

**MHD COUETTE FLOW AND HEAT TRANSFER OF A DUSTY COUPLE STRESS FLUID
WITH EXPONENTIAL DECAYING PRESSURE GRADIENT**

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ABSTRACT

In the present study, the unsteady couette flow with heat transfer of a viscous incompressible electrically conducting couple stress fluid under the influence of an exponentially decreasing pressure gradient is studied. The parallel plates are assumed to porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to plates. The equations of motion are solved by using transform technique to get the velocity distributions for the fluid and dust particles. The energy equations for both the fluid and dust particles including the viscous, couple stress parameter Joule dissipation terms are solved by using Mathematica to get temperature distributions.

Key – words: *couette flow, magnetohydrodynamics, heat transfer, dusty fluid, couple stress parameter.*

INTRODUCTION

The importance and application of solid/fluid flows and heat transfer in petroleum transport waste water treatment, combustion, power piping, corrosive particles in engine oil flow, and many other are well known in the literature [1-5]. Particularly, the flow and heat transfer of electrically conducting fluid in channels and circular pipes under the effect of transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and has possible applications in nuclear reactors filtration, geothermal systems and others.

The flows of couple stress fluids have many practical application in modern technology and industries, led various researchers to attempt diverse flow problems related to several non-Newtonian fluids one such fluid that has attracted the attention of numerous researchers in fluid mechanics during the last five decades in the theory of couple stress fluid proposed by stokes [6]. The concept of couple stress arises due to the way in which the mechanical interactions in the fluid medium are modeled. Singh and Pathak [7] have discussed unsteady flow of a dusty viscous fluid through a uniform pipe with sector of a circle as cross-section, and Pulsatile flow of blood with micro-organism through a uniform pipe with sector of a circle as cross-section, in the presence of transverse magnetic field has been investigated by Rathod and Parveen [8]. Also unsteady flow of a dusty magnetic conducting couple stress fluid through a pipe and the flow of a conducting fluid in a circular pipe has been investigate by many authors Gudiraju *et.al*, [9]. Dube and Sharma [10] and Ritter and Peddieson [11] have reported solutions for unsteady dusty gas flow in a circular pipe in the absence of a magnetic field and particle phase viscous stress. Rathod *et.al*, [12] have reported solution for couette flow of a conducting dusty visco-elastic fluid through two flat plate under the influence of transverse magnetic field. Rathod and Rasheeda [13-14] investigated unsteady flow of a dusty magnetic conducting couple stress fluid through a circular pipe, and couette flow with heat transfer of a couple stress fluid under exponential decaying pressure gradient.

The possible presence of solid particles such as ash or soot in combustion MHD generator's and plasma MHD accelerators and their effect on the performance of such device led to studied of particular suspensions in conducting fluid in the presence of magnetic fields. For example in an MHD generator coal mixed with seed is fed into a combustor the coal and seed mixture is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas mixture flowing through the MHD channel consists of a condensable vapo (slag) and a non-condensable gas mixed with seeded coil combustion products. Both the slag and the non-condensable gas are electrically conducting [1, 2]. The presence of the slag and the seeded particles significantly influence the flow and heat transfer characteristic in the MHD channel. Ignoring the effect of the slag and considering the MHD generator start up condition, the problem reduces to unsteady two-phase flow in an MHD channel [15, 16].

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In the present work, the transient couette flow with heat transfer of an electrically conducting, viscous, incompressible, dusty couple stress fluid are studied. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is acted upon by an exponentially decaying with time pressure gradient. The couple stress fluid is assumed to be incompressible and electrically conducting and particle phase is assumed to be incompressible, electrically non-conducting dusty and pressure less. The fluid is flowing between two infinite electrically insulating porous plates maintained at two constant but different temperatures while the particle phase is assumed to be electrically non-conducting. The fluid is subjected to a uniform suction from above and a uniform injection from below and mass conservation is assumed. An external uniform magnetic field is applied perpendicular to the plates while no electric field is applied and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The governing equation for both fluid and dust particles are solved numerically taking the Joule and viscous dissipation into consideration in the energy equations. The effect of the magnetic field, couple stress parameter. The Hall Current, the ion slip, and the suction velocity on the both the velocity and temperature field are reported.

DESCRIPTION OF THE PROBLEM

The couple stress dusty fluid is assumed to be flowing between two infinite horizontal porous plates located at the $y=\pm h$ planes. The upper plates is moving with a constant velocity u_0 while the lower plates is kept stationary. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the y -component of the velocity of the fluid is constant and denoted by v_0 . The dust particles are assumed to be electrically non-conducting spherical in shape and uniformly distributed throughout the fluid. So that they are not pumped out through the porous plates and have no y -component of velocity. The two plates are assumed to be electrically non-conducting and kept at two constant temperature T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$.

A uniform pressure gradient, which is taken to be exponentially decaying with time, is applied in the x -direction. A uniform magnetic field B_0 is applied in the y -direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [18]. The fluid motion starts from rest at $t=0$, and the no-slip condition at the plates implies that the fluid and dust particle velocities have neither a z nor an x -component at $y=\pm h$. The initial temperature of the fluid and dust particle are assumed to be equal to T_1 . It is required to obtain the time varying velocity and temperature distributions for both fluid and dust particles. Due to the inclusion Hall Current term, a z -component of the velocities of the fluid and dust particles, is expected to arise. Since the plates are infinite in the x and z - direction. The physical quantities do not change in these direction that $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$ and the problem is essentially one - dimensional. The governing equation for this study are based on the conservation law of mass, linear momentum and energy of both phases are treated as two interacting continua. The interaction between the phases is restricted to the inter phase drag force which is modeled by Stokes linear drag theory and inter phase heat transfer. The flow of fluid is governed by the momentum equation.

$$\rho \frac{Dv}{Dt} = \nabla p + \mu \nabla^2 v + J \times B_0 - \eta \nabla^2 (\nabla^2 v) - KN(v - v_p) \quad (1)$$

Where ρ is density of the clean fluid, μ is the viscosity of the clean fluid, v is the velocity of the fluid, $v = u(y, t) \mathbf{i} + v_0 \mathbf{j}$. v_p is the velocity of the dust particles $v_p = u_p(y, t) \mathbf{i}$, J is the Current density. N is the number of dust particles per unit volume. K is the Stokes constant $= 6\bar{\lambda}\mu a$, and a is the average radius of the dust particles. The first four terms in the right hand side of eqⁿ (1) are the pressure gradient, viscosity, Lorentz force terms couple stress terms respectively.

The last term represents the force due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of relative velocity is small, In such a case the force between dust and fluid is proportional to the relative velocity [3]. The current density J from the generalized ohm's law is given by [17]

$$J = \sigma[\sigma + v \times B_0] \quad (2)$$

Where σ is the electric conductivity of the fluid [17] solving Equation (2) for J and substituting the result in Equation (1), the two components of Equation (1)

$$\rho \frac{\partial u}{\partial t} = \rho v_0 \frac{\partial u}{\partial y} - \frac{dp}{dn} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u - \eta \nabla^2 (\nabla^2 u) - KN(u - u_p) \quad (3)$$

The motion of the dust particle is governed by Newton's Second law applied in the x and z direction.

$$m_p \frac{du_p}{dt} = KN(u - u_p) \quad (4)$$

Where m_p is the average mass of dust particles, It is assumed that the pressure gradient is applied at $t=0$ and the fluid starts its motion from rest. Thus

$$t \leq 0: u = u_p = 0. \quad (5)$$

For $t > 0$, the no-slip condition at the plates implies that
 $t > 0$: $y = -h$, $u = u_p$, $y = h$, $u = U_o$, $u_p = 0$.

(6)

Heat transfer takes place from the upper hot plate to the lower cold plate by conduction through the fluid. Since the hot plate is above, there is no natural convection, however, there is a forced convection due to the suction and injection. In addition to heat transfer, there is a heat generation due to both the Joule and viscous dissipations. The dust particle gain heat from the fluid by conduction through their spherical surface. Since the problem deals with a two – phase flow, two energy equations are required [18]. The energy equations describing the temperature distributions for both the fluid and dust particles read.

$$\rho_o c \frac{\partial T}{\partial t} + \rho_o c v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{6}{\gamma_T} B_o^2 u^2 + \frac{\rho_p C_s}{\gamma_T} (T_p - T) \quad (7)$$

$$\frac{\partial T_p}{\partial t} = \frac{1}{\gamma_T} (T_p - T) \quad (8)$$

Where T is the temperature of the fluid, T_p is the temperature of the particles. C is the specific heat capacity of the fluid at constant volume, k is the thermal conductivity of the fluid, ρ_o is the mass of dust particles per unit volume of the fluid. γ_T is the temperature relaxation time, and C_s is the specific heat capacity of the particles. The last three terms on the right hand side of equation (7) represent the viscous dissipation, the Joule dissipation, the Joule dissipation ($J^2/6$), and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends, in general, on the geometry and since the dust particles are assumed to be spherical in shape, the last term in equation (7) is equal to

$$4\pi a NK(T_p - T). \text{ Hence } \gamma_T = (3P_r \gamma_p c)/2C$$

Where γ_p is the velocity relaxation time $= 2\eta d^2/9\mu_s p_r$ is the prandtl number $= \mu c/k$, and ρ_s is the material density of dust particles $3P_r/4\pi a^3 N$.

T and T_p must satisfy the initial and boundary conditions.

$$t \leq 0: T = T_p = 0.$$

$$t > 0, y = -h, T = T_p = T_1$$

$$t > 0 y = h: T = T_p = T_2 \quad (9)$$

The problem is simplified by writing the equations in the non-dimensional form. We define the following non-dimensional quantities.

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{u}{u_o}, \hat{p} = \frac{P}{\rho u_o^2}, \hat{t} = \frac{t u_o}{h},$$

$$\hat{u}_p = \frac{u_p}{u_o}, \hat{T} = \frac{T - T_1}{T_2 - T_1}, \hat{T}_p = \frac{T_p - T_1}{T_2 - T_1},$$

$Re = \rho h u_o / \mu$, is the Reynolds number.

$S = \gamma_o / u_o$ is the suction parameter.

$P_r = \mu c / k$ is the prandtl number.

$H_a^2 = 6B_o^2 h^2 / \mu$ where H_a is the Hartmann number.

$G = m_p \mu / q h^2 k$ is the particle mass parameter.

$R = \frac{KN h^2}{\mu}$ is the particle concentration parameter.

$Ec = u_o^2 / c (T_2 - T_1)$ is the Eckert number.

$L_o = q h^2 / \mu_T$ is the temperature relaxation time parameter.

In terms of the above non-dimensional quantities the velocity and energy equations read.

$$\frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha^2} \frac{1}{Re} \frac{\partial^4 u}{\partial y^4} - \frac{1}{Re} H_a^2 u - \frac{R}{Re} (u - u_p) \quad (10)$$

$$G \frac{\partial u_p}{\partial t} = u - u_p \quad (11)$$

$$\frac{\partial T}{\partial t} + S \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2 + H_a^2 Ec u^2 + \frac{2R}{3P_r} (T_p - T) \quad (12)$$

$$\frac{\partial T_p}{\partial t} = - L_o (T_p - T) \quad (13)$$

With the initial and boundary condition,

$$t \leq 0: u = u_p = 0, t > 0; y = -1, u = u_p = 0, y = 1, u = 1, u_p = 0. \quad (14)$$

$$t \leq 0, T = T_p = 0, t > 0, T_p = 0, y = -1,$$

$$T = T_p = 0, y = 1, T = T_p = 1 \quad (15)$$

Where the pressure gradient is assumed in the form $\frac{dp}{dx} = ce^{-at}$.

Equation (10), (11), (12) and (13) represent a system of coupled and nonlinear partial differential equation which are solved by using transformation method (cosine and Hankle transformation).

$$u = \frac{4}{\alpha} \sum_{m=0}^{\infty} \frac{ca}{Re} \left(1 - \frac{1}{G}\right) \frac{(-1)^n}{\left(\frac{2m+1}{2\alpha}\right)} \frac{1}{a^2 - ax_1 + x_2} \left[e^{-at} + \frac{1}{(x_1^2 - 4x_2)^{1/2}} \{(\alpha_2 + a)e^{\alpha_1 t} - (\alpha_1 + a)e^{2t}\} \right] \sum_{i=1}^{\infty} \frac{1}{\epsilon_i} \frac{J_0(y\epsilon_i)}{J_1(\epsilon_i)} - \cos \frac{(2m+1)\bar{\alpha}y}{2\alpha} \quad (16)$$

$$u_p = \frac{4}{\alpha} \sum_{m=0}^{\infty} \frac{ca}{Re} \left(1 - \frac{1}{G}\right) \frac{(-1)^n}{\left(\frac{2m+1}{2\alpha}\right)} \left(1 - e^{-\frac{1}{G}t}\right) \frac{1}{a^2 - ax_1 + x_2} \left[\frac{1}{1-aG} \left(e^{-at} - e^{-\frac{1}{G}t}\right) + \frac{1}{(x_1^2 - 4x_2)^{1/2}} \left\{ \frac{(\alpha_2 + a)(e^{\alpha_1 t} - e^{-\frac{1}{G}t})}{(1+\alpha_1 G)} - \frac{(\alpha_1 + a)(e^{\alpha_2 t} - e^{-\frac{1}{G}t})}{(1+\alpha_2 G)} \right\} \right] \sum_{i=1}^{\infty} \frac{1}{\epsilon_i} \frac{J_0(y\epsilon_i)}{J_0(\epsilon_i)} - \cos \frac{(2m+1)\bar{\alpha}y}{2\alpha} \quad (17)$$

$$T = \frac{\beta}{(y_1^2 - 4y_2)^{1/2}} \left\{ \frac{(-2a+L_0)[(-2a-Z_2)e^{Z_1 t} - (-2a-Z_1)e^{Z_2 t} + e^{-Zat}]}{4a^2 + ay_1 + y_2} + \frac{1}{(x_1^2 - 4x_2)} \left\{ \frac{(\alpha_2 + a)^2(2\alpha_1 + L_0)[(2\alpha_1 - Z_2)e^{Z_1 t} - (2a_2 - Z_1)e^{Z_2 t} + e^{-2\alpha_1 t}]}{4\alpha_1^2 - 2x_1 y_1 + y_2} + \frac{(\alpha_1 + a)^2(2\alpha_2 + L_0)[(2\alpha_2 - Z_2)e^{Z_1 t} - (2\alpha_2 - Z_1)e^{Z_2 t} + e^{2\alpha_2 t}]}{4\alpha_2^2 - 2\alpha_2 y_1 + y_2} - \frac{2(\alpha_1 + a)(\alpha_2 + a)(\alpha_1 + \alpha_2 + L_0)[(\alpha_1 + \alpha_2 Z_2)e^{Z_1 t} - (\alpha_1 + \alpha_2 - Z_1)e^{Z_2 t} + e^{(\alpha_1 + \alpha_2)t}]}{(\alpha_1 + \alpha_2)^2 - (\alpha_1 + \alpha_2)y_1 + y_2} + \frac{2}{(x_1^2 - 4x_2)^{1/2}} \left\{ \frac{(\alpha_2 + a)(\alpha_1 - a + L_0)[(\alpha_1 - a - Z_2)e^{Z_1 t} - (\alpha_1 - a - Z_1)e^{Z_2 t} + e^{(\alpha_1 - a)t}]}{(\alpha_1 - a)^2 - (\alpha_1 - a)y_1 + y_2} - \frac{(\alpha_1 + a)(\alpha_2 - a + L_0)[(\alpha_1 - a - Z_2)e^{Z_1 t} - (\alpha_1 - a - Z_1)e^{Z_2 t} + e^{(\alpha_2 - a)t}]}{(\alpha_2 - a)^2 - (\alpha_2 - a)y_1 + y_2} \right\} \right\} \quad (18)$$

$$T_p = \frac{\beta}{(y_1^2 - 4y_2)^{1/2}} \left\{ \frac{(-2a+L_0)}{4a^2 + 2ay_1 + y_1} \left[\frac{(-2a+Z_2)(e^{Z_1 t} - e^{-L_0 t})}{(Z_1 + L_0)} - \frac{(-2a-Z_1)(e^{Z_2 t} - e^{-L_0 t})}{(Z_2 + L_0)} \right] + \frac{1}{(x_1^2 - 4x_2)} \left\{ \frac{(\alpha_2 + a)^2(2\alpha_1 + L_0)}{4\alpha_1^2 - 2x_1 y_1 + y_2} \frac{(2\alpha_1 - Z_2)(e^{Z_1 t} - e^{-L_0 t})}{(Z_1 + L_0)} - \frac{(2\alpha_2 - Z_1)(e^{Z_2 t} - e^{-L_0 t})}{(Z_2 + L_0)} + \frac{[e^{-2\alpha_1 t} - e^{-L_0 t}]}{(2\alpha_1 + L_0)} + \frac{(\alpha_1 + a)^2(2\alpha_2 + L_0)}{(4\alpha_2^2 - 2\alpha_2 y_1 + y_2)} \left(\frac{[(2\alpha_2 + Z_2)e^{Z_1 t} - e^{-L_0 t}]}{(Z_1 + L_0)} - \frac{(2\alpha_2 - Z_1)(e^{Z_2 t} - e^{-L_0 t})}{(Z_2 + L_0)} + \frac{(e^{2\alpha_2 t} - e^{-L_0 t})}{2\alpha_2 + L_0} \right) - \frac{2(\alpha_1 + a)(\alpha_2 + a)(\alpha_1 + \alpha_2 + L_0)}{(\alpha_1 + \alpha_2)^2 - (\alpha_1 + \alpha_2)y_1 + y_2} \left[(\alpha_1 + \alpha_2 - Z_2) \frac{(e^{Z_1 t} - e^{-L_0 t})}{(Z_1 + L_0)} - (\alpha_1 + \alpha_2 - Z_1) \frac{(e^{Z_2 t} - e^{-L_0 t})}{(Z_2 + L_0)} \right] + \frac{e^{(\alpha_1 + \alpha_2)t} - e^{-L_0 t}}{(\alpha_1 + \alpha_2) + L_0} \right\} + \frac{2}{(x_1^2 - 4x_2)^{1/2}} \left[\frac{(\alpha_2 + a)(\alpha_1 - a + L_0)(\alpha_1 - a - Z_2)(e^{Z_1 t} - e^{-L_0 t})}{(\alpha_1 - a)^2 - (\alpha_1 - a)y_1 + y_2} \frac{1}{(Z_1 + L_0)} - \frac{(\alpha_1 + a)(\alpha_2 - a - L_0)}{(Z_2 + L_0)} \frac{1}{(e^{Z_2 t} - e^{-L_0 t})} + \frac{(\alpha_1 - a - Z_1)(e^{Z_2 t} - e^{-L_0 t})}{(Z_2 + L_0)} + \frac{e^{(\alpha_1 + a)t} - e^{-L_0 t}}{(\alpha_1 - a) + L_0} - \frac{(\alpha_2 - a)^2 - (\alpha_2 - a)y_1 + y_2}{(e^{\alpha_2 t} - a)t - e^{-L_0 t}} \right] \right\} \quad (19)$$

$$\text{Where } B = (1 + H_a^2) \left(\frac{4}{\alpha} \sum_{m=0}^{\infty} \frac{ca}{Re} \left(1 - \frac{1}{G}\right) \frac{(-1)^m}{\left(\frac{2m+1}{2\alpha}\right)} \frac{1}{a^2 - ax_1 + x_2} \right)^2 \left(\sum_{i=1}^{\infty} \frac{1}{\epsilon_i} \frac{J_0(y\epsilon_i)}{J_0(\epsilon_i)} - \cos \frac{(2m+1)\bar{\alpha}y}{2\alpha} \right)^2$$

$$Y_1 = S - L_0 \frac{1}{Pr} - L_0 \frac{2R}{3Pr}, \quad X_1 = S + \frac{1}{Re} + \frac{1}{\bar{\alpha}^2 Re^2}$$

$$Y_2 = L_0 S - \frac{L_0}{Pr}, \quad X_2 = \frac{1}{Re} \left[H_a^2 - \frac{1}{G} + R + \frac{1}{Re G} + \frac{S}{G} - \frac{1}{\bar{\alpha}^2 Re} \right], \quad X_3 = \frac{Ca}{Re} \left(1 - \frac{1}{G}\right) \frac{(-1)^m}{\left(\frac{2m+1}{2\alpha}\right)}$$

$$\alpha_1 = \frac{1}{2} \left[-x_1 + \sqrt{x_1^2 - 4y_2} \right],$$

$$\alpha_2 = \frac{1}{2} \left[-x_1 - \sqrt{x_1^2 - 4y_2} \right],$$

$$Z_1 = \frac{y_1 + (y_1^2 - 4y_2)^{1/2}}{2}, \quad Z_2 = \frac{y_1 - \sqrt{y_1^2 - 4y_2}}{2},$$

Computations have been made for $c = -5$, $\alpha = 1$, $Re = 1$, $Pr = 1$, $R = 0.5$, $L_0 = 0.7$, $G = 0.8$, and $Ec = 0.2$. $Ha = h\bar{\alpha}^2 = q$ = couple stress parameter, Plotted the graph for different values of couple stress parameter, suction parameter, Hartman, decaying parameter and time by using Mathematica.

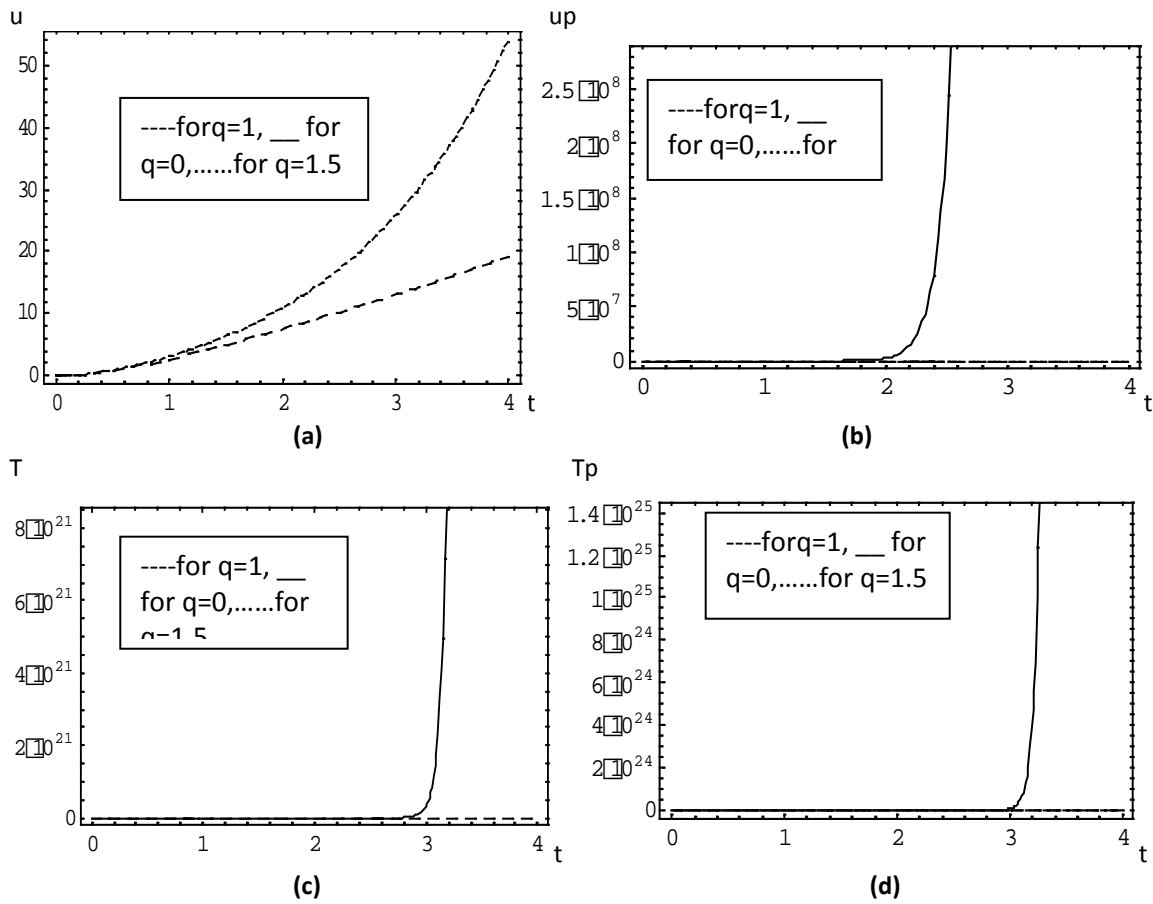


Fig. 1: Effect of q on time variation of (a) u , (b) up , (c) T , (d) Tp $\{y=0, h=0.5, s=0.5\}$

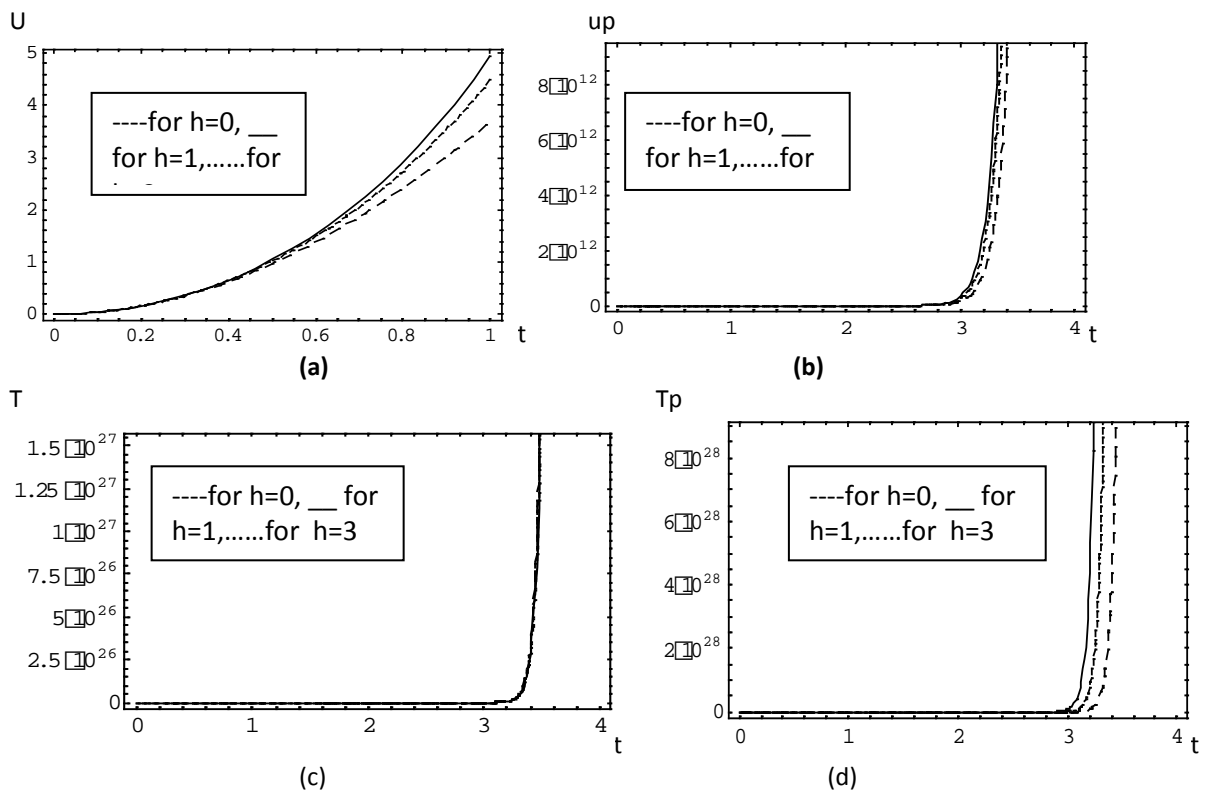


Fig. 2: Effect of h on time variation of (a) u , (b) up , (c) T , (d) Tp $\{y=0, q=0.1, s=1\}$

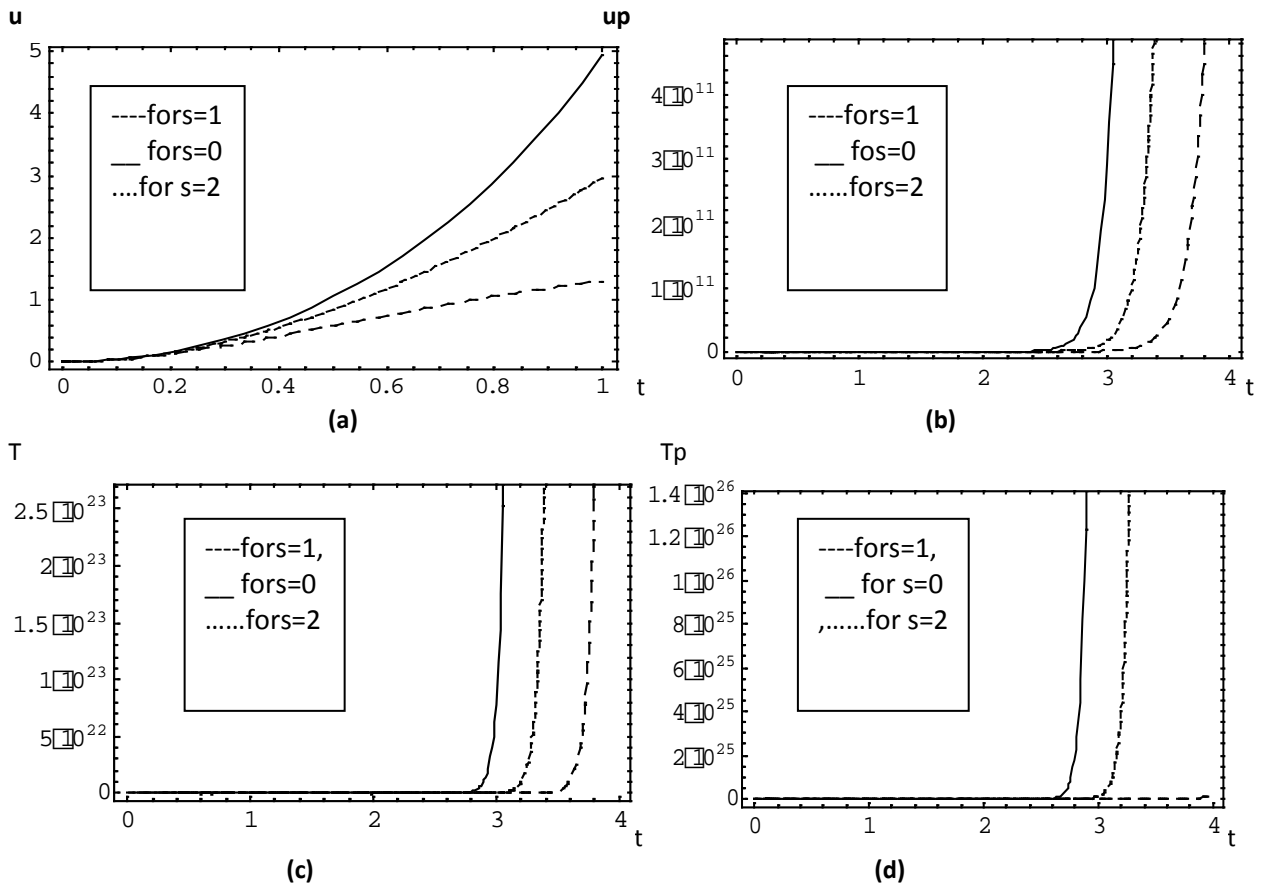


Fig. 3: Effect of s on time variation of (a) u , (b) up , (c) T , (d) Tp $\{y=0, q=0.1, h=1\}$

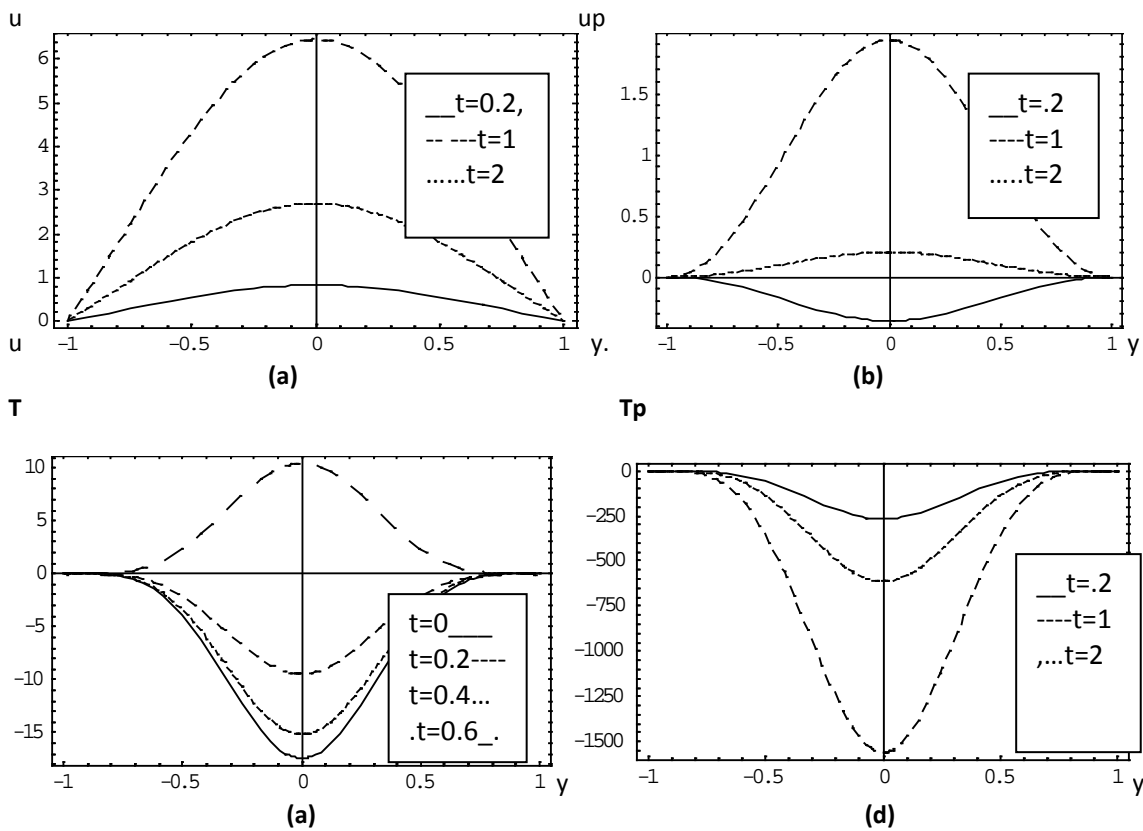


Fig. 4: Time variation of the profile of (a) u , (b) up , (c) T , (d) Tp $\{h=1, s=1, y=0\}$

RESULT AND DISCUSSION

Figure 1 presents, respectively, the profiles of the velocity components and temperature of the fluid u and T and particles u_p and T_p for various values of couple stress parameter. The figures are plotted for $H_a = 0.5$ and $S=0.5$. As shown in figures 1a and 1b the profile of u and u_p are asymmetric about the plane $y=0$ because of the suction. It is clear from figures increasing couple stress parameter decreases u and u_p for all values of t . In figures 1c and 1d the shows that increasing couple stress parameter decreases T and T_p for all values of t .

Figure 2 shows the time evolution of the velocity components and temperature at the centre of the channel $y = 0$, respectively, for the fluid and particle phases for various values of the Hartmann number H_a and $s = 0$, $q = 0.1$. In figures 2a and 2b indicate that increasing H_a decreases u and u_p for all t as a result of increasing the clamping force on u and u_p increase T and T_p due to increasing the Joule dissipation. But for large t increasing H_a decreases T as a result of decreasing the velocity u and u_p and consequently decreases the viscous and Joule dissipations.

Figure 3 presents the time evolution of the velocity components and temperature at the centre of the channel $y = 0$, respectively for the fluid and particle phases for various values of the suction parameter S and $H_a = 0$, $q = 0.1$. It is shown that Figures 3 a and b that increasing the suction parameter decreases both u and u_p due to the conversion of the fluid from regions in the lower half to the centre which has higher fluid speed. Figures 3c and d shows that increasing S decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. It is observed from Figure 2 and 3 that suction has a more pronounced effect on the particles than that of the magnetic field.

Figure 4 presents, respectively the profiles of the velocity component and temperature of the fluid u and T and particles u_p and T_p for various values of time t . The figures are plotted for $H_a=1$, $S=1$, $q=0.1$. It is observed that the velocity component and temperature of the fluid reach the steady state faster than that of the particle phase. This is because the fluid velocity is the source for the dust particles velocity. It is shown that the velocity components and temperatures of the fluid and dust particles do not reach the steady state monotonically due to the effect of the pressure gradient.

CONCLUSION

The unsteady flow with heat transfer of a dusty couple stress conducting fluid under the influence of an applied uniform magnetic field has been studied in the presence of uniform suction and injection and an exponential decaying pressure gradient. An analytic solution for the equation of the motion has been obtained while the energy equation has solved using transform technique. The effect of the magnetic field, couple stress parameter and the suction and injection velocity on the velocity and temperature distributions for both the fluid and particles phases has been investigated. It is of interest to see that the effect the magnetic field on the temperature of the fluid and particles depends on time. Also, It is observed that the suction velocity has more apparent effect than the magnetic field and couple stress parameter on the steady state time of the velocity and temperature of the dust particles.

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