A NOTE ON GENERALIZED L-CONTACT STRUCTURE

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(Received On: 10-08-15; Revised & Accepted On: 31-08-15)

ABSTRACT


Keywords: Generalized Lorentzian structure, generalized induced connection, generalized D-conformal transformation.

1. INTRODUCTION

Let \( V_n \) be an odd \((n = 2m + 1) \) dimensional differentiable manifold, on which there are defined a tensor field \( F \) of type (1, 1), contravariant vector fields \( T_i \), covariant vector fields \( A_i \), where \( i = 3, 4, 5, \ldots (n - 1) \), and a Lorentzian metric \( g \), satisfying for arbitrary vector fields \( X, Y, Z, \ldots \)

\[
(1.1) \quad \bar{X} = X - \sum_{i=3}^{n-1} A_i(X) T_i, \quad \bar{T}_i = 0, \quad A_i(T_i) = -1, \quad \bar{X} \equiv F X, \quad A_i(\bar{X}) = 0, \quad \text{rank} F = n - i
\]

\[
(1.2) \quad g(\bar{X}, \bar{Y}) = g(X, Y) + \sum_{i=3}^{n-1} A_i(X) A_i(Y), \quad \text{where} \quad A_i(X) = g(X, T_i), \quad F(X, Y) \equiv g(\bar{X}, \bar{Y}) = -g(X, Y) = -F(Y, X),
\]

Then \( V_n \) will be called a generalized Lorentzian contact manifold and the structure \((F, T_i, A_i, g)\) will be known as generalized Lorentzian contact structure.

Let \( D \) be a Riemannian connection on \( V_n \), then we have

\[(1.3) (a) \quad (D_X F)(\bar{Y}, Z) - (D_F Y)(\bar{X}, Z) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(Z) + \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(Y) = 0
\]

\%(b) \quad (D_X F)(\bar{X}, \bar{Y}) = (D_F X)(\bar{Y}, \bar{Z})
\]

\[(1.4) (a) \quad (D_X F)(\bar{Y}, \bar{Z}) + (D_F Y)(\bar{X}, \bar{Z}) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(\bar{Z}) - \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(\bar{Y}) = 0
\]

\%(b) \quad (D_X F)(\bar{X}, \bar{Z}) + (D_F X)(\bar{Y}, \bar{Z}) = 0
\]

2. GENERALIZED CONNECTION IN A GENERALIZED LORENTZIAN CONTACT MANIFOLD

Let \( V_{2m-1} \) be submanifold of \( V_{2m+1} \) and let \( c : V_{2m-1} \rightarrow V_{2m+1} \) be the inclusion map such that

\[
d \in V_{2m-1} \rightarrow cd \in V_{2m+1},
\]

Where \( c \) induces a linear transformation (Jacobian map) \( J : T_{2m-1}^* \rightarrow T_{2m+1}^* \). \( T_{2m-1} \) is a tangent space to \( V_{2m-1} \) at point \( d \) and \( T_{2m+1} \) is a tangent space to \( V_{2m+1} \) at point \( cd \) such that \( \bar{X} \) in \( V_{2m-1} \) at \( d \rightarrow J\bar{X} \) in \( V_{2m+1} \) at \( cd \)

Let \( \bar{g} \) be the induced Lorentzian metric in \( V_{2m-1} \). Then we have

\[(2.1) \quad \bar{g}(\bar{X}, \bar{Y}) \equiv g(J\bar{X}, J\bar{Y})
\]

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Let us define a type of generalized semi-symmetric non-metric connection $B$ in a generalized Lorentzian contact manifold is given by

$$(2.2)\, IBx\, Y = IdY + \sum_{i=3}^{n-1} A_i(Y)X - \sum_{i=3}^{n-1} g(X, Y)T_i + \sum_{i=3}^{n-1} g(X, Y)U_i,$$

Where $U_i$ are vector fields associated with 1-form $d_i$ defined by

$$(2.3)\, d_i(x) \equiv g(X, U_i),\quad i = 3, 4, 5, \ldots (n - 1).$$

Similarly we have

$$(2.4)\, T_i = fT_i + \rho_i M + \sigma_i N$$

(b) $U_i = fU_i + \theta_i M + \phi_i N,\quad i = 3, 4, 5, \ldots (n - 1).$

Where $t_i$ and $u_i,\ i = 3, 4, 5, \ldots (n - 1)$ are $C^\infty$ vector fields in $V_{2m-1}$ and $M, N$ are unit normal vectors to $V_{2m-1}$.

Denoting by $D$ the connection induced on the submanifold from $D$, we have Gauss equation

$$(2.5)\, D_x f\tilde{Y} = f(D_x \tilde{Y}) + p(\tilde{X}, \tilde{Y})M + q(\tilde{X}, \tilde{Y})N$$

Where $\tilde{B}$ is the connection induced on the submanifold from $B$ and $r$ and $s$ are symmetric bilinear functions in $V_{2m-1}$ Inconsequence of (2.2), we have

$$(2.7)\, IBx\, \tilde{Y} = Id\tilde{Y} + \sum_{i=3}^{n-1} A_i(\tilde{Y})X - \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})T_i + \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})U_i,$$

Using (2.5), (2.6) and (2.7), we get

$$(2.8)\, if(D_x \tilde{Y}) + ir(\tilde{X}, \tilde{Y})M + is(\tilde{X}, \tilde{Y})N = if(D\tilde{Y}) + ip(\tilde{X}, \tilde{Y})M + iq(\tilde{X}, \tilde{Y})N + \sum_{i=3}^{n-1} a_i(\tilde{Y})\tilde{X}$$

$$- \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})T_i + \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})U_i.$$}

Using (2.4) (a) and (2.4) (b), we obtain

$$(2.9)\, if(D_x \tilde{Y}) + ir(\tilde{X}, \tilde{Y})M + is(\tilde{X}, \tilde{Y})N = if(D\tilde{Y}) + ip(\tilde{X}, \tilde{Y})M + iq(\tilde{X}, \tilde{Y})N + \sum_{i=3}^{n-1} a_i(\tilde{Y})\tilde{X}$$

$$- \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})T_i + \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})U_i.$$

This gives

$$(2.10)\, iBx\tilde{Y} = iD\tilde{Y} + \sum_{i=3}^{n-1} a_i(\tilde{Y})\tilde{X} - \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})T_i + \sum_{i=3}^{n-1} g(\tilde{X}, \tilde{Y})U_i$$

If

$$(2.11)\, (a)\, ir(\tilde{X}, \tilde{Y}) = ip(\tilde{X}, \tilde{Y}) - \sum_{i=3}^{n-1} \rho_i g(\tilde{X}, \tilde{Y}) + \sum_{i=3}^{n-1} \theta_i g(\tilde{X}, \tilde{Y})$$

(b) $is(\tilde{X}, \tilde{Y}) = iq(\tilde{X}, \tilde{Y}) - \sum_{i=3}^{n-1} \sigma_i g(\tilde{X}, \tilde{Y}) + \sum_{i=3}^{n-1} \phi_i g(\tilde{X}, \tilde{Y}).$

Thus we have

**Theorem 2.1:** The connection induced on a submanifold of a generalized Lorentzian contact manifold with a generalized semi-symmetric non-metric connection with respect to unit normal vectors $M$ and $N$ is also semi-symmetric non-metric connection iff (2.11) holds.

### 3. GENERALIZED D-CONFORMAL TRANSFORMATION

Let the corresponding Jacobian map $J$ of the transformation $b$ transforms the structure $(F, T_i, A_i, g)$ to the structure $(F_i, V_i, u_i, h)$ such that

$$(3.1)\, (a) JZ = \overline{Z}$$

(b) $h(JX, JY)ob = e^a g(\overline{X}, \overline{Y}) - e^{2a} \sum_{i=3}^{n-1} A_i(X)A_i(Y)$

(c) $V_i = e^{-\alpha} Jx_i$ (d) $u_i(JX)ob = e^a A_i(X)$

Where $\sigma$ is a differentiable function on $V_i$, then the transformation is said to be generalized D-conformal transformation.

**Theorem 3.1:** The structure $(F, V_i, u_i, h)$ is generalized Lorentzian contact.

**Proof:** Inconsequence of (1.1), (1.2), (3.1) (b) and (3.1) (d), we get

$$h(J\overline{X}, J\overline{Y})ob = e^{a} g(\overline{X}, \overline{Y}) = h(JX, JY)ob + \sum_{i=3}^{n-1} e^{2a} A_i(X)A_i(Y)$$

$$= h(JX, JY)ob + \sum_{i=3}^{n-1} (u_i(JX)ob)[v_i(JY)ob]$$
This gives
\( h(fX, fY) = h(X, Y) + \sum_{i=3}^{n-1} u_i(X)u_i(Y) \)

Using (1.1), (3.1) (a), (3.1) (c) and (3.1) (d), we get
\( \overline{fX} = fX = -X - \sum_{i=3}^{n-1} A_i(X)T_i = -X - \sum_{i=3}^{n-1} [u_i(X)ob]V_i \)

Also
\( \overline{V} = e^{-\sigma} \overline{I} = 0 \)

Proof follows from equations (3.2), (3.3) and (3.4).

**Theorem 3.2:** Let \( E \) and \( D \) be the Riemannian connections with respect to \( h \) and \( g \) such that
\[
(3.5) \quad (a) \quad E_{\gamma X} Y = JD_\gamma Y + H(X, Y)
\]
and
\[
(3.5) \quad (b) \quad H(X, Y, Z) \equiv g(H(X, Y), Z)
\]

Then
\[
(3.6) \quad 2E_{\gamma X} Y = 2JD_\gamma Y - J[2\sigma \left\{ \sum_{i=3}^{n-1} (X\sigma)A_i(Y)T_i + \sum_{i=3}^{n-1} (Y\sigma)A_i(X) T_i - \sum_{i=3}^{n-1} (-G\nabla\sigma)A_i(X)A_i(Y) \right\} + e^{-\sigma} - 1] \sum_{i=3}^{n-1} (D_xA_i(Y) + (D_{YI}X) - 2A_i(H(X, Y))T_i)
\]

Proof: Inconsequence of (3.1) (b), we have
\[
JX(h(JY, JZ))ob = X\left\{ e^{\sigma} g(\overline{Y}, \overline{Z}) - \sum_{i=3}^{n-1} e^{2\sigma} A_i(Y)A_i(Z) \right\}
\]

Now
\[
(3.7) \quad h(E_{\gamma X} Y, JZ)ob + h(JY, E_{\gamma X} Z)ob = (X\sigma)e^{\sigma} g(\overline{Y}, \overline{Z}) + e^{\sigma} g(D_x\overline{Y}, \overline{Z}) + e^{\sigma} g(D_x\overline{Y}, \overline{Z}) - \sum_{i=3}^{n-1} (2(X\sigma)e^{2\sigma} A_i(Y)A_i(Z) + e^{2\sigma} (D_xA_i)A_i(Y)A_i(Z) + e^{2\sigma} (D_xA_i)A_i(Z)A_i(Y) + e^{2\sigma} A_i(Y)A_i(Z) + e^{2\sigma} A_i(D_xZ)A_i(Y))
\]

Also
\[
(3.8) \quad h(E_{\gamma X} Y, JZ)ob + h(JY, E_{\gamma X} Z)ob = e^{\sigma} g(D_x\overline{Y}, \overline{Z}) + e^{\sigma} g(H(X, Y), \overline{Z}) + e^{\sigma} g(F, D_x\overline{Z}) - \sum_{i=3}^{n-1} \left\{ e^{2\sigma} A_i(D_xY)A_i(Z) + e^{2\sigma} A_i(Y)A_i(H(X, Z)) \right\} + e^{\sigma} g(\overline{F}, \overline{D_xZ})
\]

Inconsequence of (1.3) (a), (3.7) and (3.8), we have
\[
(3.9) \quad (X\sigma)g(\overline{F}, \overline{Z}) - 2(X\sigma)e^{\sigma} \sum_{i=3}^{n-1} (A_i(Y)A_i(Z)) - (e^{\sigma} - 1) \sum_{i=3}^{n-1} ((D_xA_i)A_i(Y)A_i(Z) + (D_xA_i)A_i(Z)A_i(Y)) + e^{\sigma} \sum_{i=3}^{n-1} \left\{ e^{2\sigma} A_i(D_xY)A_i(Z) + e^{2\sigma} A_i(Y)A_i(H(X, Z)) \right\} = 2H(X, Y, Z) = \gamma H(X, Y, Z)
\]

Writing two other equations by cyclic permutation of \( X, Y, Z \) and subtracting the sum from the sum of the first two. Also using symmetry of \( 'H ' \) in the first two slots, we get
\[
(3.10) \quad 2H(X, Y, Z) = -2e^{\sigma} \sum_{i=3}^{n-1} ((X\sigma)A_i(Y)A_i(Z) + (Y\sigma)A_i(Z)A_i(X) - (Z\sigma)A_i(X)A_i(Y)) - (e^{\sigma} - 1) \sum_{i=3}^{n-1} \left\{ (D_xA_i)A_i(Y) + (D_xA_i)A_i(X) \right\} - 2A_i(H(X, Y)) + A_i(Y) \left\{ (D_xA_i)A_i(Y) \right\} + A_i(Y) \left\{ (D_xA_i)A_i(X) \right\}
\]

This implies
\[
(3.11) \quad 2H(X, Y) = -2e^{\sigma} \sum_{i=3}^{n-1} ((X\sigma)A_i(Y)YT_i + (Y\sigma)A_i(X)TI_i - (Z\sigma)A_i(X)A_i(Y)) - (e^{\sigma} - 1) \sum_{i=3}^{n-1} \left\{ (D_xA_i)A_i(Y) + (D_xA_i)A_i(X) - 2A_i(H(X, Y)) \right\} T_i + A_i(X)D_xT_i + A_i(Y)D_xT_i - A_i(X)(-G\nabla A_i)(Y) - A_i(Y)(-G\nabla A_i)(X)
\]

(3.6) follows from (3.11) and (3.5) (a).

**References**


Source of support: Nil, Conflict of interest: None Declared

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