

AN ORDERING POLICY WITH VARYING DEMAND FOR DETERIORATING ITEMS

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ABSTRACT

In this paper we have consider an order level inventory model with varying demand for deteriorating items. Here the demand is initially constant and after some period of time the demand increases exponentially due to maximum shortages we also considered deteriorating items and price breaks allowing shortages.

Key words: Order level inventory, Exponential increasing demand, deteriorating items, price breaks.

INTRODUCTION

Several researchers have attempted to obtain optimal ordering quantity for deteriorating items. In real word problems several factors will influence for obtaining optimal ordering quantity. Several Researchers attempted inventory models for price breaks for example Abad [1] has attempted to determined selling price and lot size for retailer when he is given all units discounts, further Arcelus *et al.* [2] considered the forward buying policies of an inventory model with deteriorating items and temporary price discounts. Fazal *et al.* [3] also considered the classical quantity discount model from the variety of EOQ models basically presented by Harris [4] Hwang *et al.* [5] considered an EOQ model with quantity discounts for both purchasing price and Freight cost. Matsuyama *et al.* [6] portrayed EOQ models with price discounts. Rubin *et al.* [7] also attempted to obtain EOQ with price discounts. A comparpison with JIT (Vs) EOQ with price discounts presented by Chaudhuri *et al.* [8].

In the recent literature of Inventory models, several researchers considered deterioration of Items. Deterioration place a major role for stocking of goods even in supply chain management, in this area an attempt is made by Rau *et al.* [9] by considering deterioration of Items in supply chain management. Inventory model for determining items by allowing shortages is considered by Dye *et al.* [10] chung *et al.* [11] also attempted in determining an EOQ model with deteriorating items under varying demand. The delay in payments for deteriorating items in the context of EOQ models is discussed by skouri *et al.* [12]. The deterioration of Inventory Items are classified into three categories by Ghare and Schrader [13]. These three categories are direct spoilage, physical depletion and deliration. Direct spoilage means breakage of Items which cannot be used further. where as deterioration refers to the slow but gradual loss of qualitative properties of the items in a phased manner.

2. NOTATIONS

$q(t)$ - Inventory level at time 't'

$s=q(0)$ = Stock level at the beginning of each ccycle after fulfilling back orders

$S_0 = q(T_0)$ = Stock level below which the demand rate is constant. It is an external variable beyond the control of the decision maker

Q = Stock level at the begininning the amount of shortages.

D = Constant demand rate

T_0 = Time unitl stock level reaches S_0

T_1 = Time unitl shortage begins

T = Length of the cycle time

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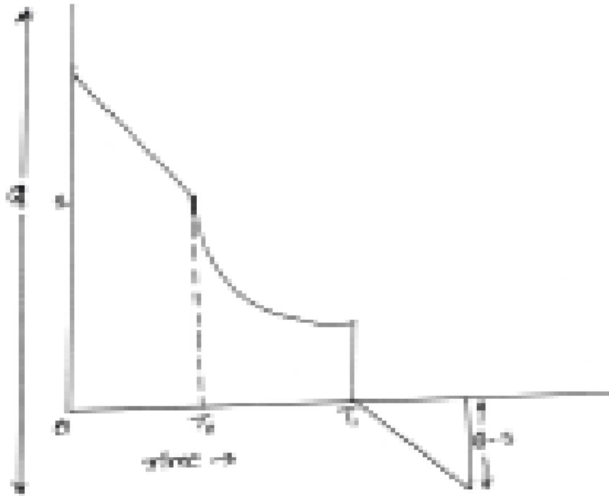
- θ = Deterioration rate, fraction of the on hand inventory
 C_2 = Shortage cost per unit per unit time
 C_3 = The ordering cost of Inventory
 P_i = The unit cost of item in $f_{i-1} < S \leq f_i$, where $i = 1$ to N and $f_0 < f_1 < f_2 \dots f_N$
 P_N = The unit cost of item in $f_N < S$
 h = Inventory holding cost per item per unit time
 U = The unit selling price of deteriorated items

3. ASSUMPTIONS

The following assumptions are considered

- i) Shortages are allowed and are fully back logged
- ii) The replenishment is instantaneous.
- iii) Deteriorated items are sold at a low price at the end of the stocks
- iv) Two - Component exponential demand rate is considered i.e. demand depends on the inventory as well as on the fixed customers
- v) price breaks are considered here i.e. the unit purchase price is discounted with the increasing ordering quantity.

4. FORMULATION OF THE MODEL



In real word, initially the demand of the product may not be high. To increase the demand of the product, the pruaent management may implement several advertising strategies. Hence demand may vary, that means initially demand is constant, when the product is launched and it may suddenly increases. Here we assume that upto certain period a constant demand and latter exponential increasing demand.

The inventory level will be depleted at a rate of during the period $(0, T_0)$ is 'D'. During the period (T_0, T_1) the Inventory level will be depleted at a exponential rate ae^{at}

The inventory falls to zero level at time $t = T_1$, shortages are than allowed for replenishment up to time $t = T$. Therefore, for a deteriorionrate the instantaneous inventory level will satisfy the following differential equation.

will the satisfy the following equation

$$\frac{dq}{dt} + \theta q = -D \quad 0 \leq t \leq T_0 \rightarrow (1)$$

with the boundary conditions are

$$q(0) = S \rightarrow (2)$$

$$q(T_0) = S_0 \rightarrow (3)$$

$$\frac{dq}{dt} + \theta q = -ae^{at} \quad T_0 \leq t \leq T_1 \rightarrow (4)$$

With the boundary conditions

$$q(T_0) = S_0 \rightarrow (5)$$

$$q(T_1) = 0 \rightarrow (6)$$

$$\frac{dq}{dt} = -ae^{\alpha t} \text{ where } T_1 \leq t \leq T \rightarrow (7)$$

With the boundary Conditions are

$$q(T_1) = 0 \rightarrow (8)$$

$$q(T) = -(Q - S) \rightarrow (9)$$

The solution of the equation (1) is

$$Y.I.F = \int Q.I.F + K_1$$

$$Q.e^{\theta t} = \int -De^{\theta t} + K_1$$

$$q(t).e^{\theta t} = -D \int e^{\theta t} dt + K_1$$

$$q(t) = e^{-\theta t} \left(-D \frac{e^{\theta t}}{\theta} + K_1 \right) \rightarrow (10)$$

By using boundary condition $q(0) = S$, and $t = 0$

$$K_1 = S + \frac{D}{\theta} \rightarrow (11)$$

Substituting (11) in (10) we get

$$q(t) = \frac{-D}{\theta} + e^{-\theta t} \left(S + \frac{D}{\theta} \right) \rightarrow (12) \quad 0 \leq t \leq T_0$$

The solution of (12) with boundary condition $t = T_0$ and $q(t_1) = 0$

$$\frac{-D}{\theta} + e^{-\theta T_0} \left(S + \frac{D}{\theta} \right) = 0$$

$$e^{-\theta T_0} \left(S + \frac{D}{\theta} \right) = \frac{D}{\theta}$$

$$-\theta T_0 = \log \left(\frac{D}{D + S\theta} \right)$$

$$T_0 = \frac{-1}{\theta} \log \left(\frac{D}{D + S\theta} \right) \rightarrow (13)$$

The solution of the equation (4) with the boundary condition $T_0 \leq t \leq T_1$ is

$$Y.I.F = \int Q.I.F + K_2$$

$$q(t).e^{\theta t} = \int -ae^{\alpha t}.e^{\theta t} + K_2$$

$$q(t) = \frac{-a}{\alpha + \theta} e^{\alpha t} + K_2 e^{-\theta t} \rightarrow (14)$$

By using the boundary condition $q(T_0) = S_0$

$$S_0 = \frac{-a}{\alpha + \theta} e^{\alpha T_0} + K_2 e^{-\theta T_0}$$

$$K_2 = e^{\theta T_0} \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \rightarrow (15)$$

Substituting (15) in (14) we get

$$S_0 = \frac{-a}{\alpha + \theta} e^{\alpha T_0} + \left(e^{\theta T_0} \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \right) e^{-\theta t}$$

$$q(t) = \frac{-a}{\alpha + \theta} e^{\alpha T_0} + \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) e^{-\theta(t-T_0)} \rightarrow (16) \quad T_0 \leq t \leq T_1$$

The boundary condition from (6) is

$$q(T_1) = 0$$

It gives

$$T_1 = T_0 + \frac{1}{\alpha + \theta} \log \left(\frac{S_0(\alpha + \theta) + a}{\alpha + \theta} \right) \rightarrow (17)$$

Substituting (13) in (17) we get

$$T_1 = \frac{-1}{\theta} \log \left(\frac{D}{D + S\theta} \right) + \frac{1}{\alpha + \theta} \log \left(\frac{S_0(\alpha + \theta) + a}{\alpha + \theta} \right) \rightarrow (18)$$

The solution of equation (7) with the boundary condition $q(T_1) = 0$ is
 $q(t) = -a.e^{\alpha(t-T_1)} \rightarrow (19) (T_1 \leq t \leq T)$

The boundary condition $q(T) = -(Q - S)$ gives

$$T = T_1 + \frac{Q - S}{ae^{\alpha T}} \rightarrow (20)$$

Substituting (18) in (20) we get

$$T = \frac{-1}{\theta} \log \left(\frac{D}{D + S\theta} \right) + \frac{1}{\alpha + \theta} \log \left(\frac{S_0(\alpha + \theta) + a}{\alpha + \theta} \right) + \frac{Q - S}{ae^{\alpha T}} \rightarrow (21)$$

Here the total variable cost is consist of fixed cost, holding cost, shortage cost, purchased cost minus the selling price of the deteriorated items. They are sumed up after evaluating the above cost individually

The ordering cost $OC = C_3$

The holding Cost

$$(HC) = h \int_0^{T_0} q(t) dt + h \int_{T_0}^{T_1} q(t) dt = h \left(\int_0^{T_0} \left\{ \frac{-D}{\theta} + e^{-\theta t} \left(S + \frac{D}{\theta} \right) \right\} dt + \int_{T_0}^{T_1} \left\{ \frac{-a}{\alpha + \theta} e^{\alpha T_0} + \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) e^{-\theta(t-T_0)} \right\} dt \right)$$

$$HC = h \left(\left\{ \frac{-D}{\theta} T_0 - \left(S + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) \right\} + \left\{ \frac{-a}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) - \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) \right\} \right) \rightarrow (22)$$

$$\text{The deteriorated items} = q(0) - \left\{ \int_0^{T_0} D dt + \int_{T_0}^{T_1} ae^{\alpha t} dt \right\}$$

$$= S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \rightarrow (23)$$

The selling price of deteriorated items

$$DP = U \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \rightarrow (24)$$

The shortage cost SHC for a cycle is

$$SHC = -C_2 \int_{T_1}^T q(t) dt$$

$$= -C_2 \int_{T_1}^T -(Q - S) dt$$

$$SHC = C_2 \frac{(Q - S)^2}{2D} \rightarrow (25)$$

The purchase cost PC for a quantity of amount $S(f_{i-1} < S < f_i)$ is
 $PC = P_i S \rightarrow (26)$

The total cost function

$$TVC = OC + HC + SHC + PC - DP$$

$$= C_3 + h \left(\left\{ \frac{-D}{\theta} T_0 - \left(S + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) \right\} + \left\{ \frac{-a}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) - \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) \right\} \right)$$

$$+ C_2 \frac{(Q - S)^2}{2D} + P_i S - U \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \rightarrow (27)$$

The selling price SR for a quantity of non- deteriorated amount is

$$SR = V_i \left(S - S_0 + DT_0 - a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \rightarrow (28)$$

The total profit function TP is defined as TP(S,Q) = SR-TVC

$$\begin{aligned} SR &= V_i \left(S - S_0 + DT_0 - a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) - \left(C_3 + h \left\{ \left(\frac{-D}{\theta} T_0 - \left(S + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) \right\} + \left(\frac{-a}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) - \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) \right\} \right) \\ &\quad + C_2 \frac{(Q-S)^2}{2D} + P_i S - U \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \\ &= \left((V_i - P_i)S - C_3 + \frac{hD}{\theta} T_0 + h \left(S_0 + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) + \frac{ha}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right. \\ &\quad \left. + h \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) + C_2 \frac{(Q-S)^2}{2D} + U \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \right. \\ &\quad \left. - V_i \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \right) \\ TP &= \left((V_i - P_i)S - C_3 + \frac{hD}{\theta} T_0 + h \left(S_0 + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) + \frac{ha}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right. \\ &\quad \left. + h \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) + C_2 \frac{(Q-S)^2}{2D} + (U - V_i) \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \right) \rightarrow (29) \end{aligned}$$

The total profit per unit time $TPU(S, Q) = \frac{TO(S, Q)}{T} - S$

$$\begin{aligned} TPU &= \frac{1}{T} \left((V_i - P_i)S - C_3 + \frac{hD}{\theta} T_0 + h \left(S_0 + \frac{D}{\theta} \right) \left(\frac{e^{-\theta T_0}}{\theta} + \frac{1}{\theta} \right) + \frac{ha}{\alpha + \theta} \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right. \\ &\quad \left. + h \left(S_0 + \frac{a}{\alpha + \theta} e^{\alpha T_0} \right) \left(\frac{e^{-\theta(T_1-T_0)}}{\theta} - \frac{1}{\theta} \right) + C_2 \frac{(Q-S)^2}{2D} + (U - V_i) \left(S_0 - \left(DT_0 + a \left(\frac{e^{\alpha T}}{\alpha} - \frac{e^{\alpha T_0}}{\alpha} \right) \right) \right) \right) \rightarrow (30) \end{aligned}$$

Here the profit function (30) is to be maximized

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

The set up cost (C_3)-8350 / order

The demand (D)- 400/ month

The shortage cost (C_2) 0.5/ unit

The holiday cost per unit (h) 12

The selling price of deteriorated items (u) = 15

Quantity	Purchase cost	Selling Price
$0 < S \leq 500$	40	50
$500 < S \leq 1000$	30	38
$1000 < S \leq 2000$	28	35
$2000 < S$	20	26

The Problem is solved by genetic algorithm and is given in appendix. The solutions for different parametric values of θ , a, b, c and so are given in Table 1 to 4

Table – 1: ($0 < S \leq 500$)

Parameters	Changing Perimeters	Change Perimeters	S	Q	TPU
$a=3$ $S_0 = 80$ $\alpha = 0.1$	θ	0.1	485.35	585.24	4895.26
		0.3	520.06	620.26	5028.16
		0.5	535.26	632.14	5320.26
		0.7	575.35	640.28	5428.24
$\theta = 0.1$ $S_0 = 90$ $\alpha = 0.1$	a	3	485.35	540.16	4895.26
		10	535.26	615.18	5215.28
		14	572.14	635.26	5425.12
		16	583.28	645.24	5616.26
$\theta = 0.1$ $a = 3$ $\alpha = 0.1$	S_0	120	485.35	575.28	4985.26
		100	565.26	590.16	5118.28
		130	585.24	620.26	5320.26
		140	600.18	640.38	5425.18
$\theta = 0.1$ $a = 3$ $S_0 = 90$	α	0.1	485.35	600.23	4895.26
		0.2	593.26	625.16	5256.12
		0.3	620.16	635.24	5358.23
		0.4	628.24	645.26	5472.28

Table – 2: ($500 < S \leq 1000$)

Parameters	Changing Perimeters	Change Perimeters	S	Q	TPU
$a = 3$ $S_0 = 100$ $\alpha = 0.1$	θ	0.1	965.24	1026.16	5986.26
		0.3	975.28	1036.38	6216.20
		0.5	985.16	1046.28	6326.54
		0.7	995.28	1056.18	6518.21
$\theta = 0.1$ $S_0 = 100$ $\alpha = 0.1$	a	3	965.24	1026.16	5986.26
		10	976.28	1056.36	6826.18
		14	986.14	1086.28	6854.02
		16	992.34	1096.24	6924.18
$\theta = 0.1$ $a = 100$ $\alpha = 0.1$	S_0	100	965.24	1026.16	5986.26
		120	972.16	1058.26	6820.16
		130	986.34	1068.14	6852.18
		140	998.28	1078.28	6926.26
$\theta = 0.1$ $a = 3$ $S_0 = 100$	α	0.1	965.24	1026.16	5986.26
		0.2	976.26	1052.28	6820.16
		0.3	986.28	1070.28	6862.28
		0.4	996.16	1089.26	6927.26

Table-3: ($1000 < s \leq 2000$)

Parameters	Changing Perimeters	Change Perimeters	S	Q	TPU
$a = 3$ $S_0 = 100$ $\alpha = 0.1$	θ	0.1	2020.25	3042.92	9325.14
		0.3	2620.18	3056.16	9526.18
		0.5	2635.16	3072.16	9828.16
		0.7	2652.08	3092.34	9878.28
$\theta = 0.1$ $S_0 = 100$ $\alpha = 0.1$	a	3	2020.28	3042.92	9325.14
		10	2720.18	3066.26	9621.28
		14	2736.24	3082.16	9921.14
		16	2758.16	4010.21	9981.26
$\theta = 0.1$ $a = 3$ $\alpha = 0.1$	S_0	100	2020.28	3042.92	9324.14
		120	2820.16	3166.26	9721.24
		130	2926.28	3182.28	9936.16
		140	2938.16	4016.26	9986.24
$\theta = 0.1$ $a = 3$ $S_0 = 100$	α	0.1	2020.28	3042.92	9324.14
		0.2	2826.24	3168.26	9721.24
		0.3	2920.18	3192.14	9936.16
		0.4	2932.12	4018.24	9994.24

Table – 4: (2000 <S)

Parameters	Changing Perimeters	Change Perimeters	S	Q	TPU
$a = 3$ $S_0 = 100$ $\alpha = 0.1$	θ	0.1	3025.26	3525.16	6325.18
		0.3	3224.14	3572.18	6628.26
		0.5	3524.16	3612.78	6721.48
		0.7	3618.14	3712.18	6832.32
$\theta = 0.1$ $S_0 = 100$ $\alpha = 0.1$	a.	3	3025.26	3562.16	6325.18
		10	3234.48	3571.16	6428.16
		14	3572.26	3628.16	6328.16
		16	3621.24	3724.08	6722.24
$\theta = 0.1$ a.=3 $\alpha = 0.1$	S_0	100	3025.26	3562.16	6325.18
		120	3526.18	3591.28	6438.26
		130	3618.26	3638.26	6348.26
		140	3724.18	3728.16	6826.18
$\theta = 0.1$ $a = 3$ $S_0 = 100$	α	0.1	3025.26	3562.16	6325.18
		0.2	3516.28	3596.26	6423.28
		0.3	3619.28	3678.18	6358.26
		0.4	3718.02	3798.26	6836.48

Sensitivity analysis of the above model reveals the following observations

- ❖ When the Parameter ‘ θ ’ increases, the corresponding order level and optimal profit will decrease.
In case of change in Parameter ‘a’ i.e., the demand change increases the optimal order quantity as well as optimal profit increases.
- ❖ When initial value of order level increases the corresponding EOQ and Profit will increase.
- ❖ From the above tables one can be observe that the total profit per minute time is maximum in range $1000 < S < 2000$ i.e., Rs.9994.24.

CONCLUSION

In this paper an attempt is made with two component demand i.e., initially the demand is constant and after certain period the demand is increasing exponentially. In real word, it is a fact that when a new product is introduced then its demand increases with constant rate initially, as the customers come to know about the product, when product is popularized when its demand rate increases exponentially. This type of situation can be seen in automobile products and electronic goods.

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APPENDIX

(D) Genetic Algorithm

Genetic Algorithm is a class of adaptive search technique based on the principle of population genetics. The algorithm is an example of a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of parameter space. Genetic Algorithm work according to the principles of natural genetics on a population of string structure representing the problem variable. All these features make genetic algorithm search robust allowing them to be applied to a wide variety of problems.

Implementing GA

The following are adopted in the proposed GA to solve the problem:

- (1) Parameters
- (2) Chromosome representation
- (3) Initial population production
- (4) Evaluation
- (5) Selection
- (6) Crossover
- (7) Mutation
- (8) Termination

Parameters

Firstly, we set the different parameters on which this GA depends. All these are the number of generation (MAXGEN), population size(POPSIZE), probability of crossover(PCROS), probability of mutation (PMUTE)

Chromosome Representation An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chrosones are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome V_i is a string of n numbers of genes G ($j = 1, 2, \dots, n$) where these n numbers of genes respectively denote n number of decision variables ($X_i, i=1, 2, \dots, n$).

Initial Population Production

The population generation technique proposed in the present GA is illustrated by the following procedure. For each chromosome V_i , every gene G_{ij} is randomly generated between its boundary (LB_j, UB_j) where LB_j , and UB_j are the lower and upper bounds of the variables $X_i, i=1, 2, \dots, n$, POPSIZE.

Evaluation

Evaluation function plays the some role in GA as that which the environment plays in natural evaluation. Now, evaluation function (EVAL) for the chromosome V_1 is equivalent to the objective function $PF(X)$. These are following steps of evaluation.

Step-1: find EVAL (V_1) by $EVAL(V_1) = f(X_1, X_2, \dots, X_n)$

Where the genes G represent the decision variable $X_1, j= 1, 2, \dots, n$, POPSIZE and f is the objective function.

Step-2: find total fitness of the population: $F = \sum_{i=1}^{popsize} EVAL(V_i)$

Step-3: calculate the probability p_i of selection for each chromosome V_1 as

$$Y_i = \sum_{i=1}^1 p_1$$

Selection

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as Stochastic random sampling, Roulette wheel selection have been proposed for various problems. In this paper we adopt roulette wheel selection process.

This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way:

- (a) Generate a random (float) number r between 0 to 1.
- (b) If $r < Y$, then the first chromosome is V_i otherwise select the i^{th} chromosome V_i ($2 \leq i \leq \text{POPSIZE}$) such that

$$T_{i-1} \leq r \leq Y_i$$

Crossover

Crossover operator is mainly responsible for the search of new string. The exploration of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection chromosomes for new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and the resulting offspring V_m^1 and V_n^1 calculated as:

$$V_m^1 = c.V_m + (1 - c).V_n$$

$$V_n^1 = c.V_n + (1 - c).V_m$$

Where c is a random number between 0 and 1.

Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome V_i the selection of a chromosome for mutation is performed in the following way:

Step-1: Set $i \leftarrow 1$

Step-2: Generate a random number u from the range $[0,1]$

Step-3: If $u < \text{PMUTE}$, then go to step 2.

Step-4: Set $i \leftarrow i+1$

Step-5: If $i \leq \text{POPSIZE}$, then go to Step 2.

Then the particular gene G_{ij} of the chromosome V_1 selected by the above mentioned steps is randomly selected in this problem, the mutation is defined as

G_y^{mut} random number from the range $(0, 1)$

Termination

If the number of iteration is less than or equal to MAXGEN then the process is going on, otherwise it terminates.

The GA's procedure is given below:

```

Begin
do {
    t ← 0
    while (all constraints are not satisfied)
    {
        Initialize Population (t)
    }
    Evaluate Population (t)
    while (not terminate)
    {
        t ← t + 1
        select Population(t) from Population (t-1)
        crossover and mutate Population(t)
        evaluate Population(t)
    }
    Print Optimum Result
}
End.
```

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