HOMOMORPHISM AND ANTIHOMOMORPHISM IN INTERVAL VALUED Q-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we study some of the properties of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, antihomomorphism and prove some results on these.

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Keywords: Interval valued fuzzy subset, interval valued Q-fuzzy subhemiring, and interval valued Q-fuzzy normal subhemiring.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring \((R, +, .)\). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (also called half rings) are algebras \((R, +, \cdot)\) share the same properties as a ring except that \((R, +)\) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra \((R, +, \cdot)\) is called a semi ring if \((R, +)\) and \((R, \cdot)\) are semi groups satisfying \(a.(b+c)=a.b+a.c\) and \((b+c).a=b.a+c.a\) for all \(a, b, c\) in \(R\). A semi ring \(R\) is said to be additively commutative if \(a+b=b+a\) for all \(a, b, c\) in \(R\). A semi ring \(R\) may have an identity \(1\), defined by \(1.a=a=a.1\) and a zero \(0\), defined by \(0+a=a=a+0\) and \(a.0=0=0.a\) for all \(a\) in \(R\). A semi ring \(R\) is said to be a hemiring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval valued membership function Jun. Y.B and Kin.K [7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju. A and Nagarajan.R [9] defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. Azriel Rosenfeld [2] defined fuzzy groups. Osman Kazanci, Sultan yamark and serife yilmaz in [11] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and Nagarajan.R [14], have given a new structure in the construction of Q-fuzzy groups and subgroups [15]. We introduced the concept of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

1. PRELIMINARIES

1.1 Definition: Let \(X\) be any nonempty set. A mapping \([M]\colon X \to D[0,1]\) is called an interval valued fuzzy subset (briefly, IVFS) of \(X\), where \(D[0,1]\) denoted the family of all closed subintervals of \([0,1]\) and \([M](x)=[M^-(x), M^+(x)],\) for all \(x\) in \(X\), where \(M^-\) and \(M^+\) are fuzzy subsets of \(X\) such that \(M^-(x) \leq M^+(x)\), for all \(x\) in \(X\). Thus \([M](x)\) is an interval (a closed subset of \([0,1]\)) and not number from the intervak \([0,1]\) as in the case of fuzzy subset. Note that \([0]=[0,0]\) and \([1]=[1,1]\).

1.2 Remark: Let \(D^X\) be the set of all interval valued fuzzy subsets of \(X\), where \(D\) means \(D[0,1]\).

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1.3 Definition: Let \([M] = \{(x, [M^{-}(x), M^{+}(x)])/x \in X]\), \([N] = \{(x, [N^{-}(x), N^{+}(x)])/x \in X\}\) be any two interval valued fuzzy subsets of \(X\). We define the following relations and operations:

(i) \([M] \subseteq [N]\) if and only if \(M^{-}(x) \leq N^{-}(x)\) and \(M^{+}(x) \leq N^{+}(x)\), for all \(x \in X\).

(ii) \([M] = [N]\) if and only if \(M^{-}(x) = N^{-}(x)\) and \(M^{+}(x) = N^{+}(x)\), for all \(x \in X\).

(iii) \([M] \cup [N] = (\max\{M^{-}(x), N^{-}(x)\}, \min\{M^{+}(x), N^{+}(x)\})/x \in X\).

(iv) \([M] \cap [N] = (\min\{M^{-}(x), N^{-}(x)\}, \max\{M^{+}(x), N^{+}(x)\})/x \in X\).

(v) \([M]^c = [1] - [M] = \{(x, [1 - M^{+}(x), 1 - M^{-}(x)])/x \in X\}\).

1.4 Definition: Let \(X\) be a non-empty set and \(Q\) be a non-empty set. A \((Q, L)\)--fuzzy subset \(A\) of \(X\) is function \(A: X \times Q \rightarrow [0,1]\).

1.5 Definition: Let \((R, +, \cdot)\) be a hemiring. A interval valued \(Q\)-fuzzy subset \([M]\) of \(R\) is said to be an interval valued \(Q\)-fuzzy subhemiring (IVFSHR) of \(R\) if the following conditions are satisfied:

(i) \([M](x + y, q) \geq \min\{[M](x, q), [M](y, q)\}\)

(ii) \([M](xy, q) \geq \min\{[M](x, q), [M](y, q)\}\), for all \(x\) and \(y\) in \(R\), and \(q\) in \(Q\).

1.6 Definition: Let \((R, +, \cdot)\) be a hemiring. A interval valued \(Q\)-fuzzy subhemiring \([A]\) of \(R\) is said to be an interval valued \(Q\)-fuzzy normal subhemiring (IVFNSHR) of \(R\) if \([A] \cap [xy, q] = [A] \cap [yx, q]\), for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

1.7 Definition: Let \(X\) and \(X'\) be any two sets. Let \(f: X \rightarrow X'\) be any function and \([A]\) be an interval valued \(Q\)-fuzzy subset of \(X\), \([V]\) be an interval valued \(Q\)-fuzzy subset in \(f(X) = X'\), defined by \([V](x, q) = \sup_{y \in \text{im}(f^{-1}(x))} [A](y, q)\) for all \(x\) in \(X\) and \(y\) in \(X'\) and \(q\) \(Q\).

Then \([A]\) is called a pre-image of \([V]\) under \(f\) and is denoted by \(f^{-1}(\text{im}(f))\).

2. PROPERTIES OF INTERVAL VALUED \(Q\)-FUZZY SUBHENDRINGS

2.1 Theorem: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. The homomorphic image of an interval valued \(Q\)-fuzzy subhemiring of \(R\) is an interval valued \(Q\)-fuzzy subhemiring of \(R'\).

Proof: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. Let \(f: R \rightarrow R'\) be a homomorphism. Then, \(f(x+y)=f(x)+f(y)\) and \(f(xy)=f(x)f(y)\), for all \(x\) and \(y\) in \(R\) Let \([V] = f([A])\), where \([A]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\). We have to prove that \([V]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R'\). Now, for \(f(x), f(y)\) in \(R' \times Q\).

\([V](x, q) = f([A])(x, q)\) \(\geq\) \([A](x, q) \geq \min\{[A](x, q), [A](y, q)\}\) Which implies that \([V](f(x), q) \geq \min\{[V](f(x), q), [V](f(y), q)\}\).

Again,
\([V](f(x), q) \geq [V](f(y), q) \geq \min\{[A](x, q), [A](y, q)\}\) Which implies that \([V](f(x), q) \geq \min\{[V](f(x), q), [V](f(y), q)\}\).

Hence \([V]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R'\).

2.2 Theorem: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. The homomorphic preimage of an interval valued \(Q\)-fuzzy subhemiring of \(R\) is interval valued \(Q\)-fuzzy subhemiring of \(R\).

Proof: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. Let \(f: R \rightarrow R'\) be a homomorphism. Then, \(f(x+y)=f(x)+f(y)\) and \(f(xy)=f(x)f(y)\), for all \(x\) and \(y\) in \(R\). Let \([V] = f([A])\), where \([V]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\). We have to prove that \([A]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\). Let \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Then
\([A](x + y, q) = [V](f(x + y, q)) \geq [V](f(x, q)) \geq \min\{[V](f(x, q)), [V](f(y, q))\} = \min\{[A](x, q), [A](y, q)\}\), which implies that \([A](x + y, q) \geq \min\{[A](x, q), [A](y, q)\}\).

Again,
\([A](x, y, q) = [V](f(x, y, q)) \geq [V](f(x, q)) \geq \min\{[V](f(x, q)), [V](f(y, q))\} = \min\{[A](x, q), [A](y, q)\}\), which implies that \([A](x, y, q) = \min\{[A](x, q), [A](y, q)\}\).

Hence \([A]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\).

2.3 Theorem: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. The anti-homomorphic image of an interval valued \(Q\)-fuzzy subhemiring of \(R\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\).

Proof: Let \((R, +, \cdot)\) and \((R', +, \cdot)\) be any two hemirings. Let \(f: R \rightarrow R'\) be a anti-homomorphism. Then, \(f(x+y)=f(x)+f(y)\) and \(f(xy)=f(x)f(y)\), for all \(x\) and \(y\) in \(R\). Let \([V] = f([A])\), where \([A]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\). We have to prove that \([V]\) is an interval valued \(Q\)-fuzzy subhemiring of \(R'\). Now, for \(f(x), f(y)\) in \(R' \times Q\).

\([V](f(x), q) + (f(y), q) = [V](f(x), q) + (f(y), q) \geq [A](x + y, q) \geq \min\{[A](y, q), [A](x, q)\}\) which implies that \([V](f(x), q) + (f(y), q) \geq \min\{[V](f(x), q), [V](f(y), q)\}\).
2.4 Theorem: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. The anti-homomorphic preimage of an interval valued Q-fuzzy subhemiring of \(R\) is an interval valued Q-fuzzy subhemiring of \(R\).

Proof: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. Let \(f: R \rightarrow R'\) be a anti-homomorphism. Then \(f(x+y)=f(y)+f(x)\) and \(f(xy)=f(y)f(x)\), for all \(x\) and \(y\) in \(R\).

Let \([A]\) is an interval valued Q-fuzzy subhemiring of \(R\). Then \([A]\) is an interval valued Q-fuzzy subhemiring of \(R\) and \([A]\) is an interval valued Q-fuzzy subhemiring of \(R\). We have to prove that \([A]\) is a interval valued Q-fuzzy subhemiring of \(R\). Let \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

Again,
\[
[A](x + y, q) = \min\{[A](x, q), [A](y, q)\}.
\]

2.5 Theorem: Let \([A]\) be an interval valued Q-fuzzy subhemiring of hemiring \(H\) and \(f\) is an isomorphism from a hemiring \(R\) onto \(H\). Then \([A]\) is an interval valued Q-fuzzy subhemiring of \(R\).

Proof: Let \(x\) and \(y\) in \(R\) and \([A]\) is an interval valued Q-fuzzy subhemiring of a hemiring \(H\). Then we have,
\[
([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(x), q) + [A](f(y), q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{[A] \circ f(x, q), [A] \circ f(y, q)\}.
\]

Therefore \([A] \circ f\) is an interval valued Q-fuzzy subhemiring of \(R\).

2.6 Theorem: Let \([A]\) be an interval valued Q-fuzzy subhemiring of hemiring \(H\) and \(f\) is an anti- isomorphism from a hemiring \(R\) onto \(H\). Then \([A] \circ f\) is an interval valued Q-fuzzy subhemiring of \(R\).

Proof: Let \(x\) and \(y\) in \(R\). Then we have,
\[
([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(x), q) + f(y), q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{[A] \circ f(x, q), [A] \circ f(y, q)\}.
\]

Therefore \([A] \circ f\) is an interval valued Q-fuzzy subhemiring of the hemiring \(R\).

2.7 Theorem: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. The homomorphic image of an interval valued Q-fuzzy normal subhemiring of \(R\) is an interval valued Q-fuzzy subhemiring of \(R\).

Proof: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. Let \(f: R \rightarrow R'\) be a homomorphism. Then \([A]\) is an interval valued Q-fuzzy subhemiring of \(R\). Since \([A]\) is an interval valued Q-fuzzy normal subhemiring of \(R\), we have to prove that \([A]\) is an interval valued Q-fuzzy normal subhemiring of \(R\). Let \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Then, clearly, \([A]\) is an interval valued Q-fuzzy subhemiring of the hemiring \(R\).
proof: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. Let \(f: R \rightarrow R'\) be an anti homomorphism. Let \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). Then clearly, \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). We have to prove that \([V]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). Now, \([V](f(x), q) \leq \text{V}(f(y), q)\), for all \(x, y \in R\) and \(q \in Q\). Hence \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of the hemiring \(R\).

2.9 Theorem: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. The antihomomorphic image of an interval valued \(Q\)-fuzzy normal subhemiring of \(R\) is an interval valued \(Q\)-fuzzy subhemiring of \(R\).

proof: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. Let \(f: R \rightarrow R'\) be an anti homomorphism. Let \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). We have to prove that \([V]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). Now, \([V](f(x), q) \leq \text{V}(f(y), q)\), for all \(x, y \in R\) and \(q \in Q\). Hence \([V]\) is an interval valued \(Q\)-fuzzy normal subhemiring of the hemiring \(R\).

2.10 Theorem: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. The anti homomorphic preimage of an interval valued \(Q\)-fuzzy normal subhemiring of \(R\) is interval valued \(Q\)-fuzzy normal subhemiring of \(R\).

proof: Let \((R, +, .)\) and \((R', +, .)\) be any two hemirings. Let \(f: R \rightarrow R'\) be an anti homomorphism. Let \([V]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). We have to prove that \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\). Let \(x \) and \(y \) in \(R\) and \(q \) in \(Q\). Then clearly, \([A]\) is an interval valued \(Q\)-fuzzy subhemiring of the hemiring \(R\). Hence \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of the hemiring \(R\).

2.11 Theorem: Let \([A]\) be an interval valued \(Q\)-fuzzy normal subhemiring of hemiring \(H\) and \(f\) is an isomorphism from a hemiring \(R\) onto \(H\). Then \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\).

proof: Let \(x \) and \(y \) in \(R\) and \([A]\) be an interval valued \(Q\)-fuzzy normal subhemiring of a hemiring \(H\). Then clearly, \((A \circ f)\) is an interval valued \(Q\)-fuzzy normal subhemiring of a hemiring \(R\). Then we have \([A \circ f](x, q) = [A](f(x), q)\) which implies that \((A \circ f)(x, q) = (A \circ f)(y, q)\), for all \(x, y \in R\) and \(q \in Q\). Hence, \([A \circ f]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\).

2.12 Theorem: Let \([A]\) be an interval valued \(Q\)-fuzzy normal subhemiring of hemiring \(H\) and \(f\) is an anti- isomorphism from a hemiring \(R\) onto \(H\). Then \([A]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\).

proof: Let \(x \) and \(y \) in \(R\) and \([A]\) be an interval valued \(Q\)-fuzzy normal subhemiring of a hemiring \(H\). Then clearly, \((A \circ f)\) is an interval valued \(Q\)-fuzzy subhemiring of the hemiring \(R\). Then we have \([A \circ f](x, q) = [A](f(x), q)\) which implies that \((A \circ f)(x, q) = (A \circ f)(y, q)\), for all \(x, y \in R\) and \(q \in Q\). Hence, \([A \circ f]\) is an interval valued \(Q\)-fuzzy normal subhemiring of \(R\).

REFERENCE


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