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HEAT AND MASS TRANSFER EFFECTS ON MHD BOUNDARY LAYER STAGNATION-POINT FLOW OVER A NONLINEAR STRETCHING/SHRINKING SHEET IN POROUS MEDIA

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ABSTRACT

An analysis is made to study two dimensional incompressible MHD boundary layer stagnation-point flow over a nonlinear stretching/shrinking sheet in porous media. The stretching/shrinking velocity and the external flow velocity imply normal to the stretching/shrinking sheet are assumed to $U(x) \cong x^m$ *, where m is a constant and x is distance from the stagnation point. The governing partial differential equations are transformed into ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved using Runge-Kutta fourth order method along with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. The numerical values of Skin-friction coefficient and Nusselt number at the stretching/shrinking sheet are derived, discussed numerically for various values of physical parameters and presented through tables.*

Key words: *MHD, boundary layer flow, stretching/shrinking sheet, porous media, similarity transformations, numerical study.*

NOMENCLATURE

a, b	Constants, [−]
B_0	Constant applied magnetic field, [Wb m^{-2}]
C_p	Specific heat at constant pressure, [J Kg^{-1} K^{-1}]
f	Dimensionless stream function, [−]
Ec	Eckert number $(= \mathrm{u}^2 / \mathrm{C_p}(T_w - T_\infty))$, [−]
K	Permeability of the porous medium, [Darcy]
M	Magnetic parameter $(= \sigma_e \mathrm{B}_0^2 / \rho \mathrm{b})$, [−]
Pr	Prandtl number $(= \mu \mathrm{C_p} / \kappa)$, [−]
S_p	Porosity parameter $(= \nu x^{1-m} / Kb)$, [−]
T	Temperature of the fluid, [K]
u, v	Velocity component of the fluid along the x and y directions, respectively, [m s^{-1}]
x, y	Cartesian coordinates along the surface and normal to it, respectively, [m]

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GREEK SYMBOLS

ρ	Density of the fluid, [Kg m^{-3}]
μ	Viscosity of the fluid, [Kg m s^{-1}]
σ_e	Electrical conductivity, [m^2 s^{-1}]
η	Dimensionless similarity variable, $[=(\nu x U_\infty)^{1/2} f(\eta)]$
α	Thermal diffusivity, [m^2 s^{-1}]
ν	Kinematic viscosity, [m^2 s^{-1}]
κ	Thermal conductivity, [W m^{-2} K^{-4}]
ψ	Stream function, $[=(U_\infty / \nu x)^{1/2} y]$
θ	Dimensionless temperature, $[=(T-T_\infty)/(T_w - T_\infty)]$
ε	Stretching/Shrinking parameter, $[=a/b]$

SUPERSCRIPT

$'$	Derivative with respect to η Subscripts
w	Properties at the plate
∞	Free stream condition

1. INTRODUCTION

The study of boundary layer flow over a stretching/shrinking sheet has generated much interest in recent years in view of its significant applications in industrial manufacturing such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the Kinematics of stretching/shrinking and simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari and Keller [12]). A stagnation flow describes that the fluid motion near the stagnation region exists on a solid body where the fluid moves towards it. The stagnation region encounters the highest pressure, the highest heat transfer are highest rate of mass deposition. The heat transfer in the flow due to a stretching sheet is very important in practically. This type of flow appears in many industrial and engineering processes and in those cases; the qualities at the final products depend to a great extent on the rate of cooling. Stokes [23] was probably the first to study the flow of a viscous incompressible fluid past to an impulsively started infinite horizontal plate. In recent years, MHD flow problem have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is significant type of flow having considerable practical application in chemical engineering electrochemistry and polymer processing. In his pioneering work, Sakiadis [21] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [1] extended the work of Sakiadis [21] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by number of researchers [5-9, 11].

The boundary layer flow of an incompressible viscous fluid over a shrinking sheet has received considerable attention of modern day researchers because of its increasing application to many engineering systems. Wang [24] first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. Later, Miklavcic and Wang [15] obtained an analytical solution for steady viscous hydrodynamic flow over a permeable shrinking sheet. Then, Hayat *et al.* [8] derived both exact and series solution describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of stagnation flow towards a shrinking sheet was studied by Wang [25]. Nadeem and Awais [17] studied thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity. Bachok and Ishak [2] studied numerically, the stagnation-point flow and heat transfer over a nonlinear stretching/shrinking sheet and they found that the solution for shrinking sheet are non-unique for m>1/3. Viscous flow over an unsteady shrinking sheet with mass transfer was studied by Fang and Zhang [5]. Fang and Zhang [6] solved the Full N-S equation analytically for two dimensional MHD viscous flow due to a shrinking sheet. Fang and Zhang [7] investigated the heat transfer characteristics of the shrinking sheet problem with a linear velocity. Later on, Noor *et al.* [18] studied the MHD viscous flow due to shrinking sheet using Adomian decomposition Method (ADM) and they obtained a series solution. Sajid and Hayat [20] applied homotopy analysis method for the MHD viscous flow due to a shrinking sheet. Midya [13] studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhaimin *et al.* [16]. Midya [14] obtained a closed form analytical solution for the distribution of reactant solute in a MHD boundary layer flow over a shrinking sheet.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of two dimensional incompressible MHD boundary layer stagnation-point flow over a nonlinear stretching/shrinking sheet with velocity $U_w = ax^m$ and free stream velocity is $U_\infty = bx^m$ in porous media, where a, b and m are constants.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider the problem of steady two-dimensional laminar MHD stagnation-point flow over a nonlinear stretching/shrinking sheet with velocity $U_w = ax^m$ and free stream velocity is $U_\infty = bx^m$ in porous media. The fluid is an electrically conducting incompressible viscous fluid. It is assumed that external field owing polarization of charges and Hall-effect are neglected. The free stream temperature T_∞ is constant, x-axis is along the sheet and y-axis perpendicular to it, the applied magnetic field is transversely to x-axis. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy (Pai [19], Schlichting [22], Bansal [3]) under the influence of externally imposed transverse magnetic field (Jeffery [10], Bansal [4]) are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0{}^2 u}{\rho} - \frac{\nu u}{K} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_e B_0{}^2 u^2}{\rho C_p} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Accompanied by the boundary conditions:

$$\begin{aligned} &y = 0 : u = U_w,\ v = 0,\ \ T = T_w \\ &y \to \infty : u \to U_\infty,\ T \to T_\infty \end{aligned} \tag{4}$$

The governing partial differential equations (1) – (3) can be reduced to ordinary differential equations by introducing the following transformation

$$\eta = \left(\frac{U_\infty}{\nu x}\right)^{1/2} y, \psi = (\nu x\, U_\infty)^{1/2} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

Where η is the similarity variable and ψ is the stream function defined in the usual way as $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$, which identically satisfies (1).

The transformed non-linear ordinary differential equations are:

$$f''' + m(1 - f'^2) + \left(\frac{m+1}{2}\right) f f'' - M f' - S_p f' = 0 \tag{6}$$

$$\frac{\theta''}{Pr} + \left(\frac{m+1}{2}\right) f \theta' + Ec(M f'^2 + f''^2) = 0 \tag{7}$$

Where,

$M = \frac{\sigma_e B_0^2}{\rho b}$ (Magnetic parameter),

$S_p = \frac{\nu x^{1-m}}{Kb}$ (Porosity parameter),

$Ec = \frac{u^2}{C_p(T_w - T_\infty)}$ (Eckert number),

$Pr = \frac{\mu C_p}{\kappa}$ (Prandtl number)

The corresponding boundary conditions are reduced to:

$$f(0)=0, f^{'}(0)=\varepsilon,\ \theta(0)=1$$

$$f^{'}(\infty)\to 1,\ \theta(\infty)\to 0. \tag{8}$$

Where $\varepsilon = a/b$ is the stretching/shrinking parameter with ε >0 for stretching and ε <0 for shrinking.

3. RESULTS AND DISCUSSIONS

The system of governing equations (6)-(7) together with the boundary condition (8) is non-linear ordinary differential equations depending on the various values of Prandtl number Pr, the Eckert number Ec, the velocity exponent parameter m, the porosity parameter S_p, magnetic parameter M, and the stretching/shrinking parameter $\in$. The system of equations (6)-(7) is solved by Runge-Kutta fourth order scheme with a systematic guessing of $f^{'}(0)$ and $\theta^{'}(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size Δη = 0.01 is used while obtaining the numerical solution and accuracy upto the seventh decimal place i.e. 1 x 10^{-4}, which is very sufficient for convergence. The computations were done by a programme which uses a symbolic and computer language Matlab.

In order to verify the accuracy of our present method, we have compared our results with those of Backok and Ishak [4]. Table 1 compares the values of $f^{''}(0)$ and $-\theta^{''}(0)$ for ε = –1.1 (shrinking case), Pr = 0.7, S_p = 0 and $M = 0$. The comparisons are found to be in excellent agreement.

Table-1: Numerical values of $f^{''}(0)$ and $-\theta^{''}(0)$ for different values of m are compared with the result obtained by Backok and Ishak [4], when ε = –1.1 (shrinking case), Pr = 0.7, S_p = 0 and M=0.

	Norfifah Bachok and Anurag Ishak [4]		**Present paper results**	
m	**f''(0)**	**-θ'(0)**	**f''(0)**	**-θ'(0)**
2/3	.8213	0.1149	.8210	0.1146
1	1.1867	0.1828	1.1861	0.1829
2	1.8719	0.2903	1.8705	0.2907

It is observed from tables 2 and 3 that shear stress and Nusselt number respectively increase due to increase in velocity exponent parameter m, for the given values of Prandtl number Pr, stretching/shrinking parameter ε, Eckert number Ec, magnetic parameter M and porosity parameter S_p.

Table-2: Numerical values Skin friction coefficient, when Pr = 0.7, ε = –1.1 (shrinking case), Ec = 0.1, M =0.1 and S_p = .06

m	**f''(0)**
2/3	.9210
1	1.2367
2	1.89476

Table-3: Numerical values Nusselt number when Pr = 0.7, ε = –1.1 (shrinking case), Ec = 0.01, M = 0.01, S_p = .0001

m	**-θ'(0)**
2/3	.0860
1	0.15340
2	0.26260

Figure 1 and Figure 2 shows the effect of velocity exponent parameter m on the velocity and temperature profiles respectively. From Figure 1, we observed that the effect of increasing values of m is to increases the velocity distribution in the flow region and it makes the momentum boundary layer thickness thinner. The dimensionless temperature profile for different values of m is demonstrated in Figure 2. From the figure it is seen that the velocity exponent parameter m affects the temperature distribution in addition to velocity field. With increasing m the temperature decreases and consequently, the thermal boundary layer thickness reduces.

The impacts of magnetic parameter M on the velocity and temperature profiles are very significant in practical point of view. In Figure 3 and Figure 4, the variations in velocity field and temperature distribution for several values of M are presented in shrinking case respectively. The dimensionless velocity increases with increasing values of M. Accordingly, the thickness of momentum boundary layer decreases. This happens due to the Lorentz force arising from the interaction of magnetic and electric field during the motion of the electrically conducting fluid. To reduce momentum boundary layer thickness the generated Lorentz force enhances the fluid motion in the boundary layer region. On the other hand, from Figure 4, it is noticed that the temperature at a point decreases with M.

Figure 5, which illustrate the porosity parameter S_p on the velocity profile. We infer from this figure that the velocity profile increase with increasing values of porosity parameter S_p but quit slowly. This phenomenon corresponds with the assumption of pure Darcy flow.

Figure 6 shows the effect of Prandtl number (Pr) on temperature profiles in stretching case. We infer from this figure that due to higher values of Pr the temperature profile shows different character in different ranges of η . First for small η ($\eta \leq 3$), the dimensionless temperature increase with increasing value of Pr and then for large η ($\eta > 3$) it decreases with increase of Pr.

Figure 7 and Figure 8 depict the effect of velocity exponent parameter m on the velocity and temperature profiles in stretching case respectively. From figure 7, we observed that the effect of increasing values of m is to decrease the velocity distribution in the flow region. The dimensionless temperature profile for different values of m is demonstrated in Figure 8, we infer from this figure that with increasing m the temperature profile increases.

4. CONCLUSION

In the present study, we have theoretically studied the two-dimensional boundary layer flow and heat transfer of stagnation-point flow over a stretching/shrinking sheet in the presence of transverse magnetic field. The governing partial differential equations are transformed into ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved using Runge-Kutta fourth order method along with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. The numerical values of Skin-friction coefficient and Nusselt number at the stretching/shrinking sheet are derived, discussed numerically for various values of physical parameters and presented through tables. From the study, following conclusions can be drawn:

- The effect of increasing values of velocity exponent parameter m is to increases the velocity distribution in the flow region and decrease the temperature profile in shrinking case, but the reverse phoneme occurs in stretching case.
- As the magnetic parameter M increases, we can find the velocity profile increases in the flow region and to decrease the temperature profile. Thus we conclude that we can control the velocity field and temperature by introducing magnetic field.
- Porosity parameter S_p has significant effect on the velocity profile.
- The higher values of Pr the temperature profile shows different character in different ranges of η . First for small η ($\eta \leq 3$), the dimensionless temperature increase with increasing value of Pr and then for large η ($\eta > 3$) it decreases with increase of Pr.
- Shear stress and Nusselt number increase due to increase in velocity exponent parameter m, for the given values of Prandtl number Pr, stretching/shrinking parameter $\in$, Eckert number Ec, magnetic parameter M and porosity parameter S_p.

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REFERENCES

1. Crane, L. (1970): Flow past a stretching plate *Angew, Z. Math Phys (ZAMP),* vol.21, pp. 645-647.
2. Bachok, N. and Ishak, A. (2011): Similarity Solution for Stagnation-Point Flow and Heat Transfer Over a Nonlinearly Stretching/Shrinking Sheet, *Sains Malaysiana 40(11)*, pp. 1297-1300.
3. Bansal, J. L. (1977): Viscous Fluid Dynamics, *Oxford & IBH Pub. Co*., New Delhi, India.
4. Bansal, J. L. (1994): Magnetofluiddynamics of Viscous Fluids, *Jaipur Publishing House*, Jaipur, India.
5. Fang T, Zhang J (2009a). Viscous flow over an unsteady shrinking sheet with mass transfer. *Chin. Phys. Lett*., 26(1):014703, pp. 1-4.
6. Fang T, Zhang J (2009). Closed form exact solutions of MHD viscous flow over a shrinking sheet. Commun. in *Nonlinear Sc. and Numer. Simul*., 14(7), pp. 2853-2857.
7. Fang T, Zhang J (2010). Thermal boundary layer over a shrinking sheet: an analytical solution. *Acta Mech*, 209, pp. 325-343.
8. Hayat T, Abbas Z, Sajid M (2007). On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet. *J. Appl. Mech. Trans ASME*, 74(6), pp. 1165-1171.
9. Jat R.N. and Chaudhary S. (2010): Hydromagnetic flow and heat transfer on a continuous moving surface, *Applied Mathematical Science*, 4, pp. 65-78.
10. Jeffery, A. (1966): Magnetohydrodynamics, *Oliver and Boyed*, New York, USA.
11. Jhankal, A. K. and Kumar, M., (2013): MHD Boundary Layer Flow Past a Stretching Plate with Heat Transfer. *International J. of Engineering and Science*, Vol 2 (3), pp. 9-13.

12. Magyari, E., Keller, B. (1999): Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *J. Phys. D: Appl. Phys*. 32, pp. 577-585.
13. Midya C (2012). Hydromagnetic boundary layer flow and heat transfer over a linearly shrinking permeable surface. *Int. J. of Appl. Math. and Mech*., 8(3), pp. 57-68.
14. Midya C (2012b). Exact Solutions of Chemically Reactive Solute Distribution in MHD Boundary Layer Flow over a Shrinking Surface. *Chin. Phys. Lett.*, 29(1):014701, pp. 1-4.
15. Miklavcic M, CY, Wang (2006). Viscous flow due to a shrinking sheet. *Quart. of Appl. Math.*, 64(2), pp. 283-290.
16. Muhaimin, Kandasamy R, Hashim I (2010). Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction. *Nuclear Engineering and Design*, 240(5), pp. 933-939.
17. Nadeem S, Awais M (2008). Thin film flow on an unsteady shrinking sheet through porous medium with variable viscosity. *Physics Letters A*, 372, pp. 4965-4972.
18. Noor, N.F.M., Abdulaziz, O. and Hashim, I. (2010): MHD fow and heat transfer in a thin liquid film on an unsteady stretching sheet by the homotopy analysis method, *International Journal for Numerical Methods in Fluids* 63, pp. 357–373.
19. Pai, S. I. (1956): Viscous Flow Theory: I – Laminar Flow, *D.Van Nostrand Co*., New York, USA.
20. Sajid M, Hayat T (2009). The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet. *Chaos, Soliton \& Fractals*, 39(3), pp. 1317-1323.
21. Sakiadis , B.C. (1961): Boundary-layer behavior on continues solid surfaces: II. The boundary-layer on continuous flat surface, *AIChE J*.vol. 7, pp. 221-225.
22. Schichting, H. (1964): Boundary Layer Theory, sixth ed., *McGraw-Hill*, New York.
23. Stokes, G.G. (1901): On the effect of the internal friction of fluids on the motion of pendulums, *Camb. Phil. Trans. IX, 8 (1851), Math. and Phys. Papers, Cambridge*, 111, pp. 1-41.
24. Wang, C.Y. (1990). Liquid film on an unsteady stretching sheet. *Quart. Appl. Math*., 48, pp. 601- 610.
25. Wang, C.Y. (2008). Stagnation flow towards a shrinking sheet, *Int. J. Nonlinear Mech*. 43, pp. 377-382.

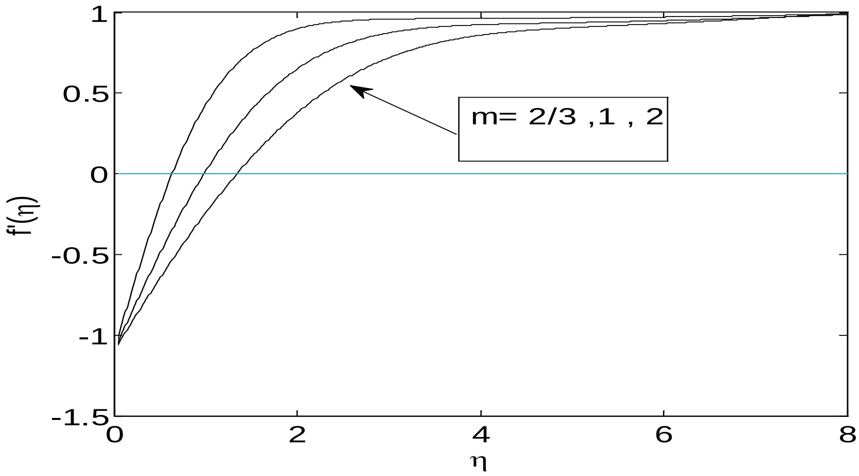

Figure-1: Velocity profile for various values of m when ε = -1.1, Ec=.1, M=.1, S_p =.06.

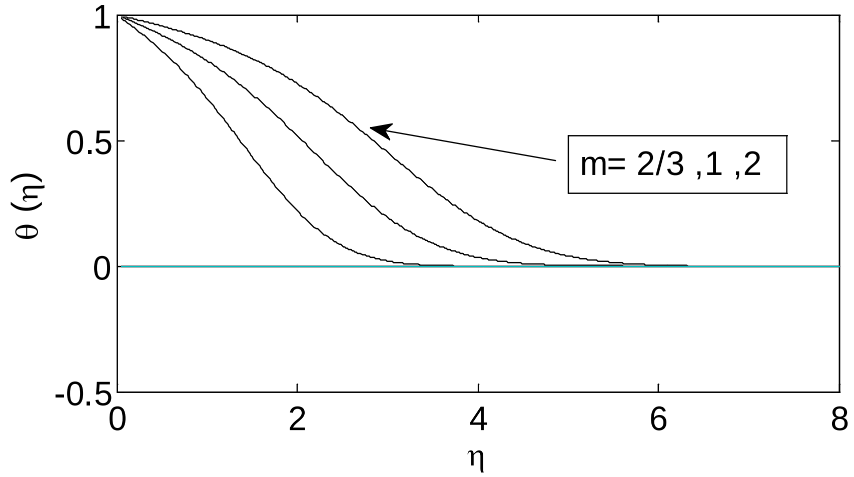

Figure-2: Temperature profile for various values of m when ε = -1.1, Ec=.01, Pr=.7, M=.01 , S_p =.0001.

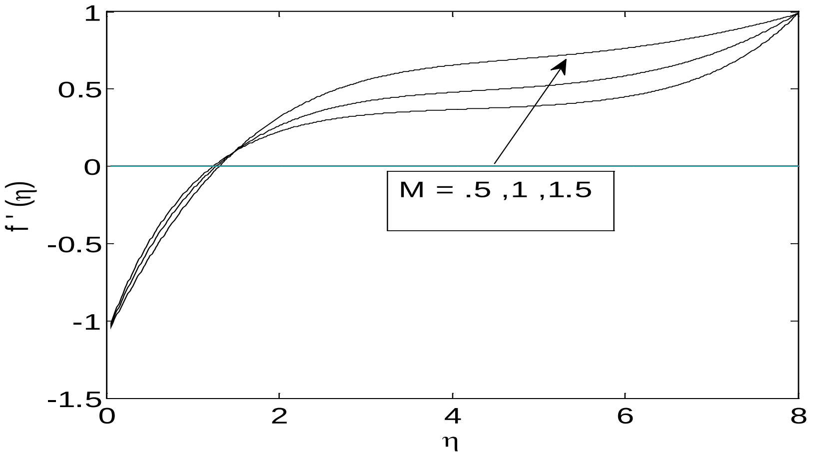

Figure-3: Velocity profile for various values of M when ε = -1.1, Ec=.1, m=2/3, S_p =.06.

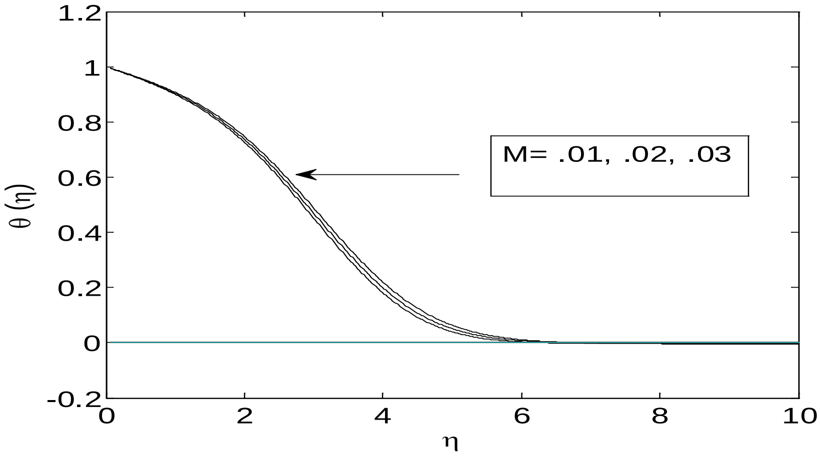

Figure-4: Temperature profile for various values of M when ε = -1.1, Ec=.01 , Pr=.7 ,m=2/3, S_p =.0001.

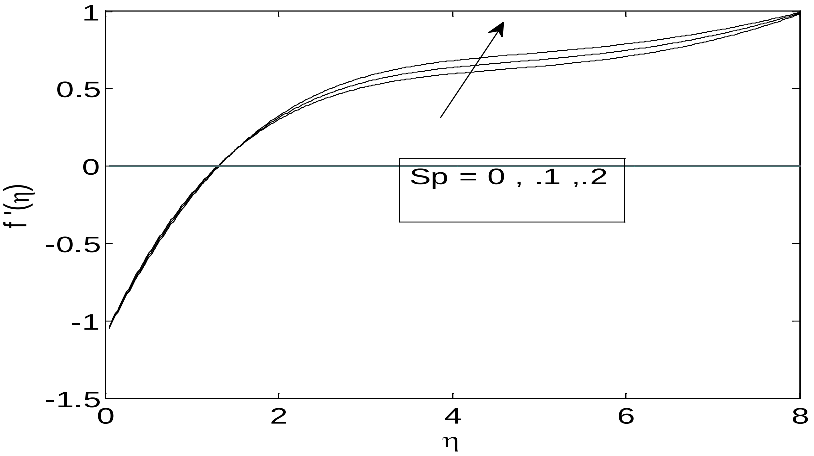

Figure-5: Velocity profile for various values of S_p when ε = -1.1, m=2/3, M=.5.

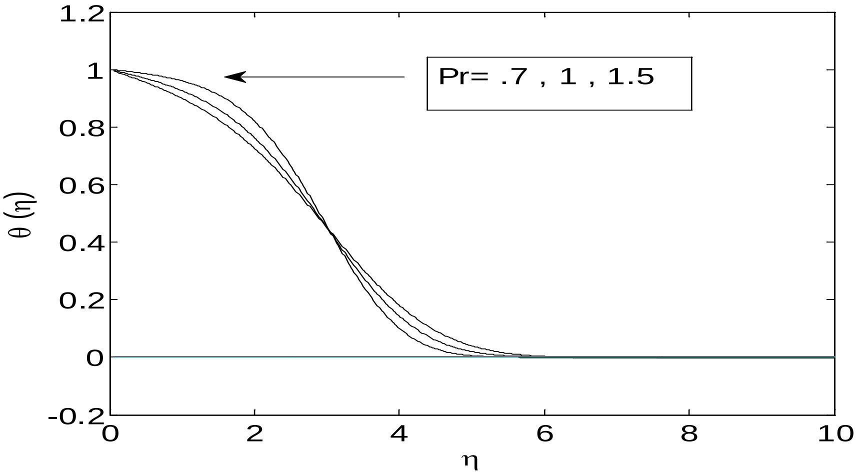

Figure-6: Temperature profile for various values of Pr when ε=-1.1, m=2/3, Ec=.01, M=.01, S_p =.0001.

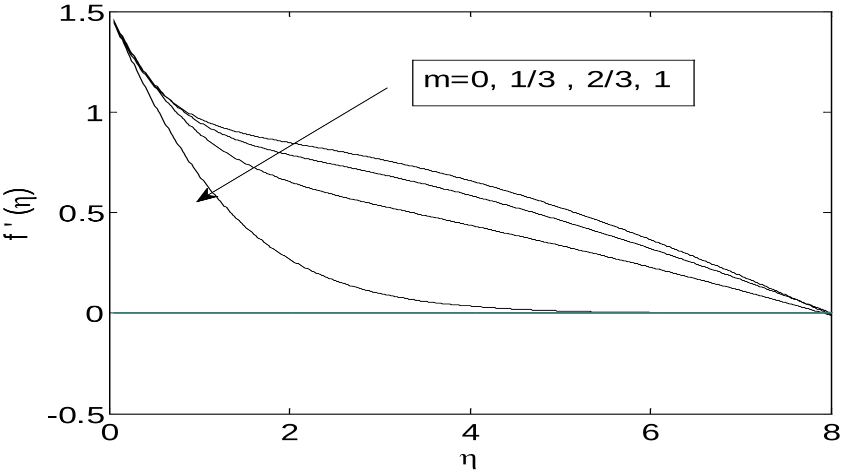

Figure-7: Velocity profile for various values of m when ε=1.5, M=.1, S_p =.06.

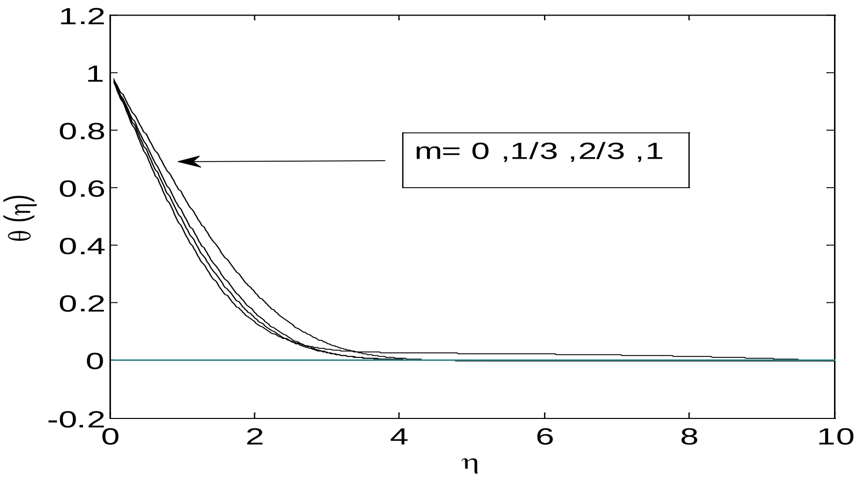

Figure-8: Temperature profile for various values of m when ε=.2, Pr=.7, Ec=.01, M=.01, S_p =.0001.

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