Pairwise $bg$ closed and Pairwise $* bg$ closed set in Bitopological Spaces

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ABSTRACT

In this paper, we introduce some new classes of sets namely Pairwise $bg$ closed set, Pairwise $* bg$ closed set. We obtain the basic properties and their relationships with other classes of sets in bitopological spaces. We devote the concept of Pairwise $bg$ and Pairwise $* bg$ continuous functions. The relationship between Pairwise $bg$ continuous and Pairwise $* bg$ continuous and other defined continuous functions are being deliberated.

Keywords and Phrases: Pairwise $bg$ closed set, Pairwise $* bg$ closed set, Pairwise $bg$ continuous function, Pairwise $* bg$ continuous function, Pairwise $bg$ irresolute function, Pairwise $* bg$ irresolute function.

I. INTRODUCTION AND PRELIMINARIES

INTRODUCTION


PRELIMINARIES

**Definition: 1.1** [15] Let $(X, \tau)$ be a topological space. A set $A$ is called semi-open set if $A \subseteq C_1(\text{Int}(A))$. The complement of semi-open set is semi-closed set.

**Definition: 1.2** [16] Let $(X, \tau)$ be a topological space. A set $A$ is called pre-open set if $A \subseteq \text{Int}(C_1(A))$. The complement of pre-open set is pre-closed set.

**Definition: 1.3** [19] Let $(X, \tau)$ be a topological space. A set $A$ is called $\alpha$-open set if $A \subseteq \text{int}(C_1(\text{int}(A)))$. The complement of $\alpha$-open set is $\alpha$-closed set.

**Definition: 1.4** [2] Let $(X, \tau)$ be a topological space. A set $A$ is called b-open set if $A \subseteq C_1(\text{Int}(A)) \cup \text{Int}(C_1(A))$. The complement of b-open set is called b-closed set.

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Definition: 1.5 [9] A subset A of a bitopological space \((X, \tau_1, \tau_2)\) is called \(\tau_1 \tau_2 - sg\) closed if \(\tau_2 - scl(A) \subseteq U\) whenever \(A \subseteq U\) and U is \(\tau_1\)-semi open in X.

Definition: 1.6 [12] A subset A of a bitopological space \((X, \tau_1, \tau_2)\) is called \(\tau_1 \tau_2 - \omega\) closed if \(\tau_2 - cl(A) \subseteq U\) whenever \(A \subseteq U\) and U is \(\tau_1\)-semi open in X.

2. Pairwise \(bg\) closed and Pairwise * \(bg\) closed set

Definition: 2.1 A set A of a bitopological space \((X, \tau_1, \tau_2)\) is called Pairwise \(bg\) closed if \(\tau_2 - bcl(A) \subseteq U\) whenever \(A \subseteq U\) and U is \(\tau_1\)-semi open in X.

Definition: 2.2 A set A of a bitopological space \((X, \tau_1, \tau_2)\) is called Pairwise * \(bg\) closed if \(\tau_2 - bcl(A) \subseteq U\) whenever \(A \subseteq U\) and U is \(\tau_1\)-\(\alpha\) open in X.

Theorem: 2.3

(a) Every Pairwise \(bg\) closed set is Pairwise * \(bg\) closed set.
(b) Every \(\tau_1 \tau_2 - \omega\) closed set is Pairwise \(bg\) closed set.
(c) Every \(\tau_1 \tau_2 - \omega\) closed set is Pairwise * \(bg\) closed set.
(d) Every \(\tau_1 \tau_2 - sg\) closed set is Pairwise \(bg\) closed set.
(e) Every \(\tau_1 \tau_2 - sg\) closed set is Pairwise * \(bg\) closed set.

Proof: a) Let A be Pairwise \(bg\) closed set. We have to prove A is Pairwise * \(bg\) closed set. Let \(A \subseteq U\) and U is \(\tau_1\)-\(\alpha\) open in X. Since every \(\alpha\) open set is semi open set then U is \(\tau_1\)-semi open in X. Also since \(A \subseteq U\) and U is \(\tau_1\)-semi open in X and A is Pairwise \(bg\) closed set, then \(\tau_2 - bcl(A) \subseteq U\). Therefore A is Pairwise * \(bg\) closed set. The other results follows from the definitions.

Remark: 2.4 The converse of the above theorems are not true and it is shown by the following examples.

Example: 2.5 Let \(X = \{a, b, c\}; \tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}; \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}\). Here \(\{a, b\}\) is Pairwise * \(bg\) closed but not Pairwise \(bg\) closed set.

Example: 2.6 Let \(X = \{a, b, c\}; \tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}; \tau_2 = \{\emptyset, X, \{b\}, \{b, c\}\}\). Here \(\{c\}\) is Pairwise \(bg\) closed but not \(\tau_1 \tau_2 - \omega\) closed set.

Example: 2.7 Let \(X = \{a, b, c\}; \tau_1 = \{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}; \tau_2 = \{\emptyset, X, \{a\}, \{a, c\}\}\). Here \(\{c\}\) is Pairwise * \(bg\) closed but not \(\tau_1 \tau_2 - \omega\) closed set.

Example: 2.8 Let \(X = \{a, b, c\}; \tau_1 = \{\emptyset, X, \{c\}, \{a, b\}\}; \tau_2 = \{\emptyset, X, \{a\}, \{a, c\}\}\). Here \(\{a, c\}\) is Pairwise * \(bg\) closed but not \(\tau_1 \tau_2 - sg\) closed set.

Remark: 2.9 From the above theorems and examples we have the following diagrammatic representation.
Proposition: 2.10 The finite union of Pairwise \( bg \) closed (Pairwise *bg closed) set is Pairwise \( bg \) closed (Pairwise *bg closed).

Proof: Let \( A \) and \( B \) be Pairwise \( bg \) closed (Pairwise *bg closed) subsets of \( X \) and let \( U \) be \( \tau_1 \)-semi open (\( \alpha \) open) in \( X \) such that \( A \cup B \subseteq U \). Then \( \tau_2 - bcl(A) \subseteq U \), \( \tau_2 - bcl(B) \subseteq U \).

Therefore \( \tau_2 - bcl(A \cup B) = \tau_2 - bcl(A) \cup \tau_2 - bcl(B) \subseteq U \). This implies \( \tau_2 - bcl(A \cup B) \subseteq U \). Hence \( A \cup B \) is Pairwise \( bg \) closed (Pairwise *bg closed) set.

Theorem: 2.11 If \( A \) is an Pairwise \( bg \) closed (Pairwise *bg closed) set of \( (X, \tau_1, \tau_2) \) such that \( A \subseteq B \subseteq \tau_2 - bcl(A) \) then \( B \) is also an Pairwise \( bg \) closed (Pairwise *bg closed) set of \( X \).

Proof: Let \( B \subseteq U \) where \( U \) is \( \tau_1 \)-semi open (\( \alpha \) open) in \( X \). Then \( A \subseteq B \) implies \( A \subseteq U \). Since \( A \) is pairwise \( bg \) closed (Pairwise *bg closed) then \( \tau_2 - bcl(A) \subseteq U \).

Given \( B \subseteq \tau_2 - bcl(A) \) then \( \tau_2 - bcl(B) \subseteq \tau_2 - bcl(\tau_2 - bcl(A)) \subseteq \tau_2 - bcl(A) \subseteq U \). Therefore \( B \) is Pairwise \( bg \) closed (Pairwise *bg closed) set.

Proposition: 2.12 If \( A \) is Pairwise \( bg \) closed (Pairwise *bg closed) subset of \( (X, \tau_1, \tau_2) \) then \([\tau_2 - bcl(A)] - A \) does not contain any non empty \( \tau_1 \)-semi closed (\( \alpha \) closed) sets.

Proof: Let \( A \) be Pairwise \( bg \) closed (Pairwise *bg closed) set. Suppose \( F \neq \phi \) is \( \tau_1 \)-semi closed (\( \alpha \) closed) set of \( [\tau_2 - bcl(A)] - A \) then \( F \subseteq \tau_2 - bcl(A) - A \). This implies \( F \subseteq \tau_2 - bcl(A) \) and \( F \subseteq X - A \). Consider \( A \subseteq X - F \) then \( F \subseteq [\tau_2 - bcl(A)] \). Therefore, \( F \subseteq [\tau_2 - bcl(A)] \cap [\tau_2 - bcl(A)] = \phi \). Hence \( F = \phi \).

Corollary: 2.13 Let \( A \) be Pairwise \( bg \) closed (Pairwise *bg closed) set in \( (X, \tau_1, \tau_2) \) then \( A \) is \( \tau_2 - b \) - closed iff \([\tau_2 - bcl(A)] - A \) is \( \tau_1 \)-semi closed (\( \alpha \) closed) set.

Proof: Let \( A \) be Pairwise \( bg \) closed (Pairwise *bg closed) set. If \( A \) is \( \tau_2 - b \) - closed we have \( \tau_2 - bcl(A) = A \) then \([\tau_2 - bcl(A)] - A = \phi \) which is \( \tau_1 \)-semi closed (\( \alpha \) closed) set.

Conversely, let \([\tau_2 - bcl(A)] - A \) is \( \tau_1 \)-semi closed (\( \alpha \) closed) set. Then by proposition 3.3, \([\tau_2 - bcl(A)] - A \) is \( \tau_1 \)-semi closed (\( \alpha \) closed) subset of itself then \([\tau_2 - bcl(A)] - A = \phi \). This implies that \( \tau_2 - bcl(A) = A \). Therefore \( A \) is \( \tau_2 - b \) - closed.

Definition: 2.14 A subset \( A \subseteq X \) is called Pairwise \( bg \) open (Pairwise *bg open) set iff its complement is Pairwise \( bg \) closed (Pairwise *bg closed) set.

Theorem: 2.15 A subset \( A \subseteq X \) is Pairwise \( bg \) open (Pairwise *bg open) set iff \( F \subseteq \tau_2 - bint(A) \) whenever \( F \) is semi closed (\( \alpha \) closed) in \( \tau_1 \) such that \( F \subseteq A \).

Proof: Necessity: Let \( A \) be Pairwise \( bg \) open (Pairwise *bg open) set and \( F \) be semi closed (\( \alpha \) closed) in \( \tau_1 \) such that \( F \subseteq A \). Then \( X - A \) is contained in \( X - F \) where \( X - F \) is semi open (\( \alpha \) open) in \( \tau_1 \). Since \( A \) is Pairwise \( bg \) open (Pairwise *bg open), \( \tau_2 - bcl(X - A) \subseteq X - F \). This implies \( X - [\tau_2 - bint(A)] \subseteq X - F \). Thus \( F \subseteq \tau_2 - bint(A) \).

Sufficiency: Suppose \( F \) is semi closed (\( \alpha \) closed) in \( \tau_1 \) and \( F \subseteq A \). This implies \( F \subseteq \tau_2 - bint(A) \). Let \( X - A \subseteq U \), where \( U \) is semi open (\( \alpha \) open) set in \( \tau_1 \). Then \( X - U \subseteq A \) where \( X - U \) is semi closed (\( \alpha \) closed) in \( \tau_1 \). By hypothesis, \( X - U \subseteq \tau_2 - bint(A) \) (i.e. \( X - [\tau_2 - bint(A)] \subseteq U \)). Then \( \tau_2 - bcl(X - A) \subseteq U \) implies \( X - A \) is Pairwise \( bg \) closed (Pairwise *bg closed) set. Therefore \( A \) is Pairwise \( bg \) open (Pairwise *bg open) set.
Theorem 2.16 If \( A \subseteq X \) is Pairwise \( bg \) closed (Pairwise \( bg \) closed) set then \([\tau_2 - bcl(A)] - A\) is Pairwise \( bg \) open (Pairwise \( bg \) open).

Proof: Let \( A \) be Pairwise \( bg \) closed(Pairwise \( bg \) closed). Let \( F \) be semi closed (\( \alpha \) closed) set in \( \tau_1 \) such that \( F \subseteq [\tau_2 - bcl(A)] - A \). Then by proposition 2.13, \( F = \phi \). So \( F \subseteq [\tau_2 - \text{bint}(\tau_2 - bcl(A)) - A] \). This implies \([\tau_2 - bcl(A)] - A\) is Pairwise \( bg \) open (Pairwise \( bg \) open).

3. Pairwise \( bg \) and Pairwise * \( bg \) continuous functions

Definition 3.1
(i) A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) where \( (X, \tau_1, \tau_2) \) and \( (Y, \sigma_1, \sigma_2) \) are bitopological space is pairwise \( bg \) continuous if \( f^{-1}(U) \) is Pairwise \( bg \) closed in \( X \) for each \( \sigma_i \) closed \( U \) in \( Y_i \neq j \) and \( i, j = 1, 2 \)

(ii) A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) where \( (X, \tau_1, \tau_2) \) and \( (Y, \sigma_1, \sigma_2) \) are bitopological space is pairwise * \( bg \) continuous if \( f^{-1}(U) \) is pairwise * \( bg \) closed in \( X \) for each \( \sigma_i \) closed \( U \) in \( Y_i \neq j \) and \( i, j = 1, 2 \)

Theorem 3.2
(a) Every pairwise \( bg \) continuous function is pairwise * \( bg \) continuous function.
(b) Every \( \tau_1 \tau_2 - \omega \) continuous function is pairwise \( bg \) continuous function.
(c) Every \( \tau_1 \tau_2 - \omega \) continuous function is pairwise * \( bg \) continuous function.
(d) Every \( \tau_1 \tau_2 - s\gamma \) continuous function is pairwise * \( bg \) continuous function.
(e) Every \( \tau_1 \tau_2 - s\gamma \) continuous function is pairwise * \( bg \) continuous function.

Proof: a) Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be pairwise \( bg \) continuous. Let \( U \) be \( \sigma_j \) closed set in \( Y \). Then \( f^{-1}(U) \) is Pairwise \( bg \) closed set in \( X \). Since every Pairwise \( bg \) closed set in \( X \) is pairwise * \( bg \) closed set in \( X \) then \( f^{-1}(U) \) is pairwise * \( bg \) closed set in \( X \). Hence \( f \) is pairwise * \( bg \) continuous function. The proof is obvious for others.

Remark: 3.3 The converse of the above theorems are not true as shown by the following examples.

Example 3.4 Let \( X = Y = \{a, b, c\} ; \tau_1 = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}, \{a, c\}; \tau_2 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}; \sigma_1 = \{\phi, Y, \{a, c\}; \sigma_2 = \{\phi, Y, \{a\} \}. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function defined by \( f(a) = c, f(b) = b \), \( f(c) = a \).
Here \( f^{-1}(b, c) = \{a, b\} \) is pairwise * \( bg \) closed but not pairwise \( bg \) closed set. Therefore \( f \) is pairwise * \( bg \) continuous but not pairwise \( bg \) continuous function.

Example 3.5 Let \( X = Y = \{a, b, c\} \); \( \tau_1 = \{\phi, X, \{a\}, \{a, c\}, \{a, b\} \}; \tau_2 = \{\phi, X, \{b\}, \{b, c\}; \sigma_1 = \{\phi, Y, \{a, c\}, \sigma_2 = \{\phi, Y, \{a, b\} \}. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be an identity function.
Here \( f^{-1}(c) = \{c\} \) is pairwise * \( bg \) closed but not \( \tau_1 \tau_2 - \omega \) closed set. Therefore \( f \) is pairwise \( bg \) continuous but not pairwise \( bg \) continuous function.

Example 3.6 Let \( X = Y = \{a, b, c\} \); \( \tau_1 = \{\phi, X, \{a\}, \{a, c\}, \{a, b\} \}; \tau_2 = \{\phi, X, \{a\}, \{a, c\}; \sigma_1 = \{\phi, Y, \{b\}, \{b, c\}; \sigma_2 = \{\phi, Y, \{a\} \}. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be an identity function.
Here \( f^{-1}(c) = \{c\} \) is pairwise * \( bg \) closed but not \( \tau_1 \tau_2 - \omega \) closed set. Therefore \( f \) is pairwise * \( bg \) continuous but not pairwise * \( bg \) continuous function.

Example 3.7 Let \( X = Y = \{a, b, c\} \); \( \tau_1 = \{\phi, X, \{a\}, \{a, b\} \}; \tau_2 = \{\phi, X, \{a\}, \{a, c\}; \sigma_1 = \{\phi, Y, \{a\}, \{a, b\}; \sigma_2 = \{\phi, Y, \{a\} \}. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function defined by \( f(a) = b, f(b) = a, f(c) = c \).
Here \( f^{-1}(b, c) = \{a, c\} \) is pairwise * \( bg \) closed but not \( \tau_1 \tau_2 - s\gamma \) closed set. Therefore \( f \) is but not pairwise * \( bg \) continuous function.

Theorem 3.8 The following are equivalent for a function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \)
(a) \( f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous).
(b) \( f^{-1}(U) \) is pairwise \( bg \) open (pairwise * \( bg \) open) in \( X \) for each \( \sigma_j \) — open set \( U \) in \( Y, i \neq j \) and \( i, j = 1, 2 \).

Proof: (a) \( \Rightarrow \) (b) Suppose that \( f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous). Let \( A \) be \( \sigma_j \) — open set in \( Y \). Then \( Y - A \) is \( \sigma_j \) — closed set in \( Y \). Since \( f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous), \( f^{-1}(Y - A) \) is pairwise \( bg \) closed (pairwise * \( bg \) closed) in \( X, i \neq j \) and \( i, j = 1, 2 \). Consequently, \( f^{-1}(A) \) is pairwise \( bg \) open (pairwise * \( bg \) open) in \( X \).
(b) \( \Rightarrow \) (a) Suppose that \( f^{-1}(A) \) is pairwise \( bg \) open (pairwise \( * bg \) open) in \( X \) for each \( \sigma_i \) -- open set \( U \) in \( Y \), \( i \neq j \) and \( i, j = 1, 2 \). Let \( V \) be \( \sigma_j \) -- closed set in \( Y \). Then \( X - V \) is \( \sigma_i \) -- open in \( Y \). Then by our assumption, \( f^{-1}(X - V) \) is pairwise \( bg \) open (pairwise \( * bg \) open) in \( X \), \( i \neq j \) and \( i, j = 1, 2 \). Then \( f^{-1}(V) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( X \). Hence \( f \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous).

Remark: 3.9 The composition of two pairwise \( bg \) continuous (pairwise \( * bg \) continuous) functions is not pairwise \( bg \) continuous (pairwise \( * bg \) continuous) functions as shown by the following example.

Example: 3.10 Let \( X = Y = Z = \{a, b, c\}; \tau_1 = \{\phi, X, \{a, b\}, \{b, c\}; \{a, b\}; \tau_2 = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\); \( \sigma_1 = \{\phi, Y, \{b\}, \{a, b\}\}; \sigma_2 = \{\phi, Y, \{c\}, \{a, c\}\}; \gamma_1 = \{\phi, Z, \{b\}, \{a, c\}\}; \gamma_2 = \{\phi, Z, \{b\}, \{b, c\}\} \) Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function defined by \( f(a) = a, f(b) = c, f(c) = b \) and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \gamma_1, \gamma_2) \) be an identity function. Then \( f \) and \( g \) are pairwise \( bg \) continuous function. But \( f^{-1}(g^{-1}(\{a\})) = \{a\} \) is not pairwise \( bg \) closed in \( (X, \tau_1, \tau_2) \). Hence \( g \circ f \) is not pairwise \( bg \) continuous function.

Definition: 3.11 A function \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is

(a) pairwise \( bg \) irresolute if \( f^{-1}(U) \) is \( \tau_i \tau_j \) -- pairwise \( bg \) closed for each \( \sigma_i \sigma_j \) -- pairwise \( bg \) closed in \( U \) in \( Y \), \( i \neq j \) and \( i, j = 1, 2 \).

(b) pairwise \( * bg \) irresolute if \( f^{-1}(U) \) is \( \tau_i \tau_j \) -- pairwise \( * bg \) closed for each \( \sigma_i \sigma_j \) -- pairwise \( * bg \) closed in \( U \) in \( Y \), \( i \neq j \) and \( i, j = 1, 2 \).

Proposition: 3.12 If \( f \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) then \( f \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function.

Proof: Let \( V \) be \( \sigma_j \) -- closed set in \( Y \). Then \( V \) is \( \sigma_i \sigma_j \) -- pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( Y \). By assumption, \( f^{-1}(V) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( X \). Hence \( f \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function.

Remark: 3.13 The converse of the above theorem is not true as shown by the following example.

Example: 3.14 Let \( X = Y = Z = \{a, b, c\}; \tau_1 = \{\phi, X, \{a, c\}\}; \tau_2 = \{\phi, X, \{c\}, \{a, c\}\}; \sigma_1 = \{\phi, Y, \{b\}, \{a, c\}\}; \sigma_2 = \{\phi, Y, \{a\}, \{a, b\}\} \) Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function defined by \( f(a) = a, f(b) = c, f(c) = b \).

Here \( f^{-1}(c) = \{b\} \) and \( f^{-1}(b, c) = \{b, c\} \) is pairwise \( bg \) closed in \( (X, \tau_1, \tau_2) \). Hence \( f \) is pairwise \( bg \) continuous. But \( f^{-1}(a, b) = \{a, c\} \) is not pairwise \( bg \) closed in \( (X, \tau_1, \tau_2) \). Hence it is not pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function.

Theorem: 3.15 Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be two functions. Then if \( f \) and \( g \) are pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) then \( g \circ f \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function.

Proof: Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute). Let \( V \) be pairwise \( bg \) closed (pairwise \( * bg \) closed) set in \( Z \). Since \( g \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function, then \( g^{-1}(v) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( Y \). Since \( f \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function, \((g \circ f)^{-1}(v) = f^{-1}(g^{-1}(v)) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( X \). Therefore \( g \circ f \) is pairwise \( bg \) irresolute function (pairwise \( * bg \) irresolute).

Theorem: 3.16 Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be two functions. Then if \( f \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function and \( g \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function. Then \( g \circ f \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function.

Proof: Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function. Let \( V \) be \( \sigma_j \) -- closed set in \( Z \). Since \( g \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function, then \( g^{-1}(v) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( Y \).

Since \( f \) is pairwise \( bg \) irresolute (pairwise \( * bg \) irresolute) function, \((g \circ f)^{-1}(v) = f^{-1}(g^{-1}(v)) \) is pairwise \( bg \) closed (pairwise \( * bg \) closed) in \( X \). Therefore \( g \circ f \) is pairwise \( bg \) continuous (pairwise \( * bg \) continuous) function.
Theorem: 3.17 Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) and \( g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2) \) be two functions. Then if \( f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous) function and \( g \) is pairwise continuous. Then \( g \circ f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous) function.

Proof: Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be pairwise \( bg \) continuous (pairwise * \( bg \) continuous) function and \( g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2) \) be pairwise continuous. Let \( V \) be \( \sigma_1 - \) closed set in \( Z \). Since \( g \) is pairwise continuous function, then \( g^{-1}(V) \) is \( \sigma_1 \) closed in \( Y \). Since \( f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous) function, \((gof)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is pairwise \( bg \) closed (pairwise * \( bg \) closed) in \( X \). Therefore \( g \circ f \) is pairwise \( bg \) continuous (pairwise * \( bg \) continuous) function.

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