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PYTHAGOREAN TRIANGLE WITH HYPOTANEOUS MINUS 8 TIMES THE RATIO (AREA/PERIMETER) AS SUM OF TWO SQUARES

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ABSTRACT

Patterns of Pythagorean triangles in each of which hypotaneous minus 8 times Area / Perimeter may be expressed as a sum of two squares.

Keywords: *Area/perimeter, Pythagorean triangle.*

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I. INTRODUCTION

The Pythagorean numbers play a significant role in the theory of higher arithmetic as they come in the majority of indeterminate problems; had a marvelous effect on a credulous people and always occupy a remarkable position due to unquestioned historical importance. The method of obtaining three non-zero integers *x*, *y* and *z* under certain relations satisfying the relation $x^2 + y^2 = z^2$ has been a matter of interest to various Mathematicians [1]-[6]. In [7]-[19], special Pythagorean problems are studied. In this communication, we search for patterns of Pythagorean triangles wherein each of which hypotaneous minus 8 times the ratio (Area / Perimeter) **is** represented as sum of two squares.

II. METHOD OF ANALYSIS

The most cited solution of the Pythagorean equation,

$$x^2 + y^2 = z^2 \tag{1}$$

is represented by

$$x = 2pq; y = p^2 - q^2; z = p^2 + q^2 \tag{2}$$

Denoting the Area and Perimeter of the Pythagorean triangle by A and P respectively, the assumption

$$Hyp - 8\left(\frac{A}{P}\right) = \alpha^2 + \beta^2 \tag{3}$$

leads to the equation

$$p^2 + 5q^2 - 4pq = \alpha^2 + \beta^2 \tag{4}$$

We present below different methods of solving (4) and thus obtain different patterns of integral solutions to (1) satisfying (3).

Pattern I: Assume equation (3) as quadratic in p then

$$p = 2q + \sqrt{\alpha^2 + \beta^2 - q^2}$$

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The square root on the right hand side of the above equation is eliminated when

$$\alpha = u + v; q = u - v; \beta = r - s$$

and thus

$$p = 2(u - v) + r + s \text{ where } uv = rs$$

Substituting the values of p & q in (2) we get the corresponding sides of the Pythagorean triangle are

$$x(u,v) = 4(u^2 - v^2) + 2(r + s)(u - v)$$
$$y(u,v) = 3(u - v)^2 + (r + s)^2 + 4(r + s)(u - v)$$
$$z(u,v) = 5(u - v)^2 + (r + s)^2 + 4(r + s)(u - v)$$

Pattern: II Assume equation (3) as quadratic in q then

$$q = \frac{1}{5}\left[2p + \sqrt{5(\alpha^2 + \beta^2) - p^2}\right] \tag{5}$$

To eliminate the square root on the right hand of (5)

Consider, $$5(\alpha^2 + \beta^2) - p^2 = A^2$$

which is written as

$$A^2 + p^2 = 5(\alpha^2 + \beta^2) \tag{6}$$

Write 5 as

$$5 = (2 + i)(2 - i) \tag{7}$$

Substituting (7) in (6) and factorizing, we have

$$(A + ip)(A - ip) = (2 + i)(2 - i)(\alpha + i\beta)(\alpha - i\beta)$$

Equating real and imaginary parts, we get

$$A = 2\alpha - \beta \;\&\; p = \alpha + 2\beta$$

Now, the value of q is

$$q = \frac{1}{5}\left[4\alpha + 3\beta\right]$$

q will be an integer when $\alpha = (5k - 2)\beta$, and we get the values of p & q as

$$\left.\begin{aligned} p &= 5k\beta \\ q &= (4k - 1)\beta \end{aligned}\right\} \tag{8}$$

Substituting (8) in (2), the corresponding sides of the pythagorean triangle are

$$x(\beta) = 10k\beta^2(4k - 1)$$
$$y(\beta) = \beta^2(9k^2 + 8k - 1)$$
$$z(\beta) = \beta^2(41k^2 - 8k + 1)$$

Remark.1: Instead of (6), write 5 as

$$5 = (-1 + 2i)(-1 - 2i) \tag{9}$$

Substituting (9) in (6) and following the above procedure, we get the corresponding sides of the Pythagorean triangle are

$$x(\beta) = \beta^2(60k^2 - 70k + 20)$$
$$y(\beta) = \beta^2(91k^2 - 88k + 21)$$
$$z(\beta) = \beta^2(109k^2 - 112k + 29)$$

Remark.2: In addition to (7) and (9), one may consider the following representations for 5

$$5 = \begin{cases} (1+2i)(1-2i) \\ \dfrac{(2+11i)(2-11i)}{25} \\ \dfrac{(2+29i)(2-29i)}{169} \end{cases}$$

Repeating the analysis presented above we obtain the corresponding sides of the Pythagorean triangle.

Pattern III: Rewrite equation (4) as

$$(p-2q)^2 + q^2 = (\alpha^2 + \beta^2) \times 1 \tag{10}$$

Write 1 as

$$1 = \frac{(m^2 - n^2 + 2imn)(m^2 - n^2 - 2imn)}{(m^2 + n^2)^2}; m \succ n \succ 0 \tag{11}$$

Substituting (11) in (10) and employing the method of factorization, define

$$(p-2q) + iq = (\alpha + i\beta)\frac{(m^2 - n^2 + 2imn)}{m^2 + n^2}$$

Equating the real and imaginary parts, we get

$$p = \frac{1}{m^2 + n^2}\left[(m^2 - n^2)(\alpha + 2\beta) + 2mn(2\alpha - \beta)\right]$$

$$q = \frac{1}{m^2 + n^2}\left[(m^2 - n^2)\beta + 2mn\alpha\right]$$

When $\alpha = (k(m^2 + n^2) - 2n^2)\beta$ then the values of p & q are

$$\left.\begin{aligned} p &= \frac{1}{m^2 + n^2}\left[(m^4 - n^4)k^2\beta - 2\beta(n^2 - 1)(m^2 - n^2) + 4kmn\beta(m^2 + n^2) - 2mn\beta(4n^2 + 1)\right] \\ q &= \frac{1}{m^2 + n^2}\left[(m^2 + n^2)2kmn\beta - 4\beta mn^3 + (m^2 - n^2)\beta\right] \end{aligned}\right\} \tag{12}$$

Substituting (12) in (2) one may get the corresponding sides of the pythagorean triangle.

3. CONCLUSION

In this paper, we have presented Pythagorean triangle with hypotaneous minus 8 times the ratio (Area/Perimeter) as sum of two squares. It is worth to note that Pythagorean problem is a treasure house and finding patterns of Pythagorean triangle is a treasure hunt.

To conclude one may search for other patterns of Pythagorean triangle with various characterizations.

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