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SUBDIVISION OF SUPER GEOMETRIC MEAN LABELING FOR TRIANGULAR SNAKE GRAPHS

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ABSTRACT

Let f: V(G) →{1,2,…,p+q} be an injective function. For a vertex labeling "f", the induced edge labeling f(e=uv) is defined by, f*(e) =* $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ *or* $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$*. Then "f" is called a **"Super Geometric mean labeling"** if {f(V(G))}∪{f(e) : e∈E(G)} = {1, 2,…,p+q}. A graph which admits Super Geometric mean labeling is called **"Super Geometric mean graph".***

In this paper we prove that S[A(T_n)], S[D(T_n)], S[A(D(T_n))], Subsivision of triple Triangular snake S[T(T_n)] and Subdivision of alternate triple Triangular snake graphs S[A(T(T_n))] are Super Geometric mean graphs.

***Key Words:** Graph, Geometric mean graph, Super Geometric mean graph, Triangular snake, Double Triangular snake and Triple Triangular snake.*

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected graph G=(V,E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2].

The concept of **"Geometric mean labeling"** has been introduced by S.Somasundaram, R. Ponraj and P. Vidhyarani in [6].

In this paper we investigate Super Geometic mean labeling behavior of S[A(T_n)], S[D(T_n)], S[A(D(T_n))], Subdivision of triple Triangular snake S[T(T_n)] and Subdivision of alternate triple Triangular snake S[A(T(T_n))].

We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition: 1.1 A graph G = (V,E) with p vertices and q edges is called a **"Geometric mean graph"** if it is possible to label the vertices $x\in V$ with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with, f(e=uv) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$ then the edge labels are distinct. In this case, "f " is called a **"Geometric mean labeling"** of G.

Definition: 1.2 Let f: V(G) → {1,2,…,p+q} be an injective function. For a vertex labeling "f", the induced edge labeling f*(e=uv) is defined by, f* (e) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then " f " is called a **"Super Geometric mean labeling"** if {f(V(G))}∪{f(e):e∈E(G)}= {1,2,…,p+q}. A graph which admits Super Geometric mean labeling is called **"Super Geometric mean graph".**

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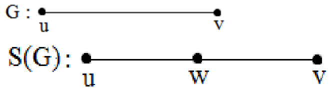
Definition: 1.3 If e=uv is an edge of G and w is not a vertex of G, then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the **Subdivision** of G and it is denoted by S(G).

For example,



Definition: 1.4 A **Triangular snake** T_n is obtained from a path $u_1u_2\dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1\le i\le n-1$. That is every edge of a path is replaced by a triangle C_3.

Definition: 1.5 An **Alternate Triangular snake** $A(T_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by a triangle C_3.

Definition: 1.6 A **Double Triangular snake** $D(T_n)$ consists of two Triangular snakes that have a common path.

Definition: 1.7 An **Alternate Double Triangular snake** $A[D(T_n)]$ consists of two Alternate Triangular snakes that have a common path.

Definition: 1.8 A **Triple Triangular snake** $T(T_n)$ consists of three Triangular snakes that have a common path.

Definition: 1.9 An **Alternate Triple Triangular snake** $A[T(T_n]$ consists of three Alternate Triangular snakes that have a common path.

Theorem 1.10: T_n, $A(T_n)$, $D(T_n)$ and $A[D(T_n)]$ are Mean graphs.

Theorem 1.11: T_n, $A(T_n)$, $D(T_n)$ and $A[D(T_n)]$ are Harmonic mean graphs.

Theorem 1.12: T_n, $A(T_n)$, $D(T_n)$, $A[D(T_n)]$, $T(T_n)$ and $A[T(T_n)]$, are Geometric mean graphs.

Theorem 1.13: T_n, $A(T_n)$, $D(T_n)$, $A[D(T_n)]$, $T(T_n)$ and $A[T(T_n)]$ are Super Geometric mean graphs.

2. MAIN RESULTS

Theorem: 2.1 Subdivision of Alternate Triangular snake $S[A(T_n)]$ is a Super Geometric mean graph.

Proof: Let $A(T_n)$ be an Alternate Triangular snake which is obtained from a path $P_n=u_1u_2\dots u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i.

Let $S[A(T_n)]=A(T_N) = G$ be a graph obtained by subdividing all the edges of $A(T_n)$.

Here we consider the following cases.

Case 1: If T_n starts from u_1,

Let t_i, $1\le i\le n-1$ be the vertices which subdivide the edges u_iu_{i+1}.

Let r_i be the vertices which subdivide the edges $u_{2i-1}\,v_{i.}$

Let s_i be the vertices which subdivide the edges $u_{2i}\,v_i$

We have to consider two subcases.

Subcase (1) (a) : If 'n' is odd, then

Define a function $f: V[A(T_N)] \to \{1,2,\dots,p+q\}$ by,

$f(u_1)=8$

$f(u_{2i-1}) = 15i-14,\ 2\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 15i-3,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_1) = 10$

$f(t_{2i-1}) = 15i-9,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(t_{2i}) = 15i-1,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(v_1)=1$
$f(v_i) = 15i-8,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(r_i) = 15i-11,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(s_1) = 5$
$f(s_i) = 15i-5,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(T_7)]$ is shown in the following figure.

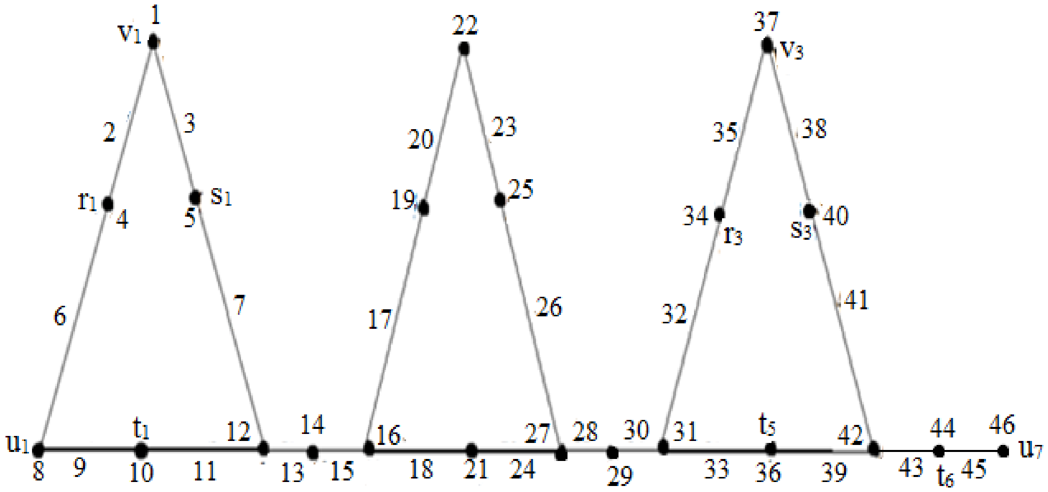


Figure: 1

From the above labeling pattern, we get, $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$

$\therefore$ In this case, "f" provides a Super Geometric mean labeling of $A(T_N)$

Subcase (1) (b): If 'n' is even, then

Define a function $f: V[A(T_N)] \to \{1,2,\ldots,p+q\}$ by,
$f(u_1)=8$
$f(u_{2i-1}) = 15i-14,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(u_{2i}) = 15i-3,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(t_1) = 10$
$f(t_{2i-1}) = 15i-9,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(t_{2i}) = 15i-1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$
$f(v_1)=1$
$f(v_i) = 15i-8,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(r_i) = 15i-11,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(s_1) = 5$
$f(s_i) = 15i-5,\ 2 \le i \le \left(\frac{n}{2}\right)$

The labeling pattern of $S[A(T_6)]$ is given below.

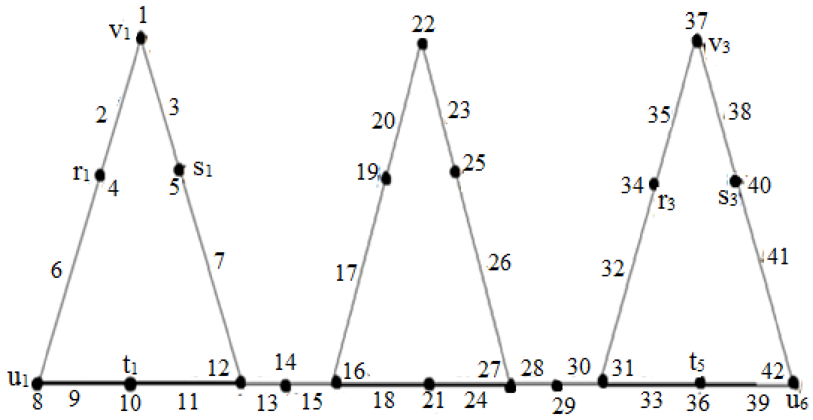


Figure: 2

From the above labeling pattern, we get, $\{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

In this case, $A(T_N)$ is a Super Geometric mean graph.

Case 2: If T_n starts from u_2,

Let t_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iu_{i+1}.

Let r_i and s_i be the vertices which subdivide the edges $u_{2i}v_i$ and $u_{2i+1}\, v_i$ respectively.

Here we have to consider two subcases.

Subcase (2) (a): If 'n' is odd, then

Define a function $f: V[A(T_N)] \rightarrow \{1,2,\ldots,p+q\}$ by,

$$f(u_{2i-1}) = 15i-14,\ 1 \le i \le \left(\frac{n-1}{2}\right) + 1$$
$$f(u_{2i}) = 15i-10,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$
$$f(t_{2i-1}) = 15i-12,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$
$$f(t_{2i}) = 15i-5,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$
$$f(r_i) = 15i-7,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$
$$f(s_i) = 15i-1,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$
$$f(v_i) = 15i-4,\ 1 \le i \le \left(\frac{n-1}{2}\right)$$

The labeling pattern of $S[A(T_7)]$ is displayed below.

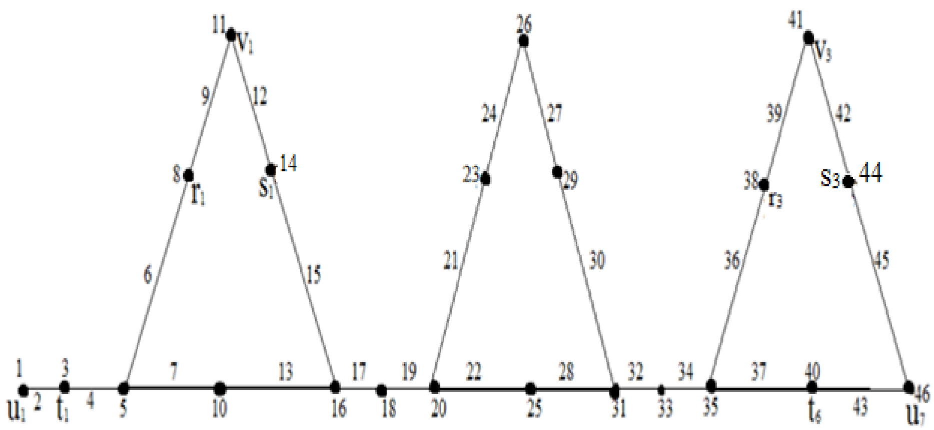


Figure: 3

From the above labeling pattern, we get $\{f(V(G))\}\cup(f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$

Hence $A(T_N)$ admits a Super Geometric mean labeling.

Subcase (2) (b): If 'n' is even, then

Define a function $f: V[A(T_N)]\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 15i-14,\ 1\le i\le\left(\frac{n}{2}\right)$
$f(u_{2i}) = 15i-10,\ 1\le i\le\left(\frac{n}{2}\right)$
$f(t_{2i-1}) = 15i-12,\ 1\le i\le\left(\frac{n}{2}\right)$
$f(t_{2i}) = 15i-5,\ 1\le i\le\left(\frac{n-2}{2}\right)$
$f(r_i) = 15i-7,\ 1\le i\le\left(\frac{n-2}{2}\right)$
$f(s_i) = 15i-1,\ 1\le i\le\left(\frac{n-2}{2}\right)$
$f(v_i) = 15i-4,\ 1\le i\le\left(\frac{n-2}{2}\right)$

The labeling pattern of $S[A(T_8)]$ is shown below.

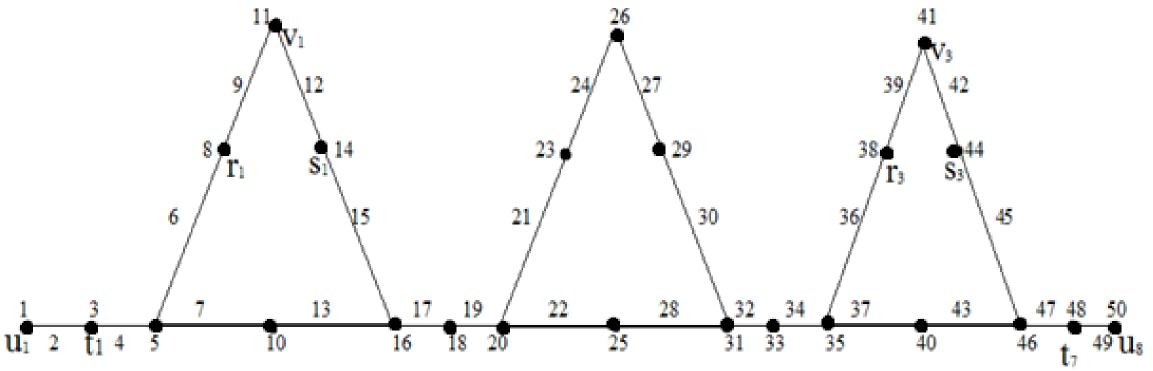


Figure: 4

From the above labeling pattern, both vertices and edges together get distinct labels from $\{1, 2, 3,\ldots,p+q\}$.

From all the above cases, we conclude that Subdivision of Alternate Triangular snake is a Super Geometric mean graph.

Theorem: 2.2 Subdivision of Double Triangular snake $S[D(T_n)]$ is a Super Geometric mean graph.

Proof: Let $D(T_n)$ be a Double Triangular snake which is obtained from a path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} with two new vertices v_i and w_i, $1\le i \le n-1$.

Let $S[D(T_n)] = D(T_N) = G$ be a graph obtained by subdividing all the edges of $D(T_n)$.

Let t_i, x_i, y_i, r_i and s_i be the new vertices which subdivide the edges $u_i u_{i+1}$, u_iv_i, $u_{i+1}v_i$, u_iw_i and $u_{i+1}w_i$, $1\le i\le n-1$ respectively.

Define a function $f: V[D(T_N)]\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_1) = 6$
$f(u_i) = 18i-17,\ 2\le i\le n$
$f(t_1) = 9$
$f(t_i) = 18i-10,\ 2\le i\le n-1$
$f(r_1) = 10$
$f(r_i) = 18i-13,\ 2\le i\le n-1$
$f(s_i) = 18i-1,\ 1\le i\le n-1$
$f(w_1) = 12$
$f(w_i)=18i-5,\ 2\le i\le n-1$
$f(x_1) = 4$

$f(x_i) = 18i-12,\ 2 \le i \le n-1$
$f(y_1) = 13$
$f(y_i) = 18i-6,\ 2 \le i \le n-1$
$f(v_1) = 1$
$f(v_i) = 18i-8,\ 2 \le i \le n-1$

From the above labeling pattern, $\{f[V(D(T_N))]\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$.

Hence $D(T_N)$ is a Super Geometric mean graph.

Example 2.3: A Super Geometric mean labeling of $S[D(T_5)]$ is displayed below.

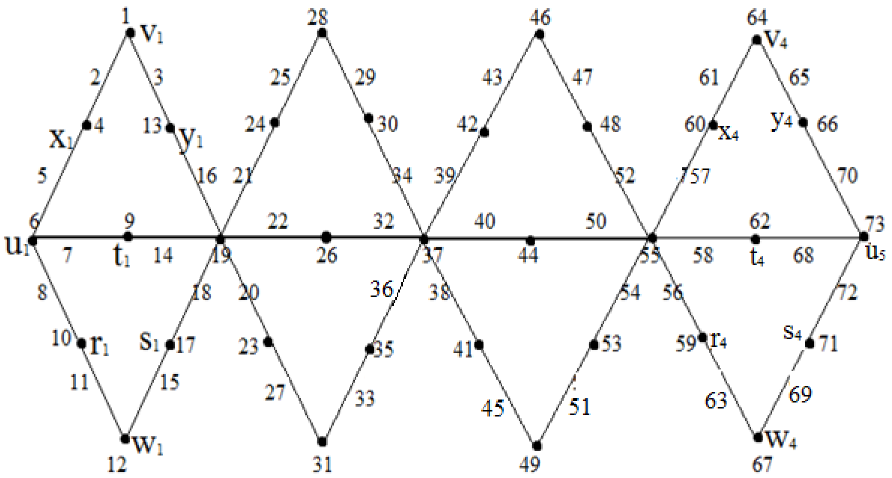


Figure: 5

Theorem: 2.4 Subdivision of Alternate Double Triangular snake $S[A(D(T_n))]$ is a Super Geometric mean graph.

Proof: Let $A[D(T_n)]$ be an Alternate Double Triangular snake which is obtained from a path $P_n=u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with two new vertices v_i and w_i.

Let $S[A(D(T_n))] = A[D(T_N)] = G$ be a graph obtained by subdividing all the edges of $A[D(T_n)]$.

Here we consider two cases.

Case 1: If $D(T_n)$ starts from u_1,

Let t_i, x_i, y_i, r_i and s_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i-1}\,v_i$, $u_{2i}v_i$, $u_{2i-1}\,w_i$ and $u_{2i}\,w_i$ respectively.

We have to consider two subcases.

Subcase (1) (a): If 'n' is odd, then

Define a function $f: V[A(D(T_N))] \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(u_1) = 6$
$f(u_{2i-1}) = 22i-21,\ 2 \le i \le \left(\frac{n-1}{2}\right) + 1$
$f(u_{2i}) = 22i-3,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(t_1) = 9$
$f(t_{2i-1}) = 22i-14,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(t_{2i}) = 22i-1,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(r_1) = 10$
$f(r_i) = 22i-17,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(s_i) = 22i-5,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(w_1) = 12$

$f(w_i) = 22i-9,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$
$f(x_1) = 4$
$f(x_i) = 22i-16,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$
$f(y_1) = 13$
$f(y_i) = 22i-10,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$
$f(v_1) = 1$
$f(v_i) = 22i-12,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(D(T_7))]$ is given below

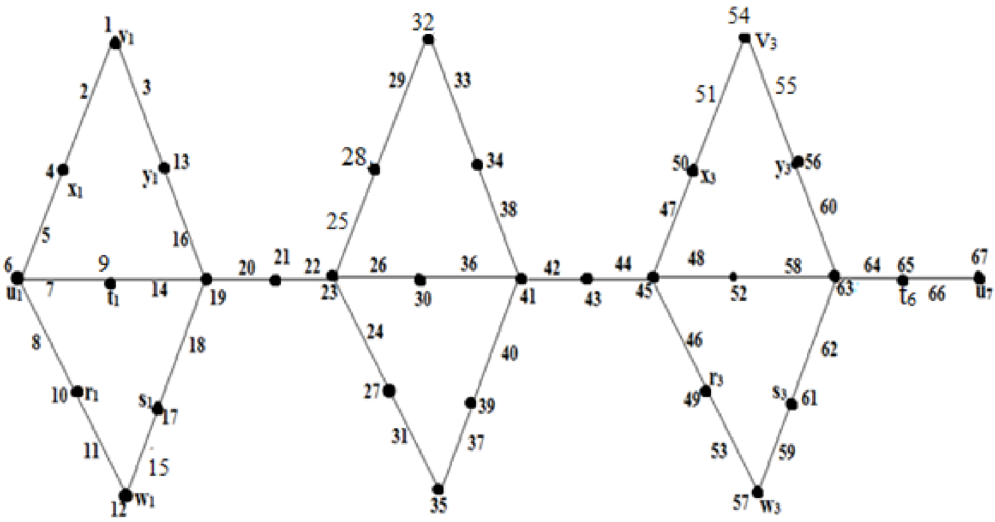


Figure: 6

$\therefore$ From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e): e \in E(G) = \{1,2,\ldots,p+q\}$,

In this case "f" provides a Super Geometric mean labeling of $A[D(T_N)]$.

Subcase (1) (b): If 'n' is even, then

Define a function $f: V[A(D(T_N))] \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(u_1) = 6$
$f(u_{2i-1}) = 22i-21,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(u_{2i}) = 22i-3,\ 1 \leq i \leq \left(\frac{n}{2}\right)$
$f(t_1) = 9$
$f(t_{2i-1}) = 22i-14,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(t_{2i}) = 22i-1,\ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
$f(r_1) = 10$
$f(r_i) = 22i-17,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(s_i) = 22i-5,\ 1 \leq i \leq \left(\frac{n}{2}\right)$
$f(w_1) = 12$
$f(w_i) = 22i-9,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(x_1) = 4$
$f(x_i) = 22i-16,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(y_1) = 13$
$f(y_i) = 22i-10,\ 2 \leq i \leq \left(\frac{n}{2}\right)$
$f(v_1) = 1$
$f(v_i) = 22i-12,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

The labeling pattern of S[A(D(T_6))] is shown below

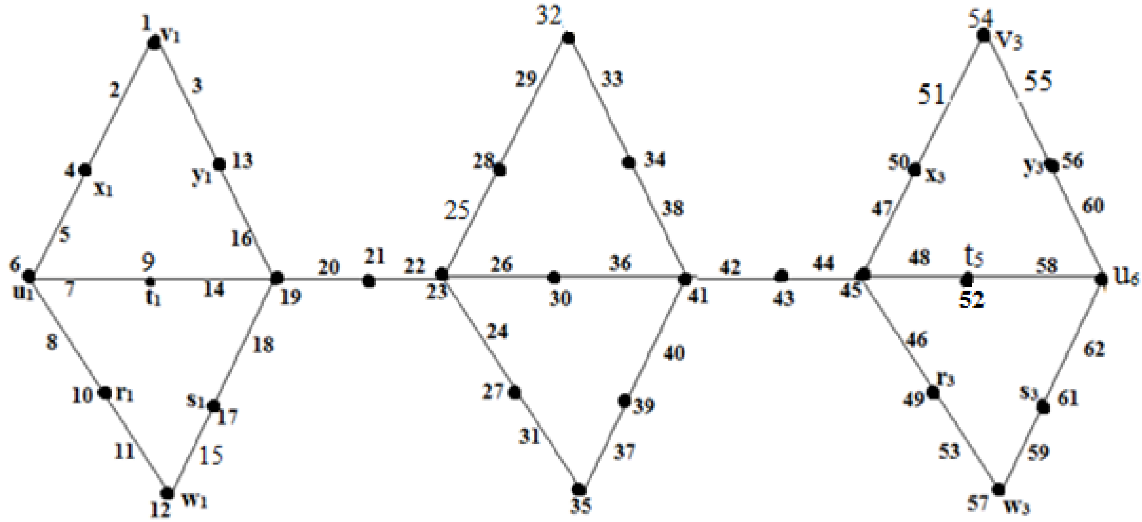


Figure: 7

From the above labeling pattern, we get

$\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1, 2,\ldots,p+q\}$.

Hence A[D(T_N)] admits Super Geometric mean labeling.

Case 2: If D (T_n) Starts from u_2.

Let t_i, x_i, y_i, r_i and s_i be the vertices which subdivide the edges $u_i\, u_{i+1}$, $u_{2i}\, v_i$, $u_{2i+1}v_i$, $u_{2i}w_i$ and $u_{2i+1}\, w_i$ respectively.

We have to consider two subcases.

Subcase (2) (a): If 'n' is odd, then

Define a function f: V[A(D(T_N))]$\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 22i-21,\ 1\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 22i-17,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 22i-19,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 22i-10\ \ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(r_i) = 22i-13,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(s_i) = 22i-1,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(w_1) = 18$

$f(w_i) = 22i-5,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(x_i) = 22i-12,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(y_i) = 22i-6,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(v_i) = 22i-8,\ 1\le i\le\left(\frac{n-1}{2}\right)$

The labeling pattern of S[A(D(T_7))] is displayed below.

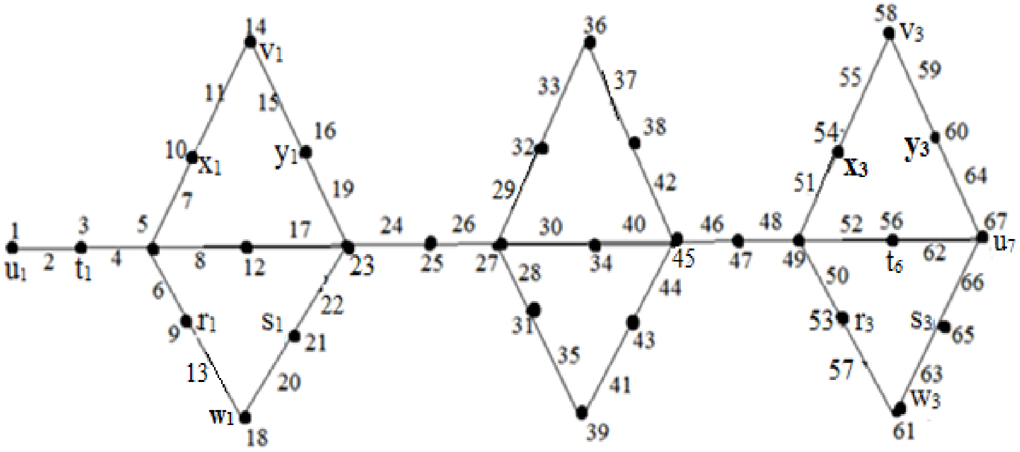


Figure: 8

From the above labeling pattern, both vertices and edges together get distinct labels from {1, 2, 3,…,p+q}.

Hence A[D(T_N)] is a Super Geometric mean graph.

Subcase (2) (b): If 'n' is even, then

Define a function f: V[A(D(T_N))] $\rightarrow$ {1,2,…,p+q} by,

$f(u_{2i-1}) = 22i-21,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(u_{2i}) = 22i-17,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 22i-19,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i}) = 22i-10\ \ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(r_i) = 22i-13,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(s_i) = 22i-1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(w_1) = 18$

$f(w_i) = 22i-5,\ 2 \le i \le \left(\frac{n-2}{2}\right)$

$f(x_i) = 22i-12,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(y_i) = 22i-6,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(v_i) = 22i-8,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

The labeling pattern of S[A(D(T_6))] is displayed below.

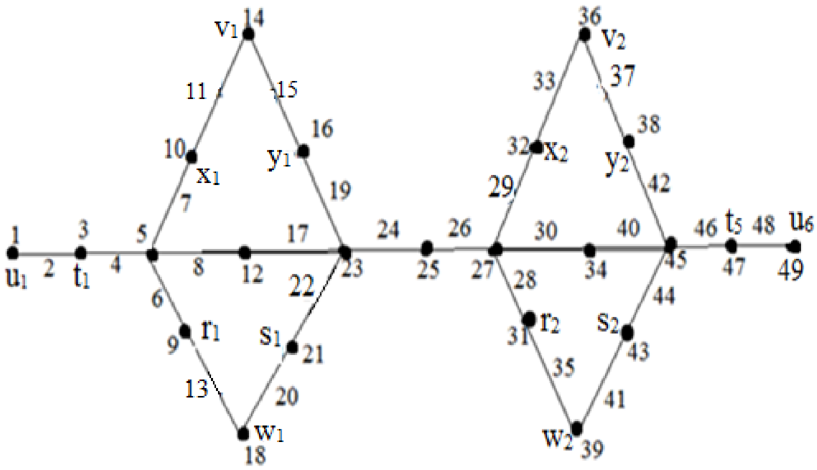


Figure: 9

From the above labeling pattern, we get $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$

This makes "f" a Super Geometric mean labeling of $A[D(T_N)]$.

From all the above cases, we conclude that Subdivision of Alternate Double Triangular snake is a Super Geometric mean graph.

Theorem: 2.5 Subdivision of Triple Triangular snake $S[T(T_n)]$ is a Super Geometric mean graph.

Proof: Let $T(T_n)$ be a Triple Triangular snake which is obtained from a path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} with three new vertices v_i, w_i and $z_i, 1\leq i\leq n-1$.

Let $S[T(T_n)] = T(T_N) = G$ be the graph obtained by subdividing all the edges of $T(T_n)$.

Let t_i, r_i, s_i, x_i, y_i, m_i and n_i be the vertices which subdivide the edges u_iu_{i+1}, u_iz_i, u_{i+1},z_i, u_iv_i, $u_{i+1}v_i$, u_iw_i and $u_{i+1}w_i$ respectively.

Define a function $f:V(G)\to\{1,2,\ldots,p+q\}$ by,

$f(u_1) = 6$
$f(u_i) = 25i-24, 2\leq i\leq n$
$f(t_i) = 25i-1, 1\leq i\leq n-1$
$f(m_1) = 9$
$f(m_i) = 25i-21, 2\leq i\leq n-1$
$f(n_1) = 22$
$f(n_i) = 25i-5, 2\leq i\leq n-1$
$f(w_1) = 19$
$f(w_i) = 25i-17, 2\leq i\leq n-1$
$f(r_1) = 4$
$f(r_i) = 25i-15, 2\leq i\leq n-1$
$f(s_1) = 10$
$f(s_i) = 25i-6, 2\leq i\leq n-1$
$f(z_1) = 1$
$f(z_i) = 25i-9, 2\leq i\leq n-1$
$f(x_1) = 11$
$f(x_i) = 25i-18, 2\leq i\leq n-1$
$f(y_i) = 25i-7, 1\leq i\leq n-1$
$f(v_1) = 15$
$f(v_i) = 25i-13, 2\leq i\leq n-1$

From the above labeling pattern, $\{f(V(G))\}\cup\{f(e): e\in E(G)\} = \{1, 2,\ldots,p+q\}$.

Hence Subdivision of Triple Triangular snake is a Super Geometric mean graph.

Example 2.6: A Super Geometric mean labeling of $S[T(T_5)]$ is shown below.

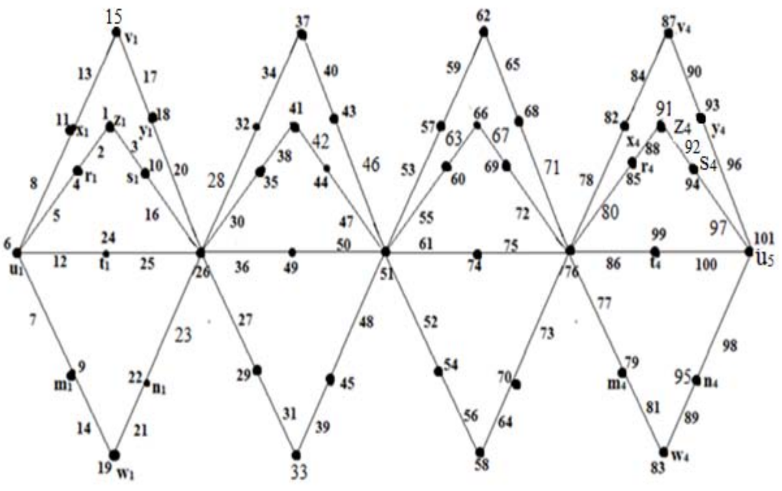


Figure: 10

Theorem: 2.7 Subdivision of Alternate Triple Triangular snake $S[A(T(T_n))]$ is a Super Geometric mean graph.

Proof: Let $A[T(T_n)]$ be an Alternate Triple Triangular snake which is obtained from a path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with three new vertices v_i, w_i and z_i.

Let $S[A(T(T_n))] = A[T(T_N)]=G$ be the graph obtained by subdividing all the edges of $A[T(T_n)]$.

Here we consider two cases.

Case: 1 If $T(T_n)$ Starts from u_1,

Let t_i, m_i, n_i, x_i. y_i, r_i and s_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i-1}w_i$, $u_{2i}w_i$, , $u_{2i-1}v_i$, u_{2i} v_i, $u_{2i-1}z_i$ and $u_{2i}z_i$ respectively.

We have to consider two subcases.

Subcase (1) (a): If 'n' is odd, then

Define a function $f: V(G)\rightarrow\{1,2,...,p+q\}$ by,

$f(u_1) = 6$
$f(u_{2i-1}) = 29i-28,\ 2\leq i\leq\left(\frac{n-1}{2}\right)+1$
$f(u_{2i}) = 29i-3,\ 1\leq i\leq\left(\frac{n-1}{2}\right)$
$f(t_{2i-1}) = 29i-5,\ 1\leq i\leq\left(\frac{n-1}{2}\right)$
$f(t_{2i}) = 29i-1,\ 1\leq i\leq\left(\frac{n-1}{2}\right)$
$f(m_1) = 9$
$f(m_i) = 29i-25,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(n_1) = 22$
$f(n_i) = 29i-9,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(w_1) = 19$
$f(w_i) = 29i-21,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(x_1) = 11$
$f(x_i) = 29i-22,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(y_i) = 29i-11,\ 1\leq i\leq\left(\frac{n-1}{2}\right)$
$f(v_1) = 15$
$f(v_i) = 29i-17,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(r_1) = 4$
$f(r_i) = 29i-19,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(s_1) =10$
$f(s_i) = 29i-10,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$
$f(z_1) = 1$
$f(z_i) = 29i-13,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(T(T_5))]$ is displayed below.

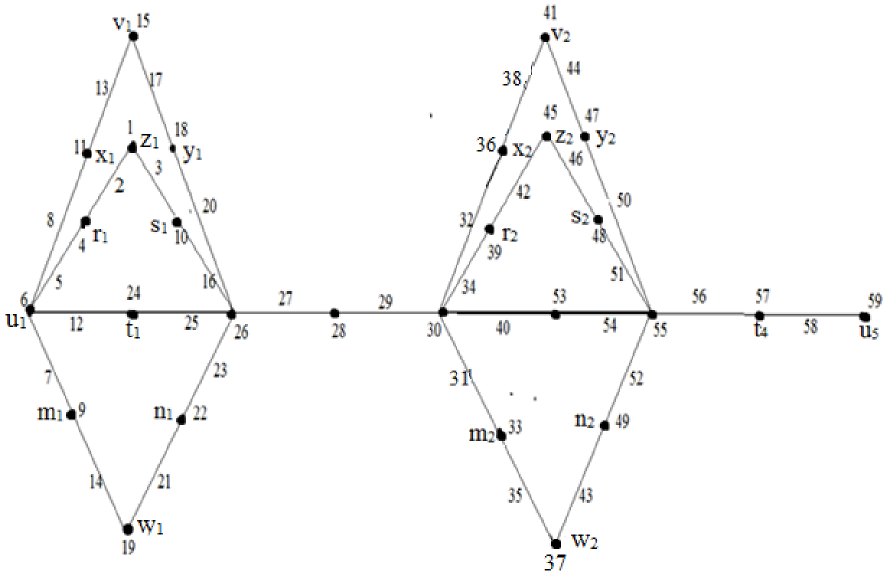


Figure: 11

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

Hence "f" provides a Super Geometric mean labeling of G.

Subcase (1) (b): If 'n' is even, then

Define a function $f: V(G) \to \{1,2,\ldots,p+q\}$ by,

$f(u_1) = 6$

$f(u_{2i-1}) = 29i-28,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(u_{2i}) = 29i-3,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 29i-5,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i}) = 29i-1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(m_1) = 9$

$f(m_i) = 29i-25,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(n_1) = 22$

$f(n_i) = 29i-9,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(w_1) = 19$

$f(w_i) = 29i-21,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(x_1) = 11$

$f(x_i) = 29i-22,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(y_i) = 29i-11,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(v_1) = 15$

$f(v_i) = 29i-17,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(r_1) = 4$

$f(r_i) = 29i-19,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(s_1) = 10$

$f(s_i) = 29i-10,\ 2 \le i \le \left(\frac{n}{2}\right)$

$f(z_1) = 1$

$f(z_i) = 29i-13,\ 2 \le i \le \left(\frac{n}{2}\right)$

The labeling pattern of $S[A(T(T_6))]$ is given below

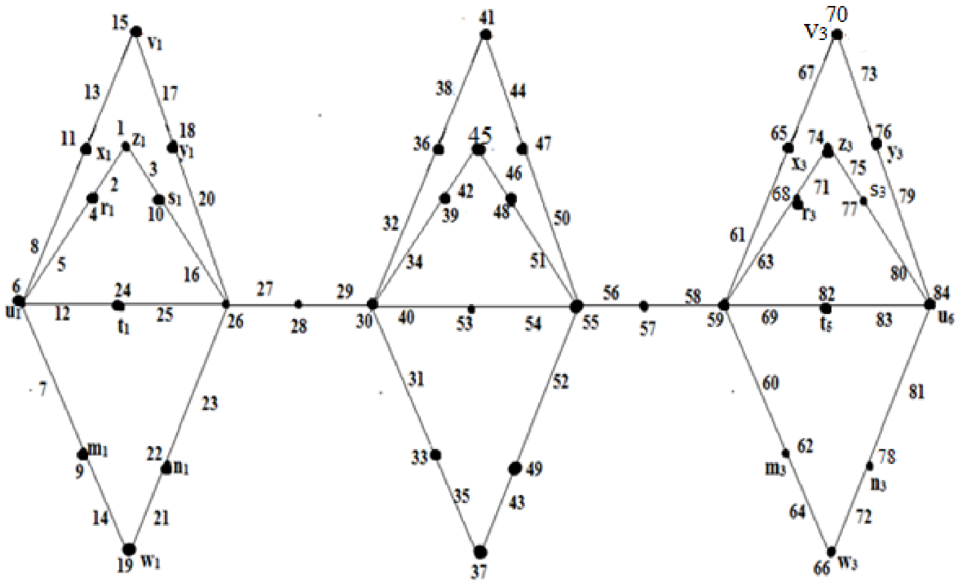


Figure: 12

From the above labeling pattern, we get $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,...,p+q\}$

Hence G admits Super Geometric mean labeling.

Case 2: If $T(T_n)$ starts from u_2,

Let t_i, m_i, n_i, x_i, y_i, r_i and s_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i}w_i$, $u_{2i+1}w_i$, $u_{2i}v_i$, $u_{2i+1}v_i$, $u_{2i}z_i$ and $u_{2i+1}z_i$ respectively.

We have to consider two subcases.

Subcase (2) (a): If ‘n’ is odd, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 29i-28,\ 1\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 29i-24,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 29i-26,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 29i-1,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(m_1) = 9$

$f(m_i) = 29i-21,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(n_i) = 29i-5,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(w_1) = 13$

$f(w_i) = 29i-17,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(x_i) = 29i-18,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(y_i) = 29i-7,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(v_i) = 29i-13,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(r_1) = 15$

$f(r_i) = 29i-15,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(s_i) = 29i-6,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(z_i) = 29i-9,\ 1\le i\le\left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(T(T_7))]$ is shown below.

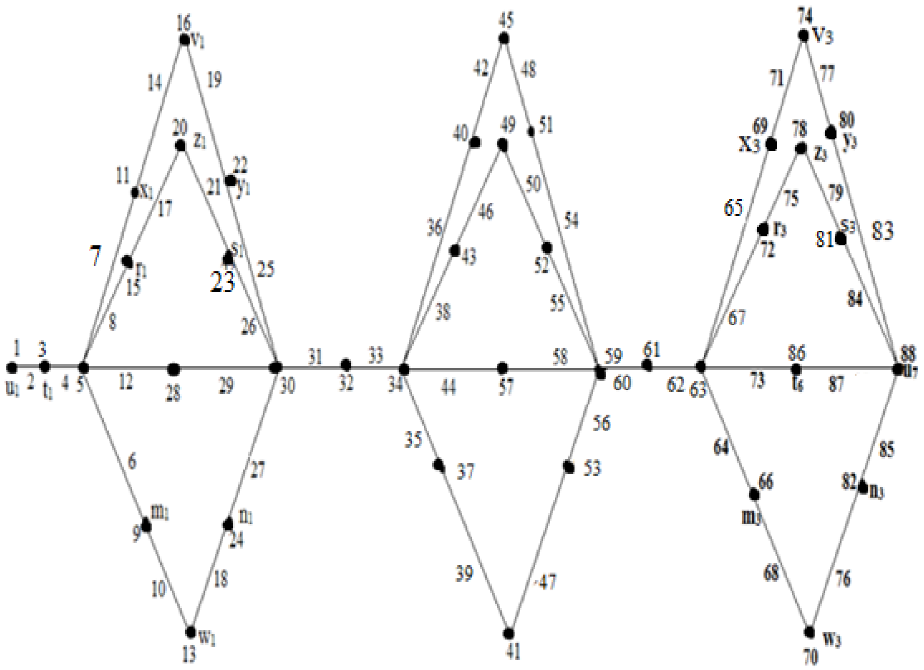


Figure: 13

From the above labeling pattern, both vertices and edges together get distinct labels from {1, 2,…,p+q}.

This makes “f” a Super Geometric mean labeling of G.

Subcase (2) (b): If ‘n’ is even, then

Define a function f: $V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 29i-28,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(u_{2i}) = 29i-24,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 29i-26,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i}) = 29i-1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(m_1) = 9$

$f(m_i) = 29i-21,\ 2 \le i \le \left(\frac{n-2}{2}\right)$

$f(n_i) = 29i-5,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(w_1) = 13$

$f(w_i) = 29i-17,\ 2 \le i \le \left(\frac{n-2}{2}\right)$

$f(x_i) = 29i-18,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(y_i) = 29i-7,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(v_i) = 29i-13,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(r_1) = 15$

$f(r_i) = 29i-15,\ 2 \le i \le \left(\frac{n-2}{2}\right)$

$f(s_i) = 29i-6,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(z_i) = 29i-9,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

The labeling pattern of S[A(T(T_6))] is displayed below.

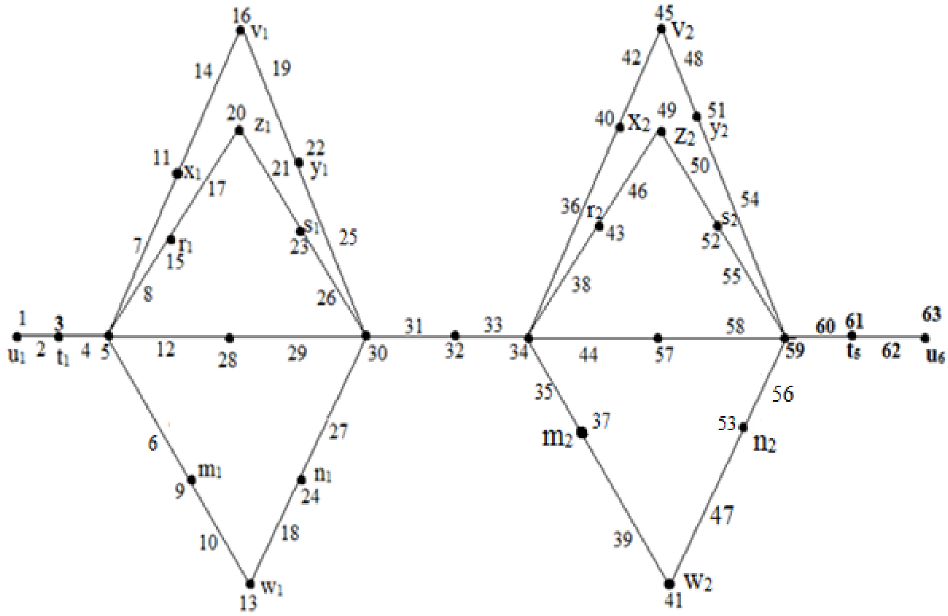


Figure: 14

From the above labeling pattern we get, $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1, 2,\ldots,p+q\}$.

Hence G admits a Super Geometric mean labeling.

From all the above cases, we conclude that Subdivision of Alternate Triple Triangular snake is a Super Geometric mean graph.

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