq n-1
\end{align*}
\]

The labeling pattern of \( S[A(T_8)] \) is shown below.

![Figure 4](image_url)

From the above labeling pattern, both vertices and edges together get distinct labels from \{1, 2, 3, \ldots, p+q\}.

From all the above cases, we conclude that Subdivision of Alternate Triangular snake is a Super Geometric mean graph.

**Theorem 2.2** Subdivision of Double Triangular snake \( S[D(T_n)] \) is a Super Geometric mean graph.

**Proof:** Let \( D(T_n) \) be a Double Triangular snake which is obtained from a path \( P_n = u_1u_2 \ldots u_n \) by joining \( u_i \) and \( u_{i+1} \) with two new vertices \( v_i \) and \( w_i \), \( 1 \leq i \leq n-1 \).

Let \( S[D(T_n)] = D(T_N) = G \) be a graph obtained by subdividing all the edges of \( D(T_n) \).

Let \( t_i, x_i, y_i, r_i \) and \( s_i \) be the new vertices which subdivide the edges \( u_i u_{i+1}, u_iv_i, u_{i+1}v_i, u_iw_i \) and \( u_{i+1}w_i \), \( 1 \leq i \leq n-1 \) respectively.

Define a function \( f: V[D(T_N)] \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
    f(u_1) &= 6 \\
    f(u_i) &= 18i - 17, 2 \leq i \leq n \\
    f(t_i) &= 9 \\
    f(x_i) &= 18i - 10, 2 \leq i \leq n-1 \\
    f(r_i) &= 10 \\
    f(s_i) &= 18i - 13, 2 \leq i \leq n-1 \\
    f(w_i) &= 12 \\
    f(y_i) &= 18i - 5, 2 \leq i \leq n-1 \\
    f(x_i) &= 4
\end{align*}
\]
From the above labeling pattern, \( \{f(V(D(T_n)))\} \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\} \).

Hence \( D(T_n) \) is a Super Geometric mean graph.

**Example 2.3:** A Super Geometric mean labeling of \( S[D(T_3)] \) is displayed below.

![Figure 5](image_url)

**Theorem 2.4** Subdivision of Alternate Double Triangular snake \( S[A(D(T_n))] \) is a Super Geometric mean graph.

**Proof:** Let \( A[D(T_n)] \) be an Alternate Double Triangular snake which is obtained from a path \( P_n = u_1u_2 \ldots u_n \) by joining \( u_i \) and \( u_{i+1} \) alternatively with two new vertices \( v_i \) and \( w_i \).

Let \( S[A(D(T_n))] = A[D(T_n)] = G \) be a graph obtained by subdividing all the edges of \( A[D(T_n)] \).

Here we consider two cases.

**Case 1:** If \( D(T_n) \) starts from \( u_1 \).

Let \( t_i, x_i, y_i, r_i \) and \( s_i \) be the vertices which subdivide the edges \( u_{2i-1}v_i, u_{2i-1}w_i, u_{2i-1}v_i, u_{2i-1}w_i \) and \( u_{2i-1}w_i \) respectively.

We have to consider two subcases.

**Subcase (1) (a):** If \( n \) is odd, then

Define a function \( f: V[A(D(T_n))] \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
f(u_1) &= 6 \\
f(u_{2i-1}) &= 22i-21, \ 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor + 1 \\
f(u_{2i}) &= 22i-3, \ 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(t_1) &= 9 \\
f(t_{2i-1}) &= 22i-14, \ 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(t_{2i}) &= 22i-1, \ 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(r_1) &= 10 \\
f(r_i) &= 22i-17, \ 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(s_1) &= 22i-5, \ 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(w_1) &= 12
\end{align*}
\]
\[ f(w_i) = 22i - 9, \quad 2 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(x_i) = 4 \]
\[ f(y_i) = 22i - 10, \quad 2 \leq i \leq \left( \frac{n-1}{2} \right) \]
\[ f(v_i) = 1 \]
\[ f(t_i) = 10 \] 
\[ f(r_i) = 22i - 17, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(s_i) = 22i - 5, \quad 1 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(w_1) = 12 \]
\[ f(x_1) = 4 \]
\[ f(y_1) = 13 \]
\[ f(v_1) = 1 \]
\[ f(t_1) = 9 \]
\[ f(r_1) = 10 \]
\[ f(s_1) = 22i - 10, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(w_1) = 22i - 9, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(x_1) = 22i - 16, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(y_1) = 22i - 12, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]

The labeling pattern of \( S[A(D(T_n))] \) is given below.

\[ f(x_i) = 22i - 16, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(y_i) = 22i - 10, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(v_i) = 22i - 12, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]

\[ f(t_i) = 22i - 17, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(s_i) = 22i - 5, \quad 1 \leq i \leq \left( \frac{n}{2} \right) \]

:. From the above labeling pattern, we get \( \{f(V(G))\} \cup \{f(e): e \in E(G)=\{1,2,\ldots,p+q\}, \)

In this case “f” provides a Super Geometric mean labeling of \( A[D(T_n)] \).

**Subcase (1) (b):** If ‘n’ is even, then

Define a function \( f: V[A(D(T_n))] \rightarrow \{1,2,\ldots,p+q\} \) by,

\[ f(u_{2i-1}) = 22i - 21, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(u_{2i}) = 22i - 3, \quad 1 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(t_i) = 9 \]
\[ f(t_{2i-1}) = 22i - 14, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(t_{2i}) = 22i - 1, \quad 1 \leq i \leq \left( \frac{n-2}{2} \right) \]
\[ f(r_i) = 10 \]
\[ f(s_i) = 22i - 5, \quad 1 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(w_i) = 12 \]
\[ f(x_i) = 4 \]
\[ f(y_i) = 13 \]
\[ f(v_i) = 1 \]
\[ f(w_1) = 22i - 9, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(x_1) = 22i - 16, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(y_1) = 22i - 10, \quad 2 \leq i \leq \left( \frac{n}{2} \right) \]
\[ f(v_1) = 1 \]
\[ f(t_1) = 9 \]
\[ f(s_1) = 22i - 5, \quad 1 \leq i \leq \left( \frac{n}{2} \right) \]
The labeling pattern of $S[A(D(T_n))]$ is shown below

From the above labeling pattern, we get

$$\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\}.$$ 

Hence $A[D(T_n)]$ admits Super Geometric mean labeling.

**Case 2:** If $D(T_n)$ starts from $u_2$.

Let $t_i, x_i, y_i, r_i$ and $s_i$ be the vertices which subdivide the edges $u_i, u_{i+1}, v_i, u_{2i+1}v_i, u_{2i}w_i$ and $u_{2i+1}w_i$ respectively.

We have to consider two subcases.

**Subcase (2) (a):** If ‘$n$’ is odd, then define a function $f: V[A(D(T_n))] \rightarrow \{1, 2, \ldots, p+q\}$ by,

- $f(u_{2i-1}) = 22i-21, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$, $+ 1$
- $f(u_{2i}) = 22i-17, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(t_{2i-1}) = 22i-19, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(t_{2i}) = 22i-10, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(r_i) = 22i-13, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(s_i) = 22i-1, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(w_1) = 18$
- $f(w_i) = 22i-5, 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(x_i) = 22i-12, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(y_i) = 22i-6, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
- $f(v_i) = 22i-8, 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$
The labeling pattern of $S[A(D(T_7))]$ is displayed below.

From the above labeling pattern, both vertices and edges together get distinct labels from \{1, 2, 3, ..., p+q\}.

Hence $A[D(T_7)]$ is a Super Geometric mean graph.

**Subcase (2) (b):** If ‘n’ is even, then

Define a function $f: V[A(D(T_n))] \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 22i-21, 1 \leq i \leq \left(\frac{n}{2}\right)$

$f(u_{2i}) = 22i-17, 1 \leq i \leq \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 22i-19, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(t_{2i}) = 22i-10, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(r_i) = 22i-13, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(s_i) = 22i-1, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(w_i) = 22i-5, 2 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(x_i) = 22i-12, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(y_i) = 22i-6, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(v_i) = 22i-8, 1 \leq i \leq \left(\frac{n-2}{2}\right)$

The labeling pattern of $S[A(D(T_n))]$ is displayed below.
From the above labeling pattern, we get \( \{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p+q\} \).

This makes “f” a Super Geometric mean labeling of \( A[D(T_n)] \).

From all the above cases, we conclude that Subdivision of Alternate Double Triangular snake is a Super Geometric mean graph.

**Theorem 2.5** Subdivision of Triple Triangular snake \( S[T(T_n)] \) is a Super Geometric mean graph.

**Proof:** Let \( T(T_n) \) be a Triple Triangular snake which is obtained from a path \( P_n = u_1u_2\ldots u_n \) by joining \( u_i \) and \( u_{i+1} \) with three new vertices \( v_i, w_i, \) and \( z_i, \) \( 1 \leq i \leq n-1.\)

Let \( S[T(T_n)] = T(T_n) = G \) be the graph obtained by subdividing all the edges of \( T(T_n) \).

Let \( t_i, r_i, s_i, y_i, m_i, \) and \( n_i \) be the vertices which subdivide the edges \( u_iu_{i+1}, \) \( u_iv_i, \) \( u_{i+1}v_i, \) \( u_iw_i, \) and \( u_{i+1}w_i \) respectively.

Define a function \( f: V(G) \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
    f(u_1) &= 6 \\
    f(u_i) &= 25i-24, \; 2 \leq i \leq n \\
    f(t_i) &= 25i-1, \; 1 \leq i \leq n-1 \\
    f(m_i) &= 9 \\
    f(n_i) &= 25i-21, \; 2 \leq i \leq n-1 \\
    f(r_i) &= 25i-5, \; 2 \leq i \leq n-1 \\
    f(w_i) &= 19 \\
    f(\tau_i) &= 25i-17, \; 2 \leq i \leq n-1 \\
    f(s_i) &= 10 \\
    f(z_i) &= 25i-6, \; 2 \leq i \leq n-1 \\
    f(y_i) &= 25i-9, \; 2 \leq i \leq n-1 \\
    f(x_i) &= 11 \\
    f(x_i) &= 25i-7, \; 1 \leq i \leq n-1 \\
    f(v_i) &= 15 \\
    f(v_i) &= 25i-13, \; 2 \leq i \leq n-1 \\
\end{align*}
\]

From the above labeling pattern, \( \{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p+q\} \).

Hence Subdivision of Triple Triangular snake is a Super Geometric mean graph.

**Example 2.6:** A Super Geometric mean labeling of \( S[T(T_3)] \) is shown below.

![Figure: 10](image-url)
Theorem: 2.7 Subdivision of Alternate Triple Triangular snake $S[A(T(T_n))]$ is a Super Geometric mean graph.

Proof: Let $A[T(T_n)]$ be an Alternate Triple Triangular snake which is obtained from a path $P_n = u_1u_2...u_n$ by joining $u_i$ and $u_{i+1}$ alternatively with three new vertices $v_i$, $w_i$ and $z_i$.

Let $S[A(T(T_n))] = A[T(T_n)]=G$ be the graph obtained by subdividing all the edges of $A[T(T_n)]$.

Here we consider two cases.

**Case: 1** If $T(T_n)$ Starts from $u_1$.

Let $t_i$, $m_i$, $n_i$, $x_i$, $y_i$, $r_i$ and $s_i$ be the vertices which subdivide the edges $u_{2i}u_{2i+1}$, $u_{2i-1}w_i$, $u_{2i}w_i$, $u_{2i}v_i$, $u_{2i}v_i$, $u_{2i-1}z_i$ and $u_{2i-1}z_i$ respectively.

We have to consider two subcases.

**Subcase (1) (a):** If ‘$n$’ is odd, then

Define a function $f: V(G) \rightarrow \{1,2,...,p+q\}$ by,

- $f(u_1) = 6$
- $f(u_{2i}) = 29i-28$, $2 \leq i \leq (n-1)/2 + 1$
- $f(u_{2i+1}) = 29i-3$, $1 \leq i \leq (n-1)/2$
- $f(t_{2i}) = 29i-5$, $1 \leq i \leq (n-1)/2$
- $f(t_{2i+1}) = 29i-1$, $1 \leq i \leq (n-1)/2 + 1$
- $f(m_i) = 9$
- $f(n_i) = 29i-9$, $2 \leq i \leq (n-1)/2$
- $f(x_i) = 11$
- $f(y_i) = 29i-11$, $1 \leq i \leq (n-1)/2$
- $f(z_i) = 1$
- $f(v_i) = 29i-22$, $2 \leq i \leq (n-1)/2$
- $f(r_i) = 4$
- $f(w_i) = 29i-21$, $2 \leq i \leq (n-1)/2$
- $f(s_i) = 10$
- $f(s_i+1) = 29i-17$, $2 \leq i \leq (n-1)/2$
The labeling pattern of $S[A(T(T_3))]$ is displayed below.

![Figure: 11](image)

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

Hence “$f$” provides a Super Geometric mean labeling of $G$.

**Subcase (1) (b):** If “$n$” is even, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

- $f(u_{i1}) = 6$
- $f(u_{i2}) = 29i-28, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(u_{i3}) = 29i-3, 1 \leq i \leq \left(\frac{n}{2}\right)$
- $f(t_{i1}) = 29i-5, 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(t_{i2}) = 29i-1, 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(m_{i1}) = 9$
- $f(m_{i2}) = 29i-25, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(n_{i1}) = 22$
- $f(n_{i2}) = 29i-9, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(w_{i1}) = 19$
- $f(w_{i2}) = 29i-21, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(x_{i1}) = 11$
- $f(x_{i2}) = 29i-22, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(y_{i1}) = 29i-11, 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(y_{i2}) = 29i-1, 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(v_{i1}) = 15$
- $f(v_{i2}) = 29i-17, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(\tau_{i1}) = 4$
- $f(\tau_{i2}) = 29i-19, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(s_{i1}) = 10$
- $f(s_{i2}) = 29i-10, 2 \leq i \leq \left(\frac{n}{2}\right)$
- $f(z_{i1}) = 1$
- $f(z_{i2}) = 29i-13, 2 \leq i \leq \left(\frac{n}{2}\right)$
The labeling pattern of \( S[A(T(T_n))] \) is given below

From the above labeling pattern, we get \( \{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p+q\} \)

Hence \( G \) admits Super Geometric mean labeling.

**Case 2:** If \( T(T_n) \) starts from \( u_2 \).

Let \( t_i, m_i, n_i, x_i, y_i, r_i \) and \( s_i \) be the vertices which subdivide the edges \( u_1u_{i+1}, u_2w_i, u_{2i+1}w_i, u_2v_i, u_{2i+1}v_i, u_{2i+1}z_i \) and \( u_{2i+1}z_i \) respectively.

We have to consider two subcases.

**Subcase (2) (a):** If \( n \) is odd, then

Define a function \( f : V(G) \rightarrow \{1, 2, \ldots, p+q\} \) by,

- \( f(u_{2i-1}) = 29i-28, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor + 1 \)
- \( f(u_{2i}) = 29i-24, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(t_{2i-1}) = 29i-26, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(t_{2i}) = 29i-1, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(m_i) = 9 \)
- \( f(m_i) = 29i-21, 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(n_i) = 29i-5, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(w_i) = 13 \)
- \( f(w_i) = 29i-17, 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(x_i) = 29i-18, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(y_i) = 29i-7, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(v_i) = 29i-13, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(r_i) = 15 \)
- \( f(r_i) = 29i-15, 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(s_i) = 29i-6, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
- \( f(z_i) = 29i-9, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor \)
The labeling pattern of $S[A(T(T_n))]$ is shown below.

![Figure: 13](image)

From the above labeling pattern, both vertices and edges together get distinct labels from \{1, 2, …, p+q\}.

This makes “f” a Super Geometric mean labeling of G.

**Subcase (2) (b):** If ‘n’ is even, then

Define a function $f: V(G) \rightarrow \{1, 2, …, p+q\}$ by,

- $f(u_{2i-1}) = 29i - 28$, $1 \leq i \leq \frac{n}{2}$
- $f(u_{2i}) = 29i - 24$, $1 \leq i \leq \frac{n}{2}$
- $f(t_{2i-1}) = 29i - 26$, $1 \leq i \leq \frac{n}{2}$
- $f(t_{2i}) = 29i - 1$, $1 \leq i \leq \frac{n}{2}$
- $f(m_i) = 9$
- $f(w_i) = 29i - 21$, $2 \leq i \leq \frac{n-2}{2}$
- $f(u_i) = 29i - 5$, $1 \leq i \leq \frac{n-2}{2}$
- $f(w_i) = 29i - 17$, $2 \leq i \leq \frac{n-2}{2}$
- $f(x_i) = 29i - 18$, $1 \leq i \leq \frac{n-2}{2}$
- $f(y_i) = 29i - 7$, $1 \leq i \leq \frac{n-2}{2}$
- $f(v_i) = 29i - 13$, $1 \leq i \leq \frac{n-2}{2}$
- $f(r_i) = 15$
- $f(s_i) = 29i - 15$, $2 \leq i \leq \frac{n-2}{2}$
- $f(s_i) = 29i - 6$, $1 \leq i \leq \frac{n-2}{2}$
- $f(z_i) = 29i - 9$, $1 \leq i \leq \frac{n-2}{2}$
The labeling pattern of $S[A(T(T_6))]$ is displayed below.

![Figure: 14](image)

From the above labeling pattern we get, $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\}$.

Hence $G$ admits a Super Geometric mean labeling.

From all the above cases, we conclude that Subdivision of Alternate Triple Triangular snake is a Super Geometric mean graph.

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Source of support: Nil, Conflict of interest: None Declared

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