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THE FULL LINE GRAPH AND THE FULL BLOCK GRAPH OF A GRAPH

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ABSTRACT

In this paper, we introduce the concepts of the full line graph of a graph and the full block graph of a graph. We establish some properties of these graphs. Also characterizations are given for graphs for which(i) the full line graph of G and the full graph of G are isomorphic ii) the full block graph of G and full graph of G are isomorphic.

Keywords: *full graph, semifull line graph, semifull block graph, full line graph, full block graph.*

Mathematics Subject Classification: *05C10.*

1. INTRODUCTION

By a graph, we mean a finite, undirected without loops or multiple lines. Any undefined term or notation in this paper may be found in Kulli [1].

If $B = \{u_1, u_2, \ldots, u_r, r \geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two blocks B_1 and B_2 of G are incident with a common cut point, then they are adjacent blocks. If $B = \{e_1, e_2, \ldots, e_s, s \geq 1\}$ is a block of a graph G, then we say that line e_1 and block B are incident with each other, as are e_2 and B and so on. This idea was introduced by Kulli in [2]. The points, lines and blocks of a graph are called its members.

The full graph $F(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding members of G are adjacent or incident. This concept was introduced by Kulli in [3].

The semifull line graph $F_l(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding points and lines of G are adjacent or one corresponds to a point of G and other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B. This concept was introduced in [4].

The semifull block graph $F_b(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding points and blocks of G are adjacent or one corresponds to a point of G and other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B. This concept was introduced in [4].

The block line forest $B_f(G)$ of a connected graph G is the graph whose point set is the union of the set of lines and the set of blocks of G in which two points are adjacent, if one corresponds to a block and other to a line incident with it. This concept was introduced by Kulli in [5]. Many other graph valued functions in graph theory were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] and also graph valued functions in domination theory were studied, for example, in [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

The following will be useful in the proof of our results.

Theorem A [4]: If G is a connected (p, q) graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the semifull line graph $F_l(G)$ of G has $q + \sum b_i + 1$ points and $2q + \sum b_i + \frac{1}{2}\sum d_i^2$ lines.

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Theorem *B* [4]: If G is a connected (p, q) graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the semifull block graph $F_b(G)$ of G has $q+\sum b_i+1$ points and $3q+\frac{1}{2}\sum b_i\left(b_i+1\right)$ lines.

Theorem C [5]: If G is a nontrivial connected (p, q) graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the block line forest $B_f(G)$ of G has $q-p+\sum b_i+1$ points and q lines.

Theorem D [3]: Let G be a graph without isolated points. Then $F(G)$ is complete if and only if G is P_2.

Theorem E [36]: Let G be a connected graph. Then $L(G) = B(G)$ if and only if G is a tree.

2. FULL LINE GRAPHS

The definition of the full graph $F(G)$ of a graph G inspired us to introduce the full line graph of a graph.

Definition 1: The full-line graph $FL(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding points and lines of G are adjacent or the corresponding members of G are incident.

Example 2: In Figure 1, a graph G and its full line graph $FL(G)$ are shown.

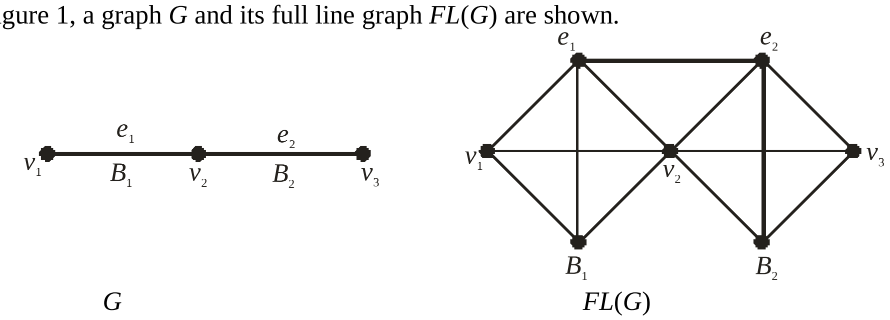

Figure-1

Remark 3: If G is a connected graph, then $FL(G)$ is also connected and conversely.

Remark 4: The full line graph $FL(G)$ of G is a spanning subgraph of the full graph $F(G)$ of G.

Remark 5: For any graph G, $F(G) = FL(G) \cup B(G)$.

Remark 6: The semifull line graph $F_l(G)$ of G is a spanning subgraph of the full line graph $FL(G)$ of G.

Remark 7: The block-line forest $B_f(G)$ of G is a subgraph of the full line graph $FL(G)$ of G.

Proposition 8: For any graph G, $FL(G) = F_l(G) \cup B_f(G)$.

We now determine the number of points and lines in $FL(G)$.

Theorem 9: If G is a (p, q) connected graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the full line graph $FL(G)$ of G has $q+\sum b_i+1$ points and $3q+\sum b_i+\frac{1}{2}\sum d_i^2$ lines.

Proof: By Remark 6, $F_l(G)$ is a spanning subgraph of $FL(G)$. Thus the number of points of $F_l(G)$ equals the number of points of $FL(G)$. By Theorem A, $F_l(G)$ has $q+\sum b_i+1$ points. Hence the number of points in

$FL(G) = q+\sum b_i+1$.

By Proposition 8, the number of lines in $FL(G)$ is the sum of the number of lines in $F_l(G)$ and the number of lines in $B_f(G)$. By Theorem A, F_l(G) has $2q+\sum b_i+\frac{1}{2}\sum d_i^2$ lines and by Theorem C, $B_f(G)$ has q lines. Hence the number of lines in

$$FL(G) = 2q+\sum b_i+\frac{1}{2}\sum d_i^2+q$$

$$= 3q+\sum b_i+\frac{1}{2}\sum d_i^2 .$$

We characterize graphs whose full line graphs and full graphs are isomorphic.

Theorem 10: Let G be a nontrivial connected graph. The graphs $FL(G)$ and $F(G)$ are isomorphic if and only if G is a block.

Proof: Let G be a block. Then the graphs $FL(G)$ and $F(G)$ have some number of points. Since G has only one block, and by definitions, we have $FL(G) = F(G)$.

Conversely suppose $FL(G) = F(G)$ and G is a nontrivial connected graph. We now prove that G is a block. On the contrary, assume G has at least two blocks. By Remark 5, the number of lines in $F(G)$ is the sum of the number of lines in $FL(G)$ and the number of lines in $B(G)$. Since G has at least two blocks, it implies that $B(G)$ has at least one line. Thus the number of lines in $FL(G)$ is less than that the number of lines in $F(G)$. Hence $FL(G)$ and $F(G)$ are not isomorphic, which is a contradiction. Thus G has no two or more blocks. Hence G is a block.

Corollary 11: Let G be a graph without isolated points. The graphs $FL(G)$ and $F(G)$ are isomorphic if and only if each component of G is a block.

We now present a characterization of graphs whose full-line graphs are complete.

Theorem 12: Let G be a graph without isolated points. Then $FL(G)$ is complete if and only if G in P_2.

Proof: Suppose $G = P_2$. Then G is a block. By Theorem 10, $FL(G) = F(G)$. By Theorem D, $F(G)$ is complete if and only if $G = P_2$ and since $FL(G) = F(G)$, this implies that $FL(G)$ is complete if and only if $G = P_2$.

Corollary 13: Let G be a graph without isolated points. Then $FL(G) = mK_4$ if and only if $G = mP_2$, $m \geq 1$.

We note that if $G = P_2$, then $FL(G) = K_4$.

3. FULL BLOCK GRAPHS

We now introduce the following graph valued function.

Definition 14: The full-block graph $FB(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding points and blocks of G are adjacent or the corresponding members of G are incident.

Example 15: In Figure 2, a graph G and its full block graph $FB(G)$ are shown.

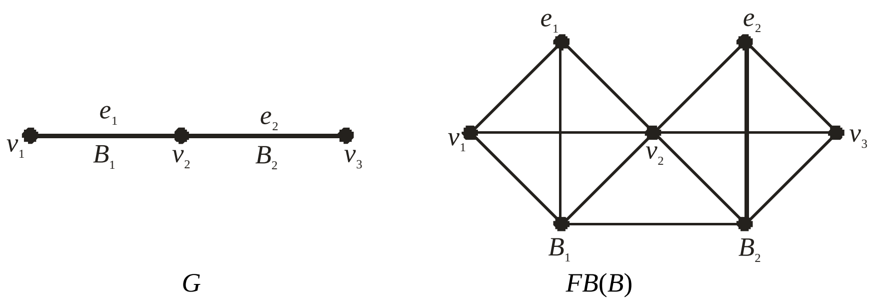

Figure-2

Remark 16: If G is a connected graph, then $FB(G)$ is also connected and conversely.

Remark 17: The full-block graph $FB(G)$ is a spanning subgraph of the full graph $F(G)$ of G.

Remark 18: For any graph G, $F(G) = FB(G) \cup L(G)$.

Remark 19: The semifull block graph $F_b(G)$ is a spanning subgraph of the full-block graph $FB(G)$ of G.

Remark 20: The block line forest $B_f(G)$ of G is a subgraph of the full block graph $FB(G)$ of G.

Proposition 21: For any graph G, $FB(G) = F_b(G) \cup B_f(G)$.

We now obtain a result which determines the number of points and lines in $FB(G)$.

Theorem 22: If G is a (p, q) connected graph whose points have degree d_i and if b_i is the number of blocks to which point v_i belongs in G, then the full block graph $FB(G)$ of G has $q+\sum b_i+1$ points and $4q+\frac{1}{2}\sum b_i\left(b_i+1\right)$ lines.

Proof: By Remark 19, $F_b(G)$ is a spanning subgraph of $FB(G)$. Thus the number of points of $F_b(G)$ equals the number of points of $FB(G)$. By Theorem B, $F_b(G)$ has $q+\sum b_i+1$ points. Thus the number of points in $FB(G)$ = $q+\sum b_i+1$.

By Proposition 21, the number of lines in $FB(G)$ is the sum of the number of lines in $F_b(G)$ and the number of lines in $B_f(G)$. By Theorem B, $F_b(G)$ has $3q+\frac{1}{2}\sum b_i\left(b_i+1\right)$ lines and by Theorem C, $B_f(G)$ has q lines. Thus the number of lines in

$$FB(G) = 3q+\frac{1}{2}\sum b_i\left(b_i+1\right)+q$$

$$= 4q+\frac{1}{2}\sum b_i\left(b_i+1\right).$$

We characterize graphs whose full block graphs and full graphs are isomorphic.

Theorem 23: Let G be a nontrivial connected graph. The graphs $FB(G)$ and $F(G)$ are isomorphic if and only if $G = P_2$.

Proof: Suppose G is P_2. Then clearly $FB(G) = F(G) = K_4$.

Conversely suppose G is a nontrivial connected graph and $FB(G) = F(G)$. We now prove that $G = P_2$. On the contrary, assume G has at least two lines. We have by Remark 18, the number of lines in $F(G)$ is the sum of the number of lines in $FB(G)$ and the number of lines in $L(G)$. Since G has at least two lines, it implies that $L(G)$ has at least one line. Hence the number of lines in $FB(G)$ is less than that the number of lines in F(G). Thus $FB(G)$ and $F(G)$ are not isomorphic, a contradiction. Thus G has no two or more lines. Hence G is P_2.

Corollary 24: Let G be a graph without isolated points. The graphs $FB(G)$ and $F(G)$ are isomorphic if and only if $G = mP_2$, $m \geq 1$.

We now characterize graphs whose full-block graphs are complete.

Theorem 25: Let G be a graph without isolated points. Then $FB(G)$ is complete if and only if $G = P_2$.

Proof: The result follows from Theorem 23 and Theorem D.

Corollary 26: Let G be a graph without isolated points. Then $FB(G)= mK_4$ if and only if $G = mP_2$, $m \geq 1$.

The following is a characterization of graphs whose full-line graphs and full-block graphs are isomorphic.

Theorem 27: Let G be a nontrivial connected graph. The graphs $FL(G)$ and $FB(G)$ are isomorphic if and only if G is a tree.

Proof: Suppose G is a tree. Then $FL(G) = FB(G)$, since lines and blocks coincide.

Conversely suppose $FL(G) = FB(G)$ and G is a nontrivial connected graph. We now prove that G is a tree. By definitions, $L(G) \subseteq FL(G)$ and $B(G) \subseteq FB(G)$. Since $FL(G) = FB(G)$, it implies that $L(G) = B(G)$. By Theorem E, G is a tree.

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