International Journal of Mathematical Archive-6(9), 2015, 32-36
Available online through www.ijma.info ISSN 2229 – 5046

ON $\hat{\mu}\beta$ - CLOSED SET IN BITOPOLOGICAL SPACES

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(Received On: 19-08-15; Revised & Accepted On: 19-09-15)

ABSTRACT

The aim of this paper is to introduce the concept of $\tau_1\tau_2\hat{\mu}\beta$ closed and open sets and study their basic properties in bitopological spaces.

Keywords: *$\tau_1\tau_2\hat{\mu}\beta$ closed sets, $\tau_1\tau_2\hat{\mu}\beta$ open sets.*

1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non-empty set τ_1 and τ_2 are topologies on X is called a bitopological space.Kelly [10] initiated the study of such spaces. In 1985, Fukutake [5] introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalization of various concepts of topology by considering bitopological spaces. Andrijevic.D[1], Levin[12], Nagaveni[16], K.Balachandran and Arokiarani[3], Gnanambal[7], Mashour.A.S[15] and Maki[14] introduced the concepts of semi open sets, weakly closed sets, generalized preclosed sets, pre regular closed sets, pre closed sets and α closed sets respectively. S.Pious Missier and E.Sucila [8] introduced $\hat{\mu}$ closed set. In this paper we introduce the notion of $\tau_1\tau_2\ \hat{\mu}\beta$ closed set and their properties.

2. PRELIMINARIES

Definition 2.1: Let A be a subset of X, then A is called $\tau_1\tau_2$ open [18, 19, 2] if $A = A_1 \cup B_1$, where A_1 is τ_1 and B_1 is τ_2.

Definition 2.2: $\tau_1\tau_2$regularopen [4] in X if $A = \tau_1int[\tau_2cl(A)]$

Definition 2.3: $\tau_1\tau_2$pre open [9] in X if $A \subseteq \tau_1int[\tau_2cl(A)]$

Definition 2.4: $\tau_1\tau_2$semi open [13] in X if $A \subseteq \tau_1cl[\tau_2int(A)]$

Definition 2.5: $\tau_1\tau_2\beta$ open [9] in X if $A \subseteq \tau_1cl[\tau_2int(\tau_1cl(A))]$,whenever $A \subseteq U$ and U is open in τ_1.

Definition 2.6: $\tau_1\tau_2$ g closed [5] in X, if $\tau_2cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_1.

Definition 2.7: $\tau_1\tau_2$w closed [6] in X, if $\tau_2cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in τ_1.

Definition 2.8: $\tau_1\tau_2$g* closed [20] in X, if $\tau_2cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_1.

Definition 2.9: $\tau_1\tau_2$sg closed [17] in X, if $\tau_2\, scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in τ_1.

Definition 2.10: $\tau_1\tau_2\alpha$g closed [17] in X, if $\tau_2\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_1.

Definition 2.11: $\tau_1\tau_2$gp closed [17] in X, if $\tau_2\, pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_1.

Definition 2.12: $\tau_1\tau_2$gsp closed [17] in X, if $\tau_2\, spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_1.

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Definition 2.13: $\tau_1\tau_2$gpr closed [17] in X, if $\tau_2$pcl(A) ⊆U whenever A⊆U and U is regular open in τ_1.

Definition 2.14: $\tau_1\tau_2\mu$ closed [17] in X, if $\tau_2$cl(A) ⊆U whenever A⊆U and U is gα* open in τ_1.

Definition 2.15: $\tau_1\tau_2$g closed [17] in X, if $\tau_2$cl(A) ⊆U whenever A⊆U and U is open in τ_1.

3. On $\tau_1\tau_2\,\hat{\mu}\beta$ closed set

Definition3.1: A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2\,\hat{\mu}\beta$ closed set in X if $\tau_2\hat{\mu}$cl(A) ⊆U whenever A ⊆U and U is $\tau_1\beta$ open in X.

Theorem 3.2: Every τ_2 closed set in (X, τ_1 ,τ_2) is $\tau_1\tau_2\,\hat{\mu}\beta$ closed set but not conversely.

Proof: Assume that A is $\tau_2$closed and whenever A ⊆U and U is $\tau_1\,\beta$ open. Every τ_2 closed set is $\tau_2\,\hat{\mu}$ closed. Therefore $\tau_2$cl(A) ⊆$\tau_2\hat{\mu}$cl(A) ⊆U. Therefore $\tau_2\hat{\mu}$cl(A) ⊆U whenever A ⊆U and U is $\tau_1\beta$ open. Hence A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.3: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c},X, ∅}. Then the set {a, d} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ closed.

Theorem 3.4: Every $\tau_1\tau_2$ regular closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be a $\tau_1\tau_2$ regular closed set. Every $\tau_1\tau_2$ regular closed set is $\tau_1\tau_2$ closed. By theorem 3.2, A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.5: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c},X,∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {d} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ regular closed.

Theorem 3.6: Every $\tau_1\tau_2$ g closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be a $\tau_1\tau_2$ g closed set. Every $\tau_1\tau_2$ g closed set is $\tau_1\tau_2$ closed. By theorem 3.2, A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.7: Let X= {a, b, c, d} be a bitopological space with topologies τ_1 = {{a}, {b},{a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {c} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ g closed.

Theorem 3.8: Every $\tau_1\tau_2$ gr closed set is $\tau_1\tau_2\beta wg^*$ closed.

Proof: Let A be $\tau_1\tau_2$ gr closed. Every $\tau_1\tau_2$ gr closed is $\tau_1\tau_2$ g closed. By theorem 3.6, A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.9: Let X= {a, b, c, d} be a bitopological space with topologies τ_1 = {{a}, {b}, {a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {a} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ gr closed.

Theorem 3.10: Every $\tau_1\tau_2$ g* closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be a $\tau_1\tau_2$ g* closed set. Every $\tau_1\tau_2$ g* closed set is $\tau_1\tau_2$ g closed. By theorem 3.6, A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.11: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c},X,∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {d} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ g* closed.

Theorem 3.12: Every $\tau_1\tau_2$ w closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be $\tau_1\tau_2$ w closed, whenever A⊆U and U is τ_1 semi open. Then $\tau_2$cl(A) ⊆ $\tau_2\,\hat{\mu}$cl(A) ⊆ U. Since every τ_1 semi open set is $\tau_1\beta$ open. Therefore every $\tau_1\tau_2$ w closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.13: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {a, d} is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ w closed.

Theorem 3.14: Every $\tau_1\tau_2\ \alpha g$ closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be $\tau_1\tau_2\ \alpha g$ closed such that τ_2αcl(A) ⊆U, whenever A⊆U and U is τ_1 open. Then τ_2αcl(A) ⊆ $\tau_2$cl(A) ⊆ $\tau_2\hat{\mu}$cl(A) ⊆ U. Since every τ_1 open set is $\tau_1\beta$ open. Therefore every $\tau_1\tau_2$ αg closed set is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Example 3.15: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c}, {a, b, c}, X, ∅}. Then the set {c} is $\tau_1\tau_2\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ αg closed.

Theorem 3.16: Every $\tau_1\tau_2$ sg closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be $\tau_1\tau_2$ sg closed such that $\tau_2$scl(A) ⊆U, whenever A⊆U and U is τ_1 semi open. Then $\tau_2$scl(A) ⊆ $\tau_2$cl(A) ⊆ $\tau_2\hat{\mu}$cl(A) ⊆ U. Since every τ_1 semi open set is $\tau_1\beta$ open. Therefore every $\tau_1\tau_2$ sg closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Example 3.17: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c},X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c},X, ∅}. Then the set {a, c, d} is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ sg closed.

Theorem 3.18: Every $\tau_1\tau_2$ $g\alpha$ closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed but not conversely.

Proof: Let A be $\tau_1\tau_2$ $g\alpha$ closed such that $\tau_2\alpha$cl(A) ⊆U, whenever A⊆U and U is τ_1 α open. Then $\tau_2\alpha$cl(A) ⊆ $\tau_2$cl(A) ⊆ $\tau_2\hat{\mu}$cl(A) ⊆ U. Since every $\tau_1\alpha$ open set is $\tau_1\beta$ open. Therefore every $\tau_1\tau_2$ gα closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Example 3.19: Let X={a, b, c, d} be a bitopological space with topologies τ_1 = {{a},{b},{a, b}, {a, b, c},X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}. Then the set {c, d} is $\tau_1\tau_2\hat{\mu}\beta$ closed but not $\tau_1\tau_2$ gα closed.

Theorem 3.20: Every $\tau_1\tau_2$ gs closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Proof: Let A be $\tau_1\tau_2$ gs closed such that $\tau_2$scl(A) ⊆U, whenever A⊆U and U is τ_1 open. Then $\tau_2$scl(A) ⊆ $\tau_2$cl(A) ⊆ $\tau_2\hat{\mu}$cl(A) ⊆ U. Since every τ_1 open set is $\tau_1\beta$ open. Therefore every $\tau_1\tau_2$ gs closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Theorem 3.21: Every $\tau_1\tau_2$ gsp closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Proof: Let A be $\tau_1\tau_2$ gsp closed such that $\tau_2$spcl(A) ⊆U, whenever A⊆U and U is τ_1 open. Then $\tau_2$spcl(A) ⊆ $\tau_2$cl(A) ⊆ $\tau_2\hat{\mu}$cl(A) ⊆ U. Since every τ_1 open set is $\tau_1\beta$ open.Therefore every $\tau_1\tau_2$ gsp closed set is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed.

Remark 3.22:.Every $\tau_1\tau_2$ θclosed, $\tau_1\tau_2$ πclosed, $\tau_1\tau_2$ δclosed set is $\tau_1\tau_2$ closed set. Therefore every $\tau_1\tau_2$ θ closedset, $\tau_1\tau_2$ πclosed, $\tau_1\tau_2$ δclosed set is $\tau_1\tau_2\hat{\mu}\beta$ closed.

Theorem 3.23: Let A be a subset of a bitopological space (X, τ_1, τ_2). If A is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed then $\tau_2\hat{\mu}$cl(A)-A does not contain non empty $\tau_1\beta$closed sets.

Proof: Suppose that A is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed. Let F be a $\tau_1\beta$ closed set such that F⊆$\tau_2\hat{\mu}$cl(A)-A. Since F⊆$\tau_2\hat{\mu}$cl(A)-A, we have F⊆$\tau_2\hat{\mu}$cl(A)-A∩A^c. Consequently F⊆A^c, we have A⊆F^c. since F is $\tau_1\beta$ closed set, we have F^c is $\tau_1\beta$ open. Since A is $\tau_1\tau_2\hat{\mu}\beta$ closed. We have $\tau_2\hat{\mu}$cl(A) ⊆F^c. Thus F⊆ $[\tau_2\hat{\mu}cl(A)\text{-}A]^c$=X- [$\tau_2\hat{\mu}$cl(A)].Hence F⊆ φ. But φ ⊆F. Therefore F= φ.

Theorem 3.24: Let A be a $\tau_1$open set in (X, τ_1, τ_2) and U be $\tau_1\beta$ open in A. Then U=A∩W for some $\tau_1\beta$ open set W in X.

Proof: Let A be a $\tau_1$open set in (X, τ_1, τ_2) and let U be $\tau_1\beta$ open in X. Since U is $\tau_1\beta$ open in A, we have

U= $\tau_1$cl[$\tau_2$int($\tau_1$cl(U))]
 = $\tau_1$cl[A ∩$\tau_2$int($\tau_1$cl(U))]
 = A ∩ {$\tau_1$cl[A ∩$\tau_2$int($\tau_1$cl(U))]}
 = A ∩{ $\tau_1$cl(A) ∩$\tau_1$cl[$\tau_2$int($\tau_1$cl(U))]}
 = A ∩{A ∩ $\tau_1$cl[$\tau_2$int($\tau_1$cl(U))]}   [since A is τ_1 open]
 = A ∩A ∩$\tau_1$cl[$\tau_2$int($\tau_1$cl(U))]
 = A ∩$\tau_1$cl[$\tau_2$int($\tau_1$cl(U))]
 = A ∩W

Where W= $\tau_1$cl[$\tau_2$int($\tau_1$cl(A))]. Then U=A ∩W, for some τ_1 β open set W in X.

Remark 3.25: If A is τ_1 open and U is τ_1 β open in X then U ∩A is τ_1 β open in A.

Theorem 3.26: If X∈$\tau_2\hat{\mu}$cl(A) if and only if U∩A≠ φ for every τ_1 β open set U containing X.

Proof: Let X∈$\tau_2\hat{\mu}$cl(A). Suppose that there exist a $\tau_1\beta$ open set U containing X such that U∩A= φ. Then A ⊆U^c and U^c is $\tau_1\beta$ closed set. Since A ⊆U^c, $\tau_2\hat{\mu}$cl(A) ⊆$\tau_2\hat{\mu}$cl(U^c). Since X∈$\tau_2\hat{\mu}$cl(A) we have X∈$\tau_2\hat{\mu}$cl(U^c), since U^c is $\tau_1\beta$ closed set ⇒X∈U^c. Hence X∉U, which is a contradiction that X∈U. Therefore U∩A≠ φ. Hence U∩A≠ φ for every $\tau_1\beta$ open set U containing X. Conversely, let U∩A≠ φ, for every $\tau_1\beta$ open set U containing X. Suppose that X∉$\tau_2\hat{\mu}$cl(A), then there exists a $\tau_1\beta$ open set U containing X such that U∩A= φ. This is contradiction to U∩A≠ φ. Hence X∈$\tau_2\hat{\mu}$cl(A).

Theorem 3.27: If A is $\tau_1\beta$ open in (X, τ_1, τ_2) then $A\cap\tau_2\hat{\mu}cl(A) \subseteq \tau_2\hat{\mu}cl(B)$ for any subset B of A.

Proof: Let A be $\tau_1\,\beta$ open in (X, τ_1, τ_2). Let $B\subseteq A$ and $x\in A\cap\tau_2\hat{\mu}cl(B)$. Then $x\in\tau_2\hat{\mu}cl(B)$. Let U be a $\tau_1\beta$ open subset of A such that $X\in U$. By theorem 3.24, there exists a $\tau_1\,\beta$ open subset W of X such that $U=A\cap W$. Since $X\in U$, we have $X\in A\cap W$.Hence $X\in A$ and $X\in W$.since $X\in\tau_2\hat{\mu}cl(B)$ and W is $\tau_1\,\beta$ open subset in X, we have $W\cap B\neq\varphi$. Now $U\cap B=(A\cap W)\cap B = W\cap(A\cap B)= W\cap B\neq\varphi$. Hence $U\cap B\neq\varphi$ for any $\tau_1\beta$ open subset U of A such that $X\in U$. Therefore $X\in\tau_2\hat{\mu}cl(B)$. Hence $A\cap\tau_2\hat{\mu}cl(B)\subseteq\tau_2\hat{\mu}cl(B)$ for any subset B of A.

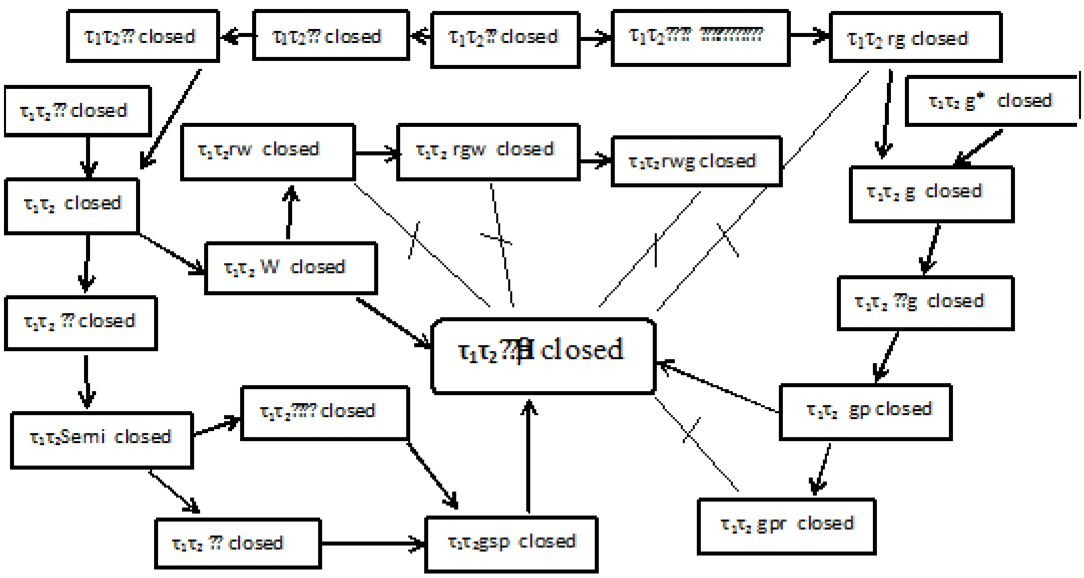
Theorem 3.28: Let A and B be subsets such that $A\subseteq B\subseteq\tau_2\hat{\mu}cl(A)$. If A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed, then B is $\tau_1\tau_2\,\hat{\mu}\beta$ closed.

Proof: Let A and B subsets such that $A\subseteq B\subseteq\tau_2\hat{\mu}cl(B)$. Suppose that A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed. Let $B\subseteq U$ and U is $\tau_1\,\beta$ open in X. Since $A\subseteq B$ and $B\subseteq U$, we have $A\subseteq U$. Hence $A\subseteq U$ and U is $\tau_1\,\beta$ open in X. Since A is $\tau_1\tau_2\,\hat{\mu}\beta$ closed we have $\tau_2\hat{\mu}cl(A)\subseteq U$. Since $B\subseteq\tau_2\hat{\mu}cl(A) \Rightarrow \tau_2\hat{\mu}cl(B)\subseteq\tau_2\hat{\mu}cl[\tau_2\hat{\mu}cl(A)]= \tau_2\hat{\mu}cl(A)\subseteq U$. Hence $\tau_2\hat{\mu}cl(B)\subseteq U$. Therefore B is $\tau_1\tau_2\,\hat{\mu}$ closed.

Example 3.29: The figure is justified with the following examples.

Let X={a, b, c, d} be a bitopological space with topologies τ_1={{a},{b},{a, b}, {a, b, c}, X, ∅} and τ_2 = {{a},{c},{a, c},{a, b, c}, X, ∅}.

1. $\tau_1\tau_2$ closed sets in X are X, ∅,{d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
2. $\tau_1\tau_2\,\hat{\mu}\beta$ closed sets in X are X, ∅,{a},{b},{c},{d},{a, d},{a, b},{b, c},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
3. $\tau_1\tau_2$ regular closed sets in X are X, ∅,{c, d},{a, c, d}
4. $\tau_1\tau_2$ gr closed sets in X are X, ∅, {d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
5. $\tau_1\tau_2$ g* closed sets in X are X, ∅,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
6. $\tau_1\tau_2$ g closed sets in X are X, ∅,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
7. $\tau_1\tau_2\alpha$ closed sets in X are X, ∅,{a},{b},{a, b},{a, b, c}
8. $\tau_1\tau_2$ semi closed sets in X are X, ∅,{b},{d},{a, b},{b, c},{b, d},{a, b, d},{b, c, d}
9. $\tau_1\tau_2$ w closed sets in X are X, ∅, {d},{b, d},{a, b, d},{b, c, d}
10. $\tau_1\tau_2$ sg closed sets in X are X, ∅,{a}{b},{c},{d},{a, b},{a, d},{b, c},{b, d},{c, d},{a, b, d},{b, c, d}
11. $\tau_1\tau_2\,\beta$ closed sets in X are X, ∅,{b},{c},{d},{a, b},{b, c},{b, d},{c, d},{a, b, d},{b, c, d}
12. $\tau_1\tau_2$ pre closed sets in X are X, ∅,{a},{b},{a, b},{a, c},{a, b, c},{a, c, d}
13. $\tau_1\tau_2$ swg closed sets in X are X, ∅,{b},{d},{b, d},{c, d},{a, c, d}
14. $\tau_1\tau_2$ gα closed sets in X are X, ∅,{b},{d},{b, c},{b, d},{b, c, d}
15. $\tau_1\tau_2$ gα* closed sets in X are X, ∅,{b},{d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
16. $\tau_1\tau_2$ μ closed sets in X are X, ∅,{d},{a, d},{b, d},{c, d},{a, c, d},{a, c, d},{b, c, d}
17. $\tau_1\tau_2$ gs closed sets in X are X, ∅,{a},{b},{c},{d},{a, d},{a, b},{b, c},{b, d},{c, d},{a, b, d}, {a, c, d},{b ,c, d}
18. $\tau_1\tau_2$ αg closed sets in X are X, ∅,{b},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
19. $\tau_1\tau_2$ gsp closed sets in X are X, ∅,{a},{b},{c},{d},{a, d},{a, b},{b, c},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}



A ⟶ B Means A implies B but not conversely	
A —/— B means A and B are independent of each other	

4. On $\tau_1\tau_2$ $\hat{\mu}\beta$ open set

Definition 4.1: A subset A of a bitopological (X, τ_1, τ_2) is called $\tau_1\tau_2$ $\hat{\mu}\beta$ open in X if its complement is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed in X.

Theorem 4.2: A subset A of a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ $\hat{\mu}\beta$ open if and only if $F \subseteq \tau_2\hat{\mu}int(A)$ whenever $F \subseteq A$ and F is $\tau_1\tau_2$ β closed in X.

Proof: Suppose that A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open. Let $A \subseteq F$ and F is τ_1 β closed in X. Then $A^c \subseteq F^c$ and F^c is $\tau_1\beta$ open in X. Since A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open, we have A^c is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed. Hence $\tau_2\hat{\mu}cl(A^c) \subseteq F^c$. Consequently, $[\tau_2\hat{\mu}int(A)]^c \subseteq F^c$. Therefore $F \subseteq \tau_2\hat{\mu}int(A)$.Conversely , suppose that $F \subseteq \tau_2\hat{\mu}int(A)$ whenever $F \subseteq A$ and F is $\tau_1\beta$ closed in X. Let $A^c \subseteq U$ and U is $\tau_1\beta$ open in X. Then $U^c \subseteq A$ and U^c is $\tau_1\beta$ closed in X. By our assumption we have $U^c \subseteq \tau_2\hat{\mu}int(A)$.Hence $[\tau_2\hat{\mu}int(A)]^c \subseteq U$. Therefore $\tau_2\hat{\mu}cl(A^c) \subseteq U$. Consequently A^c is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed. Hence A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open.

Theorem 4.3: Let A and B be subsets such that $\tau_2\hat{\mu}int(A) \subseteq B \subseteq A$. If A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open, then B is $\tau_1\tau_2$ $\hat{\mu}\beta$ open.

Proof: Suppose that A and B are subsets such that $\tau_2\hat{\mu}int(A) \subseteq B \subseteq A$. Let A be $\tau_1\tau_2$ $\hat{\mu}\beta$ open. Let $F \subseteq B$ and F is τ_1 β closed in X. Since $F \subseteq A$. Therefore $F \subseteq \tau_2\hat{\mu}int(A)$. Since $\tau_2\hat{\mu}int(A) \subseteq B$, we have
$\tau_2\hat{\mu}int[\tau_2\hat{\mu}int(A)] \subseteq \tau_2\hat{\mu}int(B) \Rightarrow \tau_2\hat{\mu}int(A) \subseteq \tau_2\hat{\mu}int(B) \Rightarrow F \subseteq \tau_2\hat{\mu}int(B) \Rightarrow$B is $\tau_1\tau_2$ $\hat{\mu}\beta$ open.

Theorem 4.4: If a subset A is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed then $\tau_2\hat{\mu}cl(A)$-A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open.

Proof: Suppose that A is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed. Let $F \subseteq \tau_2\hat{\mu}cl(A)$-A and F is τ_1 β closed. Since A is $\tau_1\tau_2$ $\hat{\mu}\beta$ closed, we have $\tau_2\hat{\mu}cl(A)$-A does not contain non empty τ_1 β closed. By theorem 3.23,
$F=\varphi \Rightarrow \varphi \subseteq \tau_2\hat{\mu}cl(A)\text{-}A \Rightarrow \tau_2\hat{\mu}int(\varphi) \subseteq \tau_2\hat{\mu}int[\tau_2\hat{\mu}cl(A)\text{-}A] \Rightarrow F \subseteq \tau_2\hat{\mu}int[\tau_2\hat{\mu}cl(A)\text{-}A]$. Therefore $\tau_2\hat{\mu}cl(A)$-A is $\tau_1\tau_2$ $\hat{\mu}\beta$ open.

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Source of support: Nil, Conflict of interest: None Declared

