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PERISTALTIC TRANSPORT OF A CONDUCTING COUPLE STRESS FLUID THROUGH A POROUS MEDIUM IN A CHANNEL

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ABSTRACT

The present paper investigates the peristaltic motion of a couple stress fluid through a porous medium in a two dimensional channel with the effect of magnetic field. The effects of various physical parameters on velocity, pressure gradient and friction force have been discussed & computed numerically. The effects of various key parameters are discussed with the help of graphs.

Keywords: *Peristaltic motion, Couple stress fluid, Porous medium, Magnetic field.*

1. INTRODUCTION

Peristalsis is known to be one of the main mechanisms of transport for many physiological fluids, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. This mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferent’s of the male reproductive organ, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Some worms like earth-worm use peristalsis for their locomotion. Some biomedical instruments such as heart-lung machine work on this principle. Mechanical devices like finger pumps, roller pumps use peristalsis to pump blood, slurries and corrosive fluids. Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. Alsaedi, Ali *et al.*, [1] studied peristaltic flow of couple stress fluid through uniform porous medium. Ayman and Sobh [2] investigated peristaltic transport of a magneto-newtonian fluid through a porous medium. Jayarami Reddy *et.al.*, [3] studied peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Gupta and Sheshadri [4] studied peristaltic transport of a Newtonian fluid in non-uniform geometries. El-dabe and El-Mohandis [5] have studied magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane. Habtu and Radhakrishnamacharya [6] studied dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition. Mekhemier [7] studied non-linear peristaltic transport a porous medium in an inclined planar channel. Mekhemier and Abd elmaboud [8] peristaltic flow of a couple stress fluid in an annulus: Application of an endoscope. Srivastava and Srivastava [9] have investigated the effect of power law fluid in uniform and non-uniform tube and channel under zero Reynolds number and long wavelength approximation. Srivastava *et.al.*, [10] peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Ramchandra and Usha [11] studied the influence of an eccentrically inserted catheter on the peristaltic pumping in a tube under long wavelength and low Reynolds numbers approximations. Raptis and Peridikis [12] are investigated flow of a viscous fluid through a porous medium bounded by a vertical surface. Rathod and Sridhar [13] studied peristaltic flow of a couple stress fluid through a porous medium in an inclined channel. Rathod and Sridhar [14] studied effects of Couple Stress fluid and an endoscope on peristaltic transport through a porous medium. Subba Reddy *et al.*, [15] studied peristaltic transport of Williamson fluid in a channel under the effect of a magnetic field. Raghunath Rao and Prasad Rao [16] studied peristaltic flow of a couple stress fluid through a porous medium in a channel at low Reynolds number. Latham [17] investigated the fluid mechanics of peristaltic pump and science. Rathod and Sridhar [18] investigated peristaltic

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transport of couple stress fluid in uniform and non-uniform annulus through porous medium. Rathod *et al*., [19] have investigated peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material. Abd elmaboud and Mekheimer [20] study non-linear peristaltic transport of a second-order fluid through a porous medium.

The present research aimed is to investigate the interaction of peristalsis for the motion of a couple stress fluid through a porous medium in a two dimensional channel with the effect of magnetic field. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

2. FORMULATION OF THE PROBLEM

Consider the peristaltic flow of a couple stress fluid through a two-dimensional channel of non-uniform thickness with a sinusoidal wave travelling down its wall. We choose a rectangular coordinate system for the channel with x^* along the centerline in the direction of wave propagation and y^* transverse to it, see in Fig. 1. The geometry of the wall surface is defined as

$$H(x^*,t^*) = a + bCos\left(\frac{2\pi}{\lambda}(x^* - ct^*)\right) \tag{1}$$

Where a is the half-width of the channel at any axial distance from inlet, b is the wave amplitude, λ is the wavelength, c is the propagation velocity and t is time.

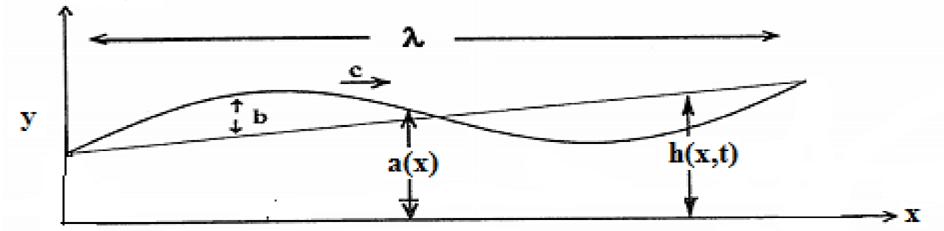

Fig.1: Peristaltic transport in a non-uniform channel.

In the absence of the body force and body couples, the equations of motion in the laboratory frame are:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{2}$$

Navier Stokes equations are:

$$\rho\left\{\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right\} = -\frac{\partial p^*}{\partial x^*} + \mu\nabla^2(u^*) - \eta\nabla^4(u^*) - \frac{\mu}{k_1}(u^*) - \sigma B_\circ^2(u^*) \tag{3}$$

$$\rho\left\{\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right\} = -\frac{\partial p^*}{\partial y^*} + \mu\nabla^2(v^*) - \eta\nabla^4(v^*) - \frac{\mu}{k_1}(v^*) - \sigma B_\circ^2(v^*) \tag{4}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial y^{*2}}$

Where u^* and v^* are velocity components in the corresponding coordinates. ρ is density, μ is viscosity, η is couple stress parameter, σ is electric conductivity, k_1 is the permeability of the porous medium and $B_\circ$ is applied magnetic field.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (x^*, y^*) by the transformation

$$x = x^* - ct^*, y = y^*, u = u - c, v = v^*, p = p^*(x^*, t^*) \tag{5}$$

Using the following non-dimensional variables

$$x = \frac{x^*}{\lambda},\ \ y = \frac{y^*}{a},\ \ u = \frac{u^*}{c},\ v = \frac{\lambda v^*}{a\ c}\ \ p = \frac{a^2}{\lambda\mu c}p^*(x^*),\ \ t = \frac{t^* c}{\lambda},\ \ \mathrm{Re} = \frac{\rho c a}{\mu},\ \ \delta = \frac{a}{\lambda},$$
$$M = B_\circ\sqrt{\frac{\sigma}{\mu a^2}},\ \ h = \frac{H}{a}, where\ \ \ \phi(amplitude) = \frac{b}{a} \le 1 \tag{6}$$

Substituting equations (5) & (6) in equations (1) to (4), these equations reduces to (after dropping asterisks)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\mathrm{Re}\,\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-$$
$$\frac{1}{\gamma^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{K}u-M^2u \tag{8}$$

$$\mathrm{Re}\,\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$-\frac{1}{\gamma^2}\delta^2\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{K}\delta^2 v-M^2\delta^2 v \tag{9}$$

Where, $\gamma^2=\dfrac{\eta}{\mu a^2}$ couple-stress parameter, $K=\dfrac{k_1}{a^2}$ porous parameter &

$M^2=B_\circ^2\dfrac{\sigma}{\mu a^2}$ Hartmann number

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y}=0;\quad \frac{\partial^2 u}{\partial y^2}=0\quad ;v=0\quad at\quad y=0$$
$$u=-1;\frac{\partial^2 u}{\partial y^2}\quad finite\quad at\quad y=\pm h=1+\phi Cos[2\pi x] \tag{10}$$

Using long wavelength approximation and neglecting the wave number δ, one can reduce governing equations:

$$\frac{\partial p}{\partial y}=0 \tag{11}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{K}u-M^2u \tag{12}$$

Solving the Eq.(12) with the boundary conditions (10), we get

$$u=\frac{\partial p}{\partial x}\left[\left(1+\frac{h^2}{\gamma^2}\right)\frac{y^2}{2}-\frac{y^2}{2}-\frac{1}{\gamma^2}\left(\frac{y^4}{4}+\frac{h^4}{4}\right)-\frac{1}{(K^{-1}+M^2)}\right]+\frac{1}{\gamma^2}\left(1-\frac{h^2}{2}+\frac{y^2}{2}\right)-1 \tag{13}$$

The volumetric flow rate in the wave frame is defined by

$$q=\int_0^h u\,dy=\frac{\partial p}{\partial x}\left[(1+\frac{h^2}{\gamma^2})\frac{h^3}{6}-\frac{h^3}{2}+\frac{h^5}{5.\gamma^2}-\frac{h}{(K^{-1}+M^2)}\right]+\left(\frac{1}{\gamma^2}\left(h-\frac{h^3}{3}\right)-h\right) \tag{14}$$

The expression for pressure gradient from Eq.(14) is given by

$$\frac{\partial p}{\partial x}=\frac{q-\left[\frac{1}{\gamma^2}\left(h-\frac{h^3}{3}\right)-h\right]}{\left(1+\frac{h^2}{\gamma^2}\right)\frac{h^3}{6}-\frac{h^3}{2}+\frac{h^5}{5.\gamma^2}-\frac{h}{(K^{-1}+M^2)}} \tag{15}$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q(x,t)=\int_0^h (u+1)dy = q+h \tag{16}$$

The average flux over one period of peristaltic wane is $\overline{Q}$

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt = q+1 \tag{17}$$

From equations (15) and (17), the pressure gradient is obtained as

$$\frac{\partial p}{\partial x}=\frac{(\overline{Q}-1)-\left[\frac{1}{\gamma^2}\left(h-\frac{h^3}{3}\right)-h\right]}{\left(1+\frac{h^2}{\gamma^2}\right)\frac{h^3}{6}-\frac{h^3}{2}+\frac{h^5}{5.\gamma^2}-\frac{h}{(K^{-1}+M^2)}} \tag{18}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

$$\Delta P = \int_0^1 \left(\frac{dp}{dx}\right)dx \tag{19}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F=\int_0^1 h^2\left(-\frac{dp}{dx}\right)dx \tag{20}$$

3. RESULT AND DISCUSSIONS

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP, friction force F for the different values of couple stress (γ), porous medium (K), amplitude (ϕ) and magnetic (M). The axial velocity is shown in Figs. (2 to 5). The Variation of u with γ, we find that u depreciates with increase in γ (Fig. 2). The Variation of u with Porous Parameter K shows that for u increases with increasing in K (Fig 3). The axial velocity u is exhibit in (fig. 4) for a different values of amplitude ϕ in the region y = 0 to y = 1. It is found that the velocity u is increases with increasing ϕ. The Variation of u with M, we find that u depreciates with increase in M (Fig. 5). The variation of pressure rise ΔP against the average volume flux $\overline{Q}$ is shown in Figs. (6 to 9) for a different values of γ, K, ϕ &M. From (Fig.6), we find that ΔP increases with increasing in γ. From (Fig. 7), we find that ΔP decreases with increasing in K. In (Fig. 8) we observe that increasing in amplitude ϕ, decrease the ΔP. From (Fig.9), we find that ΔP increases with increasing in M. The friction force F is shown in Figs. (10 to 13) for a different values of γ, K, ϕ &M. From (Fig. 10) the variation of F with γ shows that, F decreases with increase in γ. From (Fig. 11), we find that the permeable porous medium K large the F increases. From (Fig. 12), we find an increase in amplitude ϕ, increases the frictional force F in the flow region. From (Fig. 13) the variation of F with M shows that, F decreases with increase in M.

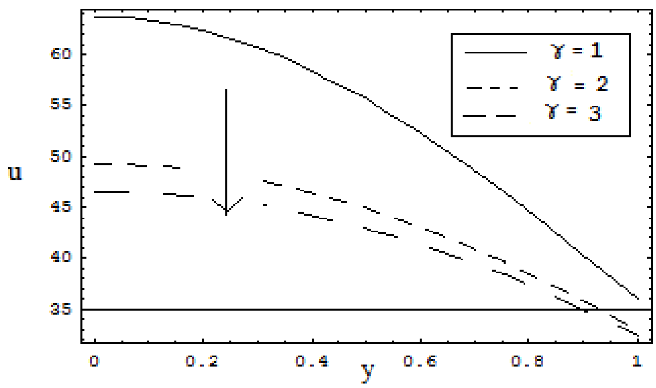

Fig. 2: Effect of γ on u ,when K = 1,M = 0.25, ϕ = 0.4,x = 0.1 & p = -25.

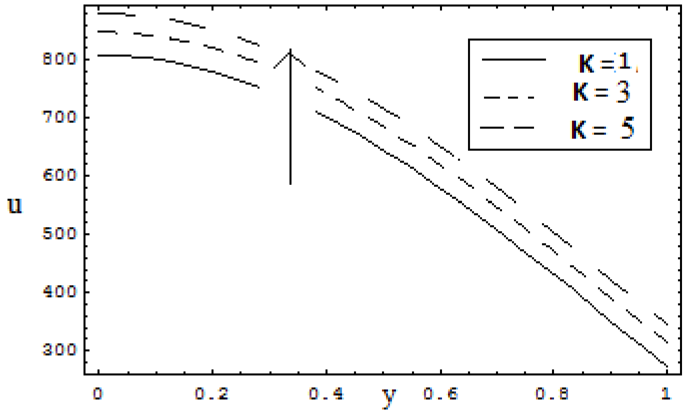

Fig. 3: Effect of K on u ,when γ =0.2, M = 0.25, ϕ = 0.6,x = 0.1 & p = -25.

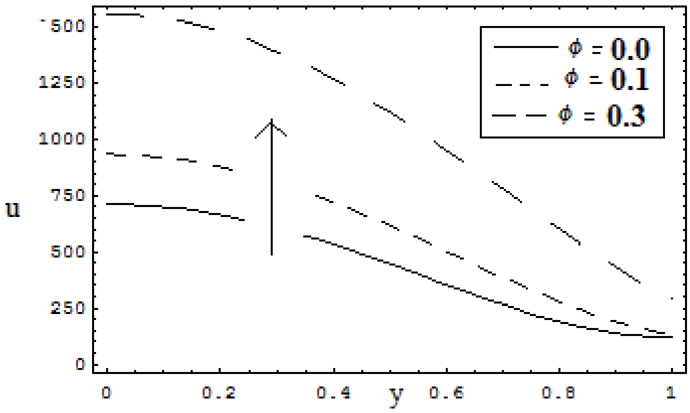

Fig. 4: Effect of ϕ on u ,when γ =0.1, M = 0.25,K = 1,x = 0.1 & p = -25.

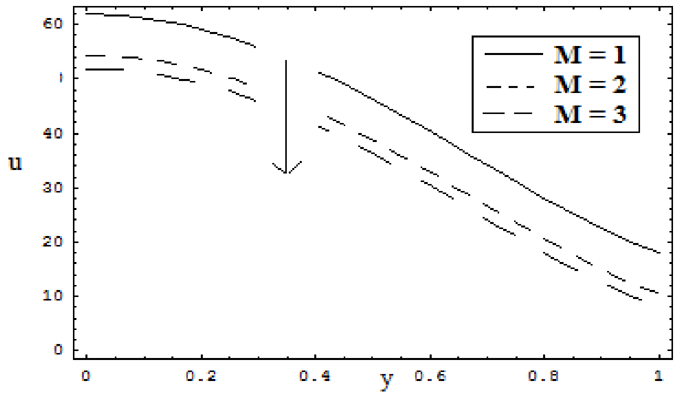

Fig. 5: Effect of M on u,when γ =0.1, ϕ = 0.1,K = 1,x = 0.1 & p = -25.

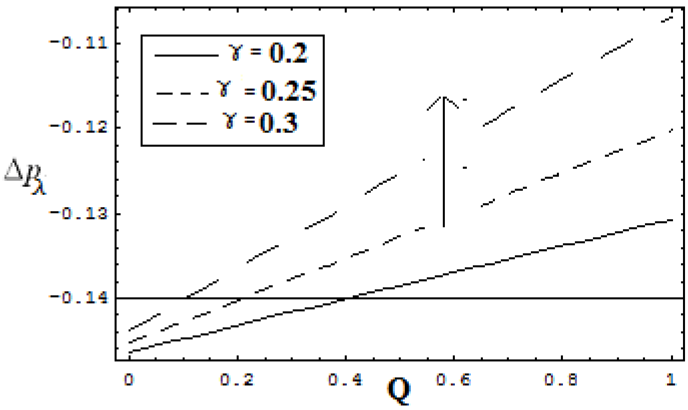

Fig. 6: Effect of γ on Δp ,when K = 1, M = 0.25, ϕ = 0.6 & x = 0.1.

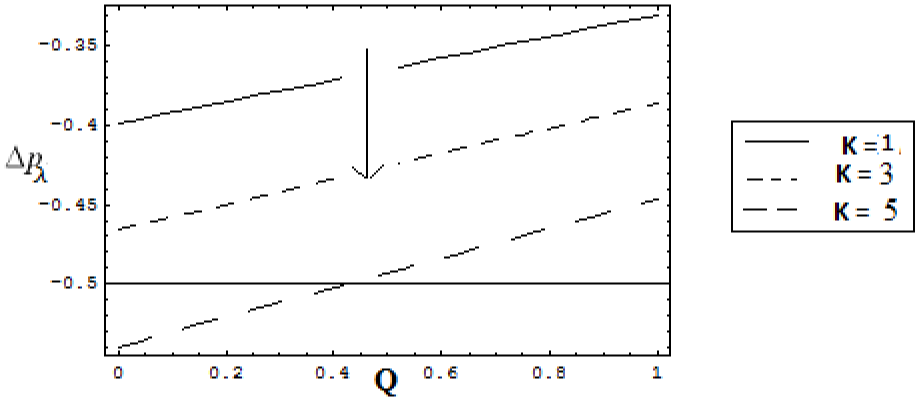

Fig. 7: Effect of K on Δp ,when γ =0.3, M = 0.25, ϕ = 0.4 & x = 0.1.

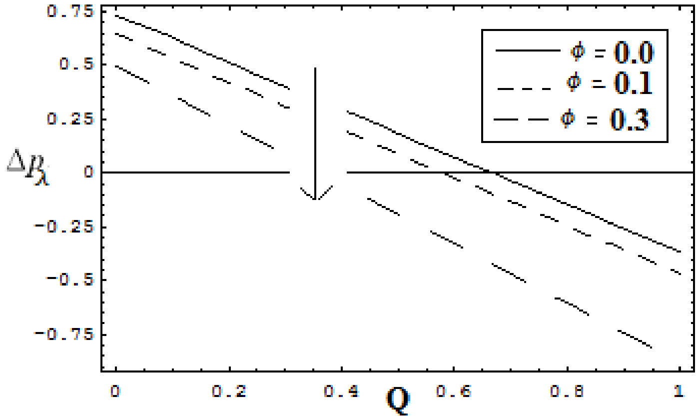

Fig. 8: Effect of ϕ on Δp ,when γ =1, M = 0.25,K = 1 & x = 0.1.

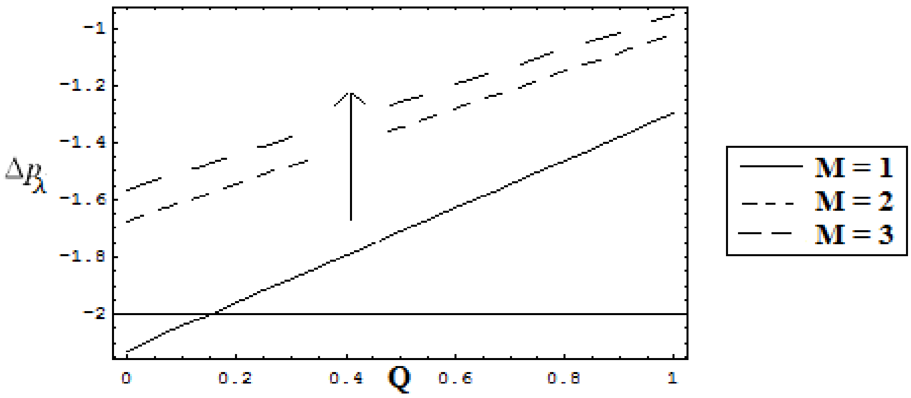

Fig. 9: Effect of M on Δp ,when γ =0.1, ϕ = 0.1,K = 1,x = 0.1 & p = -25.

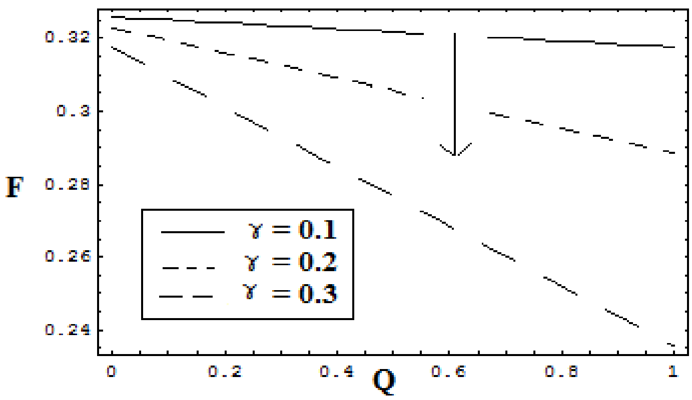

Fig. 10: Effect of γ on F ,when K = 1, M = 0.25, ϕ = 0.6 & x = 0.1.

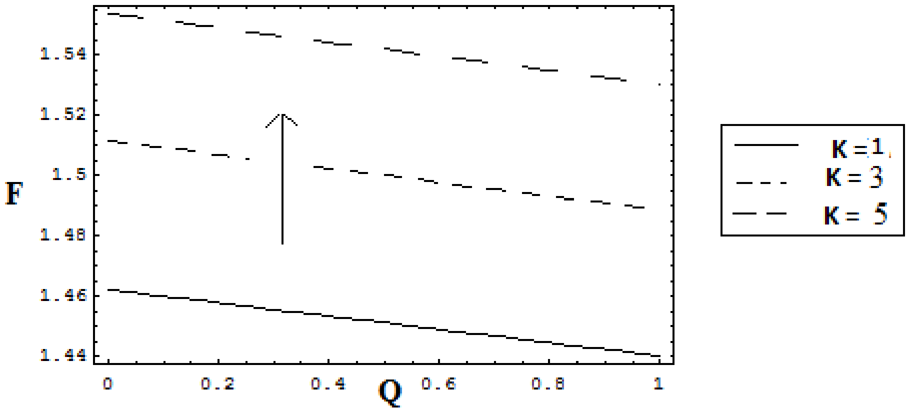

Fig. 11: Effect of K on F, when γ =0.1, M = 0.25, ϕ = 0.1 & x = 0.1.

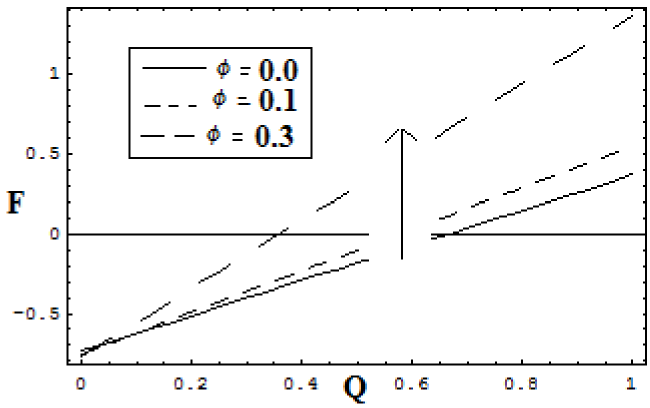

Fig. 12: Effect of ϕ on F, when γ =1, M = 0.25,K = 1 & x = 0.1.

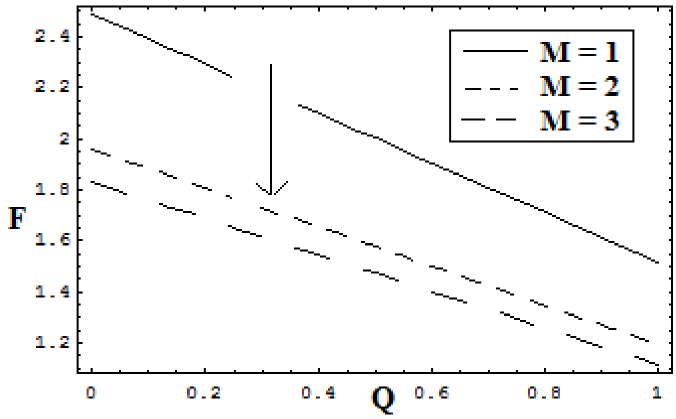

Fig. 13: Effect of M on F ,when γ =0.1, ϕ = 0.1,K = 1,x = 0.1 & p = -25.

4. CONCLUSION

In this paper we presented a theoretical approach to study the peristaltic flow of a couple stress fluid through a porous medium in a channel with the effect of magnetic field. The governing equations of motion are solved analytically. Furthermore, the effect of various values of parameters on Velocity, Pressure rise and Friction force have been computed numerically and explained graphically. We conclude the following observations:

1. The velocity u increases with increase in porous medium K & amplitude ϕ and decreases with increase in couple stress parameter γ & magnetic field M.
2. Pressure rise ΔP decreases with increase in porous medium K & amplitude ϕ and increases with increasing in couple stress parameter γ & magnetic field M.
3. The friction force F has increases with increase in porous medium K & amplitude ϕ and decreases with increase in couple stress parameter γ & magnetic field M.

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