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HOMOMORPHISM AND ANTIHOMOMORPHISM IN INTERVAL VALUED Q-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we study some of the properties of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, antihomomorphism and prove some results on these.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R, +, .). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras (R,+ ,.) share the same properties as a ring except that (R, +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R, +, .) is said to be a semi ring (R, +) and (R, .) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semi ring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semi ring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semi ring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function Jun. Y.B and Kin.K [7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju. A and Nagarajan.R [9] defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. Azriel Rosenfeld [2] defined fuzzy groups. Osman Kazanci, Sultan yamark and serife yilmaz in [11] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and Nagarajan.R [14], have given a new structure in the construction of Q-fuzzy groups and subgroups [15]. We introduced the concept of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

1. PRELIMINARIES

1.1 Definition: Let X be any nonempty set. A mapping $[M]: X \to D[0,1]$ is called an interval valued fuzy subset(briefly, IVFS) of X, where D[0,1] denoted the family of all closed subintervals of [0,1] and $[M](x)=[M^-(x), M^+(x)]$, for all x in X, where $M^-$and M^+ are fuzzy subsets of X such that $M^-(x) \le M^+(x)$, for all x in X. Thus [M](x) is an interval (a closed subset of [0,1]) and not number from the intervak [0,1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

1.2 Remark: Let D^X be the set of all interval valued fuzzy subsets of X, where D means D[0,1].

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1.3 Definition: Let $[M] = \{\langle x, [M^-(x), M^+(x)]\rangle / x \in X\}, [N] = \{\langle x, [N^-(x), N^+(x)]\rangle / x \in X\}$ be any two interval valued fuzzy subsets of x. We define the following relations and operations:

(i) $[M] \subseteq [N]$ if and only if $M^-(x) \le N^-(x)$ and $M^+(x) \le N^+(x)$, for all x in X.
(ii) [M]= [N] if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$, for all x in X.
(iii) $[M] \cap [N] = \{\langle x, [\min\{M^-(x), N^-(x)\}, \min\{M^+(x), N^+(x)\}]\rangle / x \in X\}$.
(iv) $[M] \cup [N] = \{\langle x, [\max\{M^-(x), N^-(x)\}, \max\{M^+(x), N^+(x)\}]\rangle / x \in X\}$.
(v) $[M]^C = [1] - [M] = \{\langle x, [1 - M^+(x), 1 - M^-(x)\ \rangle / x \in X\}$

1.4 Definition: Let X be a non-empty set and Q be a non-empty set. A (Q, L)--fuzzy subset A of X is function $A: X \times Q \to [0,1]$.

1.5 Definition: Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subset [M] of R is said to be an interval valued Q -fuzzy subhemiring (IVFSHR) of R if the following conditions are satisified:
(i) $[M](x + y, q) \ge \min\big([M](x, q), [M](y, q)\big)$
(ii)$[M](xy, q) \ge \min\big([M](x, q), [M](y, q)\big)$, for all x and y in R, and q in Q.

1.6 Definition: Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subhemiring [A] of R is said to be an interval valued Q-fuzzy normal subhemiring (IVFNSHR) of R if [A] (xy, q) = [A] (yx, q), for all x and y in R and q in Q.

1.7 Definition: Let X and X' be any two sets. Let $f: X \to X'$ be any function and [A] be an interval valued Q-fuzzy subset in X, [V] be an interval valued Q-fuzzy subset in $f(X) = X'$, defined by $[V](y, q) = \sup_{x \in f^{-1}(y)} [A](x, q)$ for all x in X and y in X' and q in Q. Then [A] is called a pre-image of [V] under f and is denoted by $f^{-1}([V])$.

2. PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

2.1 Theorem: Let (R, +,.) and (R$'$, +, .) be any two hemirings. The homomorphic image of an interval valued Q-fuzzy subhemiring of R is an interval valued Q-fuzzy subhemiring of R$'$.

Proof: Let (R, +,.) and (R$'$, +, .) be any two hemirings. Let $f: R \to R'$ be a homomorphism. Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let [V] = f ([A]), where [A] is an interval valued Q-fuzzy subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy subhemiring of R$'$. Now, for f(x), f(y) in R$'$ & q *in* Q .
$[V]\big(\ (f(x), q)\ +\ (\ f(y), q)\big) = \ [V](\ f(x + y,\ q) \ge\ [A](x + y, q) \ge \min\{[A](x, q), [A](y, q))\}$ Which implies that
$[V]\big((f(x)\ ,\ q) +\ (f(y)\ , q)\big) \ge \min\big([V](f(x), q), [V](f(y), q)\big)$.
Again,
$[V]\big((f(x), q)(f(y), q)\big) =\ [V]((f(xy)\ , q)) \ge [A](xy\ , q) \ge \min\{[A](\ x\ , q), [A]\ (y\ , q))\}$ Which implies that
$[V]\big((f(x), q)(f(y), q)\big) \ge\ \min\big([V](f(x), q), [V](f(y), q)\big)$.Hence [V] is an interval valued Q-fuzzy subhemiring of R$'$.

2.2 Theorem: Let (R, +,.) and (R$'$, +,.) be any two hemirings. The homomorphic preimage of an interval valued Q-fuzzy subhemiring of R$'$is interval valued Q-fuzzy subhemiring of R.

Proof: Let (R, +, .) and (R$'$, +, .) be any two hemirings. Let $f: R \to R'$ be a homomorphism. Then, Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let [V] = f([A]), where [V] is an interval valued Q-fuzzy subhemiring of R$'$. We have to prove that [A] is an interval valued Q – fuzzy subhemiring of R. Let x and y in R and q in Q. Then
$[A](x + y, q) = [V]\ (f(x + y\ , q)) \ge [V](f(x, q) + f(y, q)) \ge \min\{[V](f(x), q), [V](f(y, q))\} = \min\{[A](x, q), [A](y, q)\}$,
which implies that $[A]\ (x + y\ , q) = \min\{[A]\ (x\ , q)\ , [A]\ (y\ , q)\}$

Again,
$[A]\ (\ xy\ , q) = [V]\big(f(xy\ , q)\big) \ge [V]\big(f(x, q)f(y, q)\big) \ge \min\big\{[V](f(x), q), [V]\big(f(y), q)\big)\big\} = \min\{[A](x, q), [A](y, q)\}$
which implies that $[A](xy, q) = \min\{[A](x\ , q), [A](\ y\ , q)\}$.
Hence [A] is an interval valued Q-fuzzy subhemiring of R.

2.3 Theorem: Let (R, +, .) and (R$'$, +, .) be any two hemirings. The anti-homomorphic image of an interval valued Q-fuzzy subhemiring of R is an interval valued Q-fuzzy subhemiring of **R**$'$.

Proof: Let (R, +,.) and (R$'$, +,.) be any two hemirings.Let $f: R \to R'$ be a anti-homomorphism. Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let [V] = f([A]), where [A] is an interval valued Q-fuzzy subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy subhemiring of R$'$. Now, for f(x), f(y) in R$'$& q *in* Q $[V]\big((f(x), q) + (f(y), q)\big) = [V]\big((f(y), q) + (f(x), q)\big) \ge [A](y + x, q) \ge \min\{[A](y, q), [A](x, q))\}$ whichimplies that $[V]\big((f(x), q) + (f(y), q)\big) \ge min\big([V](f(x), q), [V](f(y), q)\big)$.

Again,
$[V]((f(x),q)(f(y),q)) = [V](f(yx),q) \geq [A](yx,q) \geq \min\{[A](y,q),[A](x,q))\} = \min\{[A](x,q),[A](y,q))\}$ which implies that $[V]((f(x),q)(f(y),q)) \geq min([V](f(x),q),[V](f(y),q))$.
Hence [V] is an interval valued Q-fuzzy subhemiring of R'.

2.4 Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The anti-homomorphic preimage of an interval valued Q-fuzzy subhemiring of R' is an interval valued Q-fuzzy subhemiring of R.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings. Let $f: R \to R'$ be a anti-homomorphism. Then f(x+y)=f(y)+f(x) and f(xy)=f(y)f(x),for all x and y in R.

Let $[V] = f([A])$where V is an anti $(Q,L) -$ fuzzysubhemiring of R'. Let x and y in R & q in Q. where [V] is an interval valued Q-fuzzy subhemiring of R'. We have to prove that [A] is an interval valued Q – fuzzy subhemiring of R. Let x and y in R and q in Q. Then
$[A]\ (x + y\ ,\ q) = [V]\ (f(x + y\ ,q)) = [V]\ (f(y + x\ ,q) \geq \min\{[V](f(y),q),[V](f(x),q))\} \geq$
$\min\{[V](f(x),q),[V](f(y),q))\} = \min\{[A](x,q),[A](y,q)\}$, which implies that
$[A]\ (x + y\ ,q) = \min\{[A]\ (x\ ,q)\ ,[A]\ (y\ ,q)\}$ .Again,$[A]\ (\ xy\ ,q) = [V]((f(xy)\ ,q)) \geq [V](f(x),q)(f(y),q)) \geq$
$\min\{[V](f(y),q),[V](f(x),q))\} \geq \min\{[V](f(x),q),[V](f(y),q))\} = \min\{[A](x,q),[A](y,q)\}$ which implies that $[A](xy,q) = \min\{[A](x\ ,q),[A](y,q)\}$. Hence [A] is an interval valued Q-fuzzy subhemiring of R.

2.5 Theorem: Let [A] be an interval valued Q-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then [A]∘ f is an interval valued Q-fuzzy subhemiring of R.

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy subhemiring of a hemiring H. Then we have,
$([A] \circ f)(\ x + y,q\ ) = [A]\ (\ f(x + y),q)) = [A](f(x),q) + (f(y),q)) \geq \min\{([A](\ f(x)\ ,\ q\ )\ ,[A](f(y),q)\} =$
$min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$. which implies that
$([A] \circ f)(\ x\ +\ y\ ,q)\ \geq min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$. And, $([A] \circ f)(\ xy,q\ ) = [A]\ (\ f(xy),q)) =$
$[A]((f(x),q)(f(y),q)) \geq \min\{([A](\ f(x)\ ,\ q\ ),[A](f(y),q)\} = min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$. which implies that $([A] \circ f)(\ xy\ ,q)\ \geq min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$.Therefore $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of R.

2.6 Theorem: Let [A] be an interval valued Q-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then [A]∘ f is an interval valued Q-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $([A] \circ f)(\ x + y,q\ ) = [A]\ (\ f(x + y),q)) = [A](f(y,q) + f(x,q)) \geq \min\{([A](\ f(x)\ ,q\ )\ ,[A](f(y),q)\} = min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$. which implies that $([A] \circ f)(\ x\ +\ y\ ,q)\ \geq min\{([A] \circ f)(x,q),([A] \circ f)(y,q)\}$. and,
$([A] \circ f)(\ xy,q\ ) = [A]\ (\ f(yx),q)) = [A](f(y),q)f(x),q)) \geq \min\{([A](\ f(x)\ ,\ q\ )\ ,[A](f(y),q)\} = min\{([A] \circ f)(x,q),$
A∘fy,q. which implies that *[A]*∘f x*y*,*q*≥*min*A∘fx,q,*[A]*∘fy,q.
Therefore $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R.

2.7 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. The homomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of R'.

Proof: Let (R, +, .) and (R', +, .) be any two hemirings.Let $f: R \to R'$ be a homomorphism. Let [A] is an interval valued Q-fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x)$, $f(y)$ in R' & q in Q . clearly, [V] is an interval valued Q-fuzzy subhemiring of $f(R) = R'$. since [A] is an interval valued Q-fuzzy subhemiring of R.

Now, $[V]((f(x),q)(f(y),q)) = [V](f(xy),q) \geq [A](xy,q) = [A](yx,q) \leq [V]\{f(yx),q)\} = [V]((f(y),q)(f(x),q))$ which implies that $[V]((f(x),q)(f(y),q)) = [V]((f(y),q)(f(x),q))$. Hence [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R'.

2.8 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. The homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of R' is interval valued Q-fuzzy normal subhemiring of R.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings.Let $f: R \to R'$ be a homomorphism. Let [V] is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. we have to prove that [A] is an interval valued Q – fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then, clearly, [A] is an interval valued Q-fuzzy subhemiring of the hemiring R.

Now, $[A](xy,q) = [V](f(xy),q) = [V]\{(f(x),q)(f(y),q)\} = [V]\big((f(y),q)(f(x),q)\big) = [V](f(yx),q) = [A](yx,q)$ which implies that $[A](xy,q) = [A](yx,q)$, for all x and y in R and q in Q. Hence [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.9 Theorem: Let (R, +, .) and (R$'$, +, .) be any two hemirings. The antihomomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of R$'$.

Proof: Let (R, +,.) and (R$'$, +, .) be any two hemirings.Let $f: R \to R'$ be a anti homomorphism. Let [A] is an interval valued Q-fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$.Now, for f(x), f(y) in R$'$ & q in Q . clearly, [V] is an interval valued Q-fuzzy subhemiring of R^'. since [A] is an interval valued Q − fuzzy subhemiring of R. Now, $[V]\big((f(x),q)(f(y),q)\big) = [V](f(yx),q) \geq [A](yx,q) = [A](xy,q) \leq [V]\{f(xy),q)\} = [V]\big((f(y),q)(f(x),q)\big)$ which implies that $[V]\big((f(x),q)(f(y),q)\big) = [V]\big((f(y),q)(f(x),q)\big)$. Hence [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R$'$.

2.10 Theorem: Let (R, +, .) and (R$'$, +,.) be any two hemirings. The anti homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of R$'$ is interval valued Q-fuzzy normal subhemiring of R.

Proof: Let (R, +,.) and (R$'$, +,.) be any two hemirings. Let $f: R \to R'$ be anti homomorphism. Let [V] is an interval valued Q-fuzzy normal subhemiring of f(R)=R$'$. We have to prove that [A] is an interval valued Q – fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly, [A] is an interval valued Q-fuzzy subhemiring of the hemiring R. since [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R$'$.

Now, $[A](xy,q) = [V](f(xy),q) = [V]\{(f(y),q)(f(x),q)\} = [V]\big((f(x),q)(f(y),q)\big) = [V](f(yx),q) = [A](yx,q)$ which implies that $[A](xy,q) = [A](yx,q)$, for all x and y in R and q in Q. Hence [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.11 Theorem: Let [A] be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then [A]∘ f is an interval valued Q-fuzzy normal subhemiring of R.

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy normal subhemiring of a hemiring H. Then clearly, $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R.$Then\ we\ have$
$([A] \circ f)(xy,q) = [A]\ (f(xy),q)) = [A]\big((f(x),q)(f(y),q)\big) = [A]\Big((f(y),q)(f(x),q)\Big) = [A](\ f(yx),q)\Big) = ([A] \circ f)(\ yx,q\ )$
which implies that $([A] \circ f)(\ xy\ ,q) = ([A] \circ f)(\ yx\ ,q)$ therefore $([A] \circ f)$ is an interval valued Q-fuzzy normal subhemiring of R.

2.12 Theorem: Let [A] be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then [A]∘ f is an interval valued Q-fuzzy normal subhemiring of R.

Proof: Let x and y in R and [A] be an interval valued Q-fuzzy normal subhemiring of a hemiring H. Then clearly, $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R. Then we have
$([A] \circ f)(xy,q) = [A](f(xy),q)) = [A]\big((f(y),q)(f(x),q)\big) = [A]\Big(\big((f(x),q)(f(y),q)\big) = [A](f(yx),q)\Big) = ([A] \circ f)(yx,q)$
which implies that
$([A] \circ f)(\ xy\ ,q) = ([A] \circ f)(\ yx\ ,q)$, for all x and y in R and q in Q. Hence, $([A] \circ f)$ is an interval valued Q-fuzzy normal subhemiring of hemiring R.

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