A STUDY ON $(T, S)$-INTUITIONISTIC FUZZY SUBNEARRINGS OF A NEARRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of a $(T, S)$-intuitionistic fuzzy subnearring of a nearring.

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Key Words: T-fuzzy subnearring, anti S-fuzzy subnearring, $(T, S)$-intuitionistic fuzzy subnearring, product.

INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[4, 5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined the fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [13, 14]. In this paper, we introduce the some Theorems in $(T, S)$-intuitionistic fuzzy subnearring of a nearring.

1.PRELIMINARIES:

1.1 Definition: A $(T, S)$-norm is a binary operations $T: [0, 1] \times [0, 1] \rightarrow [0, 1]$ and $S: [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) $T(0, x) = 0$, $T(1, x) = x$ (boundary condition)
(ii) $T(x, y) = T(y, x)$ (commutativity)
(iii) $T(x, T(y, z)) = T(T(x, y), z)$ (associativity)
(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$ (monotonicity).
(v) $S(0, x) = x$, $S(1, x) = 1$ (boundary condition)
(vi) $S(x, y) = S(y, x)$ (commutativity)
(vii) $S(x, S(y, z)) = S(S(x, y), z)$ (associativity)
(viii) if $x \leq y$ and $w \leq z$, then $S(x, w) \leq S(y, z)$ (monotonicity).

1.2 Definition: Let $(R, +, .)$ be a near ring. A fuzzy subset $A$ of $R$ is said to be a T-fuzzy sub nearring (fuzzy subnearring with respect to $T$-norm) of $R$ if it satisfies the following conditions:

(i) $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))$
(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$ for all $x$ and $y$ in $R$.

1.3 Definition: Let $(R, +, .)$ be a nearring. A fuzzy subset $A$ of $R$ is said to be an $(T, S)$-intuitionistic fuzzy subnearring if it satisfies the following conditions:

(i) $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))$
(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$
(iii) $\nu_A(x - y) \leq S(\nu_A(x), \nu_A(y))$
(iv) $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$ for all $x$ and $y$ in $R$.

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1.4 Definition: Let $A$ and $B$ be intuitionistic fuzzy subsets of sets $G$ and $H$, respectively. The product of $A$ and $B$, denoted as $A \times B$, is defined as $A \times B = \{(x, y), \mu_A(x, y), \nu_A(x, y)\}$ for all $x \in G$ and $y \in H$, where $\mu_A(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\nu_A(x, y) = \max\{\nu_A(x), \nu_B(y)\}$.

1.5 Definition: Let $A$ be an intuitionistic fuzzy subset in a set $S$, the strongest intuitionistic fuzzy relation on $S$, that is an intuitionistic fuzzy relation on $A$ is $V$ given by $\mu_V(x, y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_V(x, y) = \max\{\nu_A(x), \nu_A(y)\}$ for all $x$ and $y$ in $S$.

1.6 Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two nearrings. Let $f : R \rightarrow R^1$ be any function and $A$ be an $(T, S)$-intuitionistic fuzzy subnearring in $R$, $V$ be an $(T, S)$-intuitionistic fuzzy subnearring in $R$, $f$ defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$ for all $x$ in $R$ and $y$ in $R^1$. Then $A$ is called a preimage of $V$ under $f$ and is denoted by $f^{-1}(V)$.

1.7 Definition: Let $A$ be an $(T, S)$-intuitionistic fuzzy subnearring of a nearring $(R, +, \cdot)$ and $a$ in $R$. Then the pseudo $(T, S)$-intuitionistic fuzzy coset $(aA)^p$ is defined by $((\nu_{aA})^p)(x) = p(a)\mu_A(x)$ and $((\nu_{aA})^p)(x) = p(a)\nu_A(x)$ for every $x$ in $R$ and for some $p$ in $P$.

2- PROPERTIES

2.1 Theorem: Intersection of any two $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ is a $(T, S)$-intuitionistic fuzzy subnearring of a nearring $R$.

Proof: Let $A$ and $B$ be any two $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ and $x$ and $y$ in $R$. Let $A = \{(x, \mu_A(x), \nu_A(x)) / x \in R\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) / x \in R\}$ and also let $C = A \cap B = \{(x, \mu_C(x), \nu_C(x)) / x \in R\}$ where $\mu_C(x) = \mu_A(x) \wedge \mu_B(x)$ and $\nu_C(x) = \nu_A(x) \vee \nu_B(x)$. Now $\mu_C(x) = \mu_A(x) \wedge \mu_B(x)$ and $\nu_C(x) = \nu_A(x) \vee \nu_B(x)$.

2.2 Theorem: The intersection of a family of $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ is an $(T, S)$-intuitionistic fuzzy subnearring of a nearring $R$.

Proof: It is trivial.

2.3 Theorem: If $A$ and $B$ are any two $(T, S)$-intuitionistic fuzzy subnearrings of the nearrings $R_1$ and $R_2$, respectively, then $A \times B$ is an $(T, S)$-intuitionistic fuzzy subnearring of a nearring $R_1 \times R_2$.

Proof: Let $A$ and $B$ be two $(T, S)$-intuitionistic fuzzy subnearrings of the nearrings $R_1$ and $R_2$, respectively. Let $x_1$ and $x_2$ be in $R_1$, $y_1$ and $y_2$ be in $R_2$. Then $(x_1, y_1)$ and $(x_2, y_2)$ are in $R_1 \times R_2$. Now $\mu_{A \times B}((x_1, y_1) - (x_2, y_2)) = \mu_{A \times B}(x_1 - x_2, y_1 - y_2) = \min\{\mu_A(x_1 - x_2), \mu_B(y_1 - y_2)\}$ and $\nu_{A \times B}(x_1 - x_2, y_1 - y_2) = \max\{\nu_A(x_1 - x_2), \nu_B(y_1 - y_2)\}$.
2.4 Theorem: If A is a (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +), then 
\( \mu_A(x) \leq \mu_A(0) \) and 
\( \nu_A(x) \geq \nu_A(0) \) for x in R, the identity element 0 in R.

Proof: For x in R and 0 is the identity element of R. Now \( \mu_A(0) = \mu_A(x-x) \geq T(\mu_A(x), \mu_A(x)) \geq \mu_A(x) \) for all x in R. So 
\( \mu_A(x) \leq \mu_A(0) \). And 
\( \nu_A(0) = \nu_A(x-x) \leq S(\nu_A(x), \nu_A(x)) \leq \nu_A(x) \) for all x in R. So 
\( \nu_A(x) \geq \nu_A(0) \).

2.5 Theorem: Let A and B be (T, S)-intuitionistic fuzzy subnearring of the nearrings R1 and R2 respectively. Suppose that 0 and 0 are the identity element of R1 and R2 respectively. If A\times B is an (T, S)-intuitionistic fuzzy subnearring of R1\times R2, then at least one of the following two statements must hold. (i) \( \mu_A(0) \geq \mu_A(x) \) and \( \nu_A(0) \leq \nu_A(x) \) for all x in R1. 
(ii) \( \mu_A(0) \geq \mu_B(y) \) and \( \nu_A(0) \leq \nu_B(y) \) for all y in R2.

Proof: Let A\times B be an (T, S)-intuitionistic fuzzy subnearring of R1\times R2. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R1 and b in R2 such that \( \mu_A(a) > \mu_A(0) \), \( \nu_A(a) < \nu_A(0) \) and 
\( \mu_B(b) > \mu_B(0) \), \( \nu_B(b) < \nu_B(0) \). We have \( \mu_A(a, b) = \min\{\mu_A(a), \mu_A(b)\} > \min\{\mu_A(0), \mu_A(0)\} = \mu_A(0, 0) \) and 
\( \nu_A(a, b) = \max\{\nu_A(a), \nu_A(b)\} < \max\{\nu_A(0), \nu_A(0)\} = \nu_A(0, 0) \). Thus A\times B is not an (T, S)-intuitionistic fuzzy subnearring of R1\times R2. Hence either \( \mu_A(0) \geq \mu_A(x) \) and \( \nu_A(0) \leq \nu_A(x) \) for all x in R1 or 
\( \mu_A(0) \geq \mu_B(y) \) and \( \nu_A(0) \leq \nu_B(y) \) for all y in R2.

2.6 Theorem: Let A and B be two intuitionistic fuzzy subsets of the nearrings R1 and R2 respectively and A\times B is an (T, S)-intuitionistic fuzzy subnearring of R1\times R2. Then the following are true:
(i) if \( \mu_A(x) \leq \mu_B(0) \) and \( \nu_A(x) \geq \nu_B(0) \), then A is an (T, S)-intuitionistic fuzzy subnearring of R1.
(ii) if \( \mu_A(x) \leq \mu_B(0) \) and \( \nu_A(x) \geq \nu_B(0) \), then B is an (T, S)-intuitionistic fuzzy subnearring of R2.
(iii) either A is an (T, S)-intuitionistic fuzzy subnearring of R1 or B is an (T, S)-intuitionistic fuzzy subnearring of R2.

Proof: Let A\times B be an (T, S)-intuitionistic fuzzy subnearring of R1\times R2 and x and y in R1 and 0 in R2. Then \( x(0) \) and 
\( y(0) \) are in R1\times R2. Now using the property that \( \mu_A(x) \leq \mu_B(0) \) and \( \nu_A(x) \geq \nu_B(0) \) for all x in R1. We get 
\( \mu_A(x) = \min\{\mu_A(x), \mu_B(0)\} = \mu_B(0) \) and 
\( \nu_A(x) = \max\{\nu_A(x), \nu_B(0)\} = \nu_B(0) \) for all x in R1. So 
\( \mu_A(x-y) \geq \mu_A(0-x) \geq \mu_A(0) \) and 
\( \nu_A(x-y) \leq \nu_A(0-x) \leq \nu_A(0) \). Therefore 
\( \mu_A(x) \times \mu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \nu_B(0) \) for all x and y in R1. And 
\( \nu_A(x-y) \geq \nu_A(0-x) \geq \nu_A(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1. Therefore 
\( \mu_A(x) \times \nu_B(y) \geq \mu_A(x) \times \nu_B(0) \) and 
\( \nu_A(x) \times \mu_B(y) \leq \nu_A(x) \times \mu_B(0) \) for all x and y in R1.
max \{v_A(x_1y_1), v_A(x_2y_2)\} ≤ max(S(v_A(x_1), v_A(y_1)), S(v_A(x_2), v_A(y_2))) ≤ S(max(v_A(x_1), v_A(x_2)), max(v_A(y_1), v_A(y_2))) = S(v_A(x_1y_1), v_A(x_2y_2)) = S(v_A(x_1), v_A(y_1)) = S(v_A(x_2), v_A(y_2)). Therefore, v_A(xy) ≤ S(v_A(x), v_A(y)), for all x and y in \(R\times R\). This proves that V is an (T, S)-intuitionistic fuzzy subnearring of R. Conversely assume that V is an (T, S)-intuitionistic fuzzy subnearring of R×R, then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R, we have min\{\mu(x_1–y_1), \mu(x_2–y_2)\} = \mu(x_1–y_1, x_2–y_2) = \mu(x_1, x_2)−(y_1, y_2) = \mu(x−y) \geq T(\mu(x), \mu(y)) = T(\mu(x_1, x_2), \mu(y_1, y_2)) = T(\min(\mu(x_1, x_2)), \min(\mu(y_1, y_2))). If x_2 = 0, y_2 = 0, we get, \(\mu(x_1–y_1) \geq T(\mu(x_1), \mu(y_1))\), for all x_1 and y_1 in R. And min\{\mu(x_1, x_2), \mu(x_1y_1)\} = \mu(x_1y_1, x_2y_2) = \mu(x_1y_1, x_2y_2) = \mu(x−y) \geq T(\mu(x), \mu(y)) = T(\mu(x_1, x_2), \mu(y_1, y_2)) = T(\min(\mu(x_1, x_2)), \min(\mu(y_1, y_2))). If x_2 = 0, y_2 = 0, we get \(\mu(x_1y_1) \geq T(\mu(x_1), \mu(y_1))\), for all x_1 and y_1 in R. We have max \{v_A(x_1y_1), v_A(x_2y_2)\} = v_A(x_1y_1, x_2y_2) = v_A(x_1, x_2)−(y_1, y_2) = v_A(x−y) \leq S(v_A(x), v_A(y)) = S(v_A(x_1), v_A(x_2)) \geq S(v_A(x), v_A(y_1)) = S(v_A(x_1), v_A(y_2)) = S(max(v_A(x_1), v_A(x_2)), max(v_A(y_1), v_A(y_2))). If x_2 = 0, y_2 = 0, we get v_A(x_1y_1) \leq S(v_A(x_1), v_A(y_1)) for all x_1 and y_1 in R. Therefore V is an (T, S)-intuitionistic fuzzy subnearring of R.

2.8 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring \(R, +, \cdot\), then H = \{x / x ∈ R: \mu_A(x) = 1, v_A(x) = 0\} is either empty or is a subnearring of R.

Proof: It is trivial.

2.9 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring \(R, +, \cdot\), then

(i) if \(\mu_A(x−y) = 0\), then either \(\mu_A(x) = 0\) or \(\mu_A(y) = 0\) for all x and y in R.

(ii) if \(\mu_A(xy) = 0\), then either \(\mu_A(x) = 0\) or \(\mu_A(y) = 0\) for all x and y in R.

(iii) if \(v_A(x−y) = 1\), then either \(v_A(x) = 1\) or \(v_A(y) = 1\) for all x and y in R.

(iv) if \(v_A(xy) = 1\), then either \(v_A(x) = 1\) or \(v_A(y) = 1\) for all x and y in R.

Proof: It is trivial.

2.10 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring \(R, +, \cdot\), then \(\Box A\) is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Consider \(\Box A = \{x / \mu_A(x), v_A(x)\}\), for all x in R, we take \(\Box A = B = \{x / \mu_B(x), v_B(x)\}\), where \(\mu_B(x) = \mu_A(x), v_B(x) = 1–\mu_A(x)\). Clearly \(\mu_A(x−y) \geq T(\mu_B(x), \mu_B(y))\) for all x and y in R and \(\mu_B(xy) \geq T(\mu_A(xy), \mu_A(y))\) for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have \(\mu_A(x−y) \geq T(\mu_A(x), \mu_A(y))\) for all x and y in R, which implies that \(1–\mu_A(x−y) \geq T(1–\mu_A(x), 1–\mu_A(y))\), which implies that \(v_A(x−y) \leq 1–T(1–\mu_A(x), 1–\mu_A(y))\), which implies that \(v_A(x−y) \leq S(v_A(x), v_A(y))\). Therefore \(v_A(xy) \leq S(v_A(x), v_A(y))\), for all x and y in R. And \(\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))\) for all x and y in R, which implies that \(1–\mu_A(xy) \leq T(1–\mu_A(x), 1–\mu_A(y))\), which implies that \(v_A(xy) \leq 1–T(1–\mu_A(x), 1–\mu_A(y)) \leq S(v_A(x), v_A(y))\). Therefore \(v_A(xy) \leq S(v_A(x), v_A(y))\) for all x and y in R. Hence \(\Box A\) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.11 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring \(R, +, \cdot\), then \(\Delta A\) is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. That is \(\Delta A = \{x / \mu_A(x), v_A(x)\}\) for all x in R. Let \(\Delta A = B = \{x / \mu_B(x), v_B(x)\}\), where \(\mu_B(x) = 1–v_A(x), v_B(x) = 1–v_A(x)\). Clearly \(v_A(x–y) \leq S(v_B(x), v_B(y))\) for all x and y in R and \(v_B(xy) \leq S(v_A(x), v_A(y))\) for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have \(v_A(x–y) \leq S(v_A(x), v_A(y))\) for all x and y in R, which implies that \(1–\mu_A(x–y) \leq S(1–\mu_A(x), 1–\mu_A(y))\) which implies that \(\mu_A(x–y) \geq 1–S(1–\mu_A(x), 1–\mu_A(y)) \geq T(\mu_A(x), \mu_A(y))\). Therefore \(\mu_A(x–y) \geq T(\mu_A(x), \mu_A(y))\) for all x and y in R. And \(\mu_A(xy) \leq T(\mu_A(x), \mu_A(y))\) for all x and y in R, which implies that \(1–\mu_A(xy) \geq T(1–\mu_A(x), 1–\mu_A(y)) \leq S(v_A(x), v_A(y))\). Therefore \(v_A(xy) \leq S(v_A(x), v_A(y))\) for all x and y in R. Hence \(\Delta A\) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.12 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring \(R, +, \cdot\), then the pseudo (T, S)-intuitionistic fuzzy coset (aA)\(\Box\) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R, for every a in R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. For every x and y in R, we have \((a\mu_A)(x–y) = p(a)\mu_A(x–y) \geq p(a)T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(a\mu_A(x), (a\mu_A(y)))\). Therefore \((a\mu_A)(x–y) \geq T((a\mu_A)(x), (a\mu_A)(y))\). Now \((a\mu_A)(xy) = p(a)\mu_A(xy) \geq p(a)T(\mu_A(xy), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T((a\mu_A)(x), (a\mu_A)(y))\). Therefore \((a\mu_A)(xy) \geq T((a\mu_A)(x), (a\mu_A)(y))\). Therefore \((a\mu_A)(x–y) \leq S((a\mu_A)(x), (a\mu_A)(y))\). Therefore \((a\mu_A)(x–y) \leq S((a\mu_A)(x), (a\mu_A)(y))\).
Proof: Let x and y in R and A be an (T, S) -intuitionistic fuzzy subnearring of a nearring H. Then we have \( µ(A(x−y)) ≥ T(µ(A(x)), µ(A(y))) \) which implies that \( µ(A(x−y)) ≥ T(µ(A(x)), µ(A(y))) \). And \( µ(A(x+y)) = µ(A(f(x+y))) \) which implies that \( µ(A(x+y)) ≥ T(µ(A(x)), µ(A(y))) \). Therefore \(((a ◦ f)(x+y)) = S((a ◦ f)(x), (a ◦ f)(y))) \) which implies that \((a ◦ f)(x+y) ≤ S((a ◦ f)(x), (a ◦ f)(y)))\). Hence \((a ◦ f)) \) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.14 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an isomorphism from a nearring R onto H. Then \( A ◦ f \) is an (T, S)-intuitionistic fuzzy subnearring of nearring R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H. Then we have \( µ(A◦f(x−y)) = µ(A◦f((x−y))) \) which implies that \( µ(A◦f(x−y)) ≥ T(µ(f(x)), µ(f(y))) \). And \( µ(A◦f(xy)) = µ(A◦f(f(xy))) \) which implies that \( µ(A◦f(xy)) ≥ T(µ(f(x)), µ(f(y))) \). Therefore \( µ(A◦f(xy)) ≥ T(µ(f(x)), µ(f(y))) \). And \( µ(A◦f(x+y)) = µ(A◦f(f(x+y))) \) which implies that \( µ(A◦f(x+y)) ≥ T(µ(f(x)), µ(f(y))) \). Therefore \( µ(A◦f(x+y)) ≥ T(µ(f(x)), µ(f(y))) \). Hence \( A ◦ f \) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.15 Theorem: Let \( R, +, \cdot \) and \( R^1, +, \cdot \) be any two nearrings. The homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \).

Proof: Let \( R, +, \cdot \) and \( R^1, +, \cdot \) be any two nearrings. Let \( f : R → R^1 \) be a homomorphism. Let \( V = f(A) \) where \( A \) is an (T, S)-intuitionistic fuzzy subnearring of R. We have to prove that \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \). Now for \( f(x), f(y) \) in \( R^1 \), \( µ(A(x−y)) = µ(A((x−y))) \) which implies that \( µ(A(x−y)) ≥ T(µ(A(x)), µ(A(y))) \). Again \( µ(A(x+y)) = µ(A(f(x+y))) \) which implies that \( µ(A(x+y)) ≥ T(µ(A(x)), µ(A(y))) \). Therefore \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Again \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Hence \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \).

2.16 Theorem: Let \( R, +, \cdot \) and \( R^1, +, \cdot \) be any two nearrings. The homomorphic preimage of an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \) is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let \( V = f(A) \), where \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \). We have to prove that \( A \) is an (T, S)-intuitionistic fuzzy subnearring of R. Let \( x \) and \( y \) in R. Then \( µ(A(x−y)) = µ(A((x−y))) \) which implies that \( µ(A(x−y)) ≥ T(µ(A(x)), µ(A(y))) \). And \( µ(A(x+y)) = µ(A(f(x+y))) \) which implies that \( µ(A(x+y)) ≥ T(µ(A(x)), µ(A(y))) \). Therefore \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Again \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Hence \( A \) is an (T, S)-intuitionistic fuzzy subnearring of R.

2.17 Theorem: Let \( R, +, \cdot \) and \( R^1, +, \cdot \) be any two nearrings. The anti-homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \).

Proof: Let \( R, +, \cdot \) and \( R^1, +, \cdot \) be any two nearrings. Let \( f : R → R^1 \) be an anti-homomorphism. Then \( f(x+y) = f(y) + f(x) \) and \( f(xy) = f(y)f(x) \) for all x and y in R. Let \( V = f(A) \), where \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \). We have to prove that \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \). Now for \( f(x), f(y) \) in \( R^1 \), \( µ(A(−x)) = µ(A((x−y))) \) which implies that \( µ(A(−x)) ≥ T(µ(A(x)), µ(A(y))) \). And \( µ(A(x+y)) = µ(A(f(x+y))) \) which implies that \( µ(A(x+y)) ≥ T(µ(A(x)), µ(A(y))) \). Therefore \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Again \( ν(A(f(x)+f(y))) = ν(A((f(x)+f(y)))) \). Hence \( V \) is an (T, S)-intuitionistic fuzzy subnearring of \( R^1 \).
2.18 Theorem: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two nearrings. The anti-homomorphic preimage of an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\).

Proof: Let \(V = f(A)\), where \(V\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\). We have to prove that \(A\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\). Let \(x, y \in R\). Then \(\mu_A(x - y) = \mu_A(f(x - y)) = \mu_A(f(x) - f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) = T(\mu_A(f(x)), \mu_A(f(y))) = T(\mu_A(x), \mu_A(y))\) which implies that \(\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))\). Again \(\nu_A(x - y) = \nu_A(f(x) - f(y)) = \nu_A(f(x) - f(y)) \leq S(\nu_A(f(x)), \nu_A(f(y))) = S(\nu_A(x), \nu_A(y))\) which implies that \(\nu_A(x - y) \leq S(\nu_A(x), \nu_A(y))\). Hence \(A\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\).

REFERENCE


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