International Journal of Mathematical Archive-6(10), 2015, 30-38
Available online through www.ijma.info ISSN 2229 – 5046

SOME OPERATORS DEFINED OVER INTUITIONISTIC FUZZY SETS
AND INTERVAL VALUED INTUITIONISTIC FUZZY SETS

1,2S. SUDHARSAN*, 3D. EZHILMARAN

1Department of Mathematics, C. Abdul Hakeem College of Engineering & Technology,
Melvisharam, Vellore – 632 509. Tamilnadu, India.

2Research Scholar, Bharathiar University, Coimbatore -641046, India.

3School of Advanced Sciences, VIT University, Vellore – 632 014. Tamilnadu, India.

(Received On: 06-09-15; Revised & Accepted On: 14-10-15)

ABSTRACT

In this paper, two operators defined over IFS, that will be an analogous as of Operations “extraction” as well as of
operation “Multiplication of an IFS $A_n$ with $\frac{1}{n}$ and multiplication of an IFS with $A_n^\frac{1}{n}$ with the natural number $n$ are
proved, also we extended the same operators over IVIFS are proved.

AMS Classification: 03E72.

Keywords: Intuitionistic fuzzy sets, Interval valued intuitionistic fuzzy sets, Operations over intuitionistic fuzzy sets,
Operations over Interval valued intuitionistic fuzzy sets.

1. INTRODUCTION

The notion of intuitionistic fuzzy sets was introduced by Atanassov. K (1986) [1] as a generalization of the concept of
fuzzy sets was introduced by Zadeh. L. A (1965) [23]. Intuitionistic fuzzy sets are characterized by two functions
expressing the degree of membership and the degree of non-membership respectively. The concept of intuitionistic
fuzzy sets has been successfully applied in numerous fields, such as pattern recognition, machine learning, image
processing and decision making, and etc. A lot of operations are introduced and proved over the intuitionistic fuzzy
sets. Atanassov. K (1994) [3] proposed new operations defined over the intuitionistic fuzzy sets. Supriya Kumar De,
Ranjit Biswas and Akhil Ranjan Roy(2000) [17,18] proposed Some operations on intuitionistic fuzzy sets and also
Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy (2001) proposed an application of intuitionistic fuzzy sets in
connected with intuitionistic fuzzy sets. Verma, R. K and Sharma, B. D (2011) [20] proposed Intuitionistic fuzzy sets:
sets which is a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets. After the introduction of
IVIFS, many researchers have shown interest in the IVIFS theory and its application. Atanassov.K (1994) [6]proposed
Operators over interval-valued intuitionistic fuzzy sets. Xu. Z.S (2007) [22] proposed methods for aggregating interval-
valued intuitionistic fuzzy information and their application to decision making. Rangasamy Parvathi, Beloslav Riecan
MonoranjanBhowmik and Madhumangal Pal (2012) [12] proposed Some Results on Generalized Interval-Valued
Intuitionistic Fuzzy Weighted Entropy (IVIFWE) method for selection of vendor and also application of generalized
interval valued intuitionistic fuzzy relation with fuzzy max- min composition technique in medical diagnosis. In 2014,
Definition 2.1: Intuitionistic fuzzy set. Let a set \( X \) be fixed. An intuitionistic fuzzy set \( A \) in \( X \) is an object having the form \( A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \} \), where the functions \( \mu_A : X \to [0, 1] \) and \( \gamma_A : X \to [0, 1] \) define the degree of membership and the degree of non-membership of the element \( x \in X \) to the set \( A \) which is a subset of \( X \), respectively, and for every \( x \in X \): \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \). Let for every \( x \in X \): \( \pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \). Therefore, Function determines the degree of uncertainty. Let us define the empty IFS, the totally uncertain IFS, and the unit IFS by: \( O^* = \{ (x, 0.1) \mid x \in X \} \), \( U^* = \{ (x, 0,0) \mid x \in X \} \) and \( E^* = \{ (x, 1,0) \mid x \in X \} \).

Definition 2.2: Interval valued intuitionistic fuzzy set. An interval valued intuitionistic fuzzy set \( A \) in the finite universe \( X \) is defined as \( A = \{ x, [\mu_A(x), \gamma_A(x)] \mid x \in X \} \), where \( \mu_A : X \to [0, 1] \) and \( \gamma_A : X \to [0, 1] \) with the condition \( 0 \leq \sup(\mu_A(x)) + \sup(\gamma_A(x)) \leq 1 \), for any \( x \in X \). The intervals \( \mu_A(x) \) and \( \gamma_A(x) \) denote the degree of membership function and the degree of non-membership of the element \( x \) to the set \( A \). For every \( x \in X, \mu_A(x) \) and \( \gamma_A(x) \) are closed intervals and their Left and Right end points are denoted by \( \mu^L_A(x), \mu^R_A(x), \gamma^L_A(x), \gamma^R_A(x) \). Let us denote \( A = \{ [\mu^L_A(x), \mu^R_A(x)], [\gamma^L_A(x), \gamma^R_A(x)] \mid x \in X \} \) where \( 0 \leq \mu^L_A(x) + \gamma^L_A(x) \leq 1, \mu^R_A(x) \geq 0, \gamma^R_A(x) \geq 0 \).

We call the interval \( [1 - \mu^L_A(x) - \gamma^L_A(x), 1 - \mu^R_A(x) - \gamma^R_A(x)] \), abbreviated by \( [\pi^L_A(x), \pi^R_A(x)] \) and denoted by \( \pi_A(x) \), the interval-valued intuitionistic index of \( x \) in \( A \), which is a hesitancy degree of \( x \) to \( A \). Especially if \( \mu_A(x) = \mu^L_A(x) = \mu^R_A(x) \) and \( \gamma_A(x) = \gamma^L_A(x) = \gamma^R_A(x) \) then the given IVIFS \( A \) is reduced to an ordinary IF.

Definition 2.3: Set operations on IFS. Let A and B be two IFSs on the universe X, where

\( A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \} \), \( B = \{ (x, \mu_B(x), \gamma_B(x)) \mid x \in X \} \)

Here, we define some set operations for IFSs:

\[ A^T = \{ (x, \pi_A(x), \mu_A(x)) \mid x \in X \} \]

\[ A \cap B = \{ (x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))) \mid x \in X \} \]

\[ A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))) \mid x \in X \} \]

\[ A + B = \{ (x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)) \mid x \in X \} \]

\[ \Delta A = \{ (x, \pi_A(x), 1 - \mu_A(x)) \mid x \in X \} \]

\[ \bar{A} = \{ (x, \mu_A(x)\gamma^R_A(x), 1 - \gamma_A(x)) \mid x \in X \} \]

\[ \sqrt[n]{A} = \{ x, n \frac{\ln(\mu_A(x)) + \ln(1 - \mu_A(x))}{n} \mid x \in X \} \]

\[ \frac{1}{\sqrt[n]{A}} = \{ x, n \frac{\ln(1 - \mu_A(x))}{n} \mid x \in X \} \]

\[ \n A = \{ x, 1 - (1 - \mu_A(x))^n, (\gamma_A(x))^n \mid x \in X \} \]

\[ A^T = \{ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \mid x \in X \} \]

Definition 2.4: Set operations on IVIFS. Let A and B be two IVIFSs on the universe X, where

\( A = \{ (x, [\mu^L_A(x), \mu^R_A(x)], [\gamma^L_A(x), \gamma^R_A(x)]) \mid x \in X \} \)

\( B = \{ (x, [\mu^L_B(x), \mu^R_B(x)], [\gamma^L_B(x), \gamma^R_B(x)]) \mid x \in X \} \)

Here, we define some set operations for IVIFSs:

\[ A \cup B = \{ x, \max(\mu^L_A(x), \mu^L_B(x)), \max(\mu^R_A(x), \mu^R_B(x)), \min(\gamma^L_A(x), \gamma^L_B(x)), \min(\gamma^R_A(x), \gamma^R_B(x)) \mid x \in X \} \]

\[ A \cap B = \{ x, \min(\mu^L_A(x), \mu^L_B(x)), \min(\mu^R_A(x), \mu^R_B(x)), \max(\gamma^L_A(x), \gamma^L_B(x)), \max(\gamma^R_A(x), \gamma^R_B(x)) \mid x \in X \} \]

\[ A + B = \{ x, (\mu^L_A(x) + \mu^L_B(x) - \mu^L_A(x)\mu^L_B(x))\mu^L_A(x)\mu^L_B(x), (\mu^R_A(x)\mu^R_B(x) - \mu^R_A(x)\mu^R_B(x))\gamma^R_A(x)\gamma^R_B(x) \mid x \in X \} \]
Because from \( n \geq 1 \), and for each \( x \in X \)

\[
\Delta A \equiv \{ \{ x, (\mu_A(x), \mu_A(x)), (1 - \mu_A(x), 1 - \mu_A(x)) \} \mid x \in X \}
\]

\( \Box A \equiv \{ \{ x, (\mu_A(x), \mu_A(x)), (1 - \mu_A(x), 1 - \mu_A(x)) \} \mid x \in X \}
\]

\( nA \equiv \{ \{ x, (1 - (1 - \mu_A(x))^n, 1 - (1 - \mu_A(x))^n) \} \mid x \in X \}
\]

\( A^n \equiv \{ \{ x, (1 - (1 - \mu_A(x))^n, 1 - (1 - \mu_A(x))^n) \} \mid x \in X \}
\]

\( \nabla A \equiv \{ \{ x, (1 - (1 - \mu_A(x))^n, 1 - (1 - \mu_A(x))^n) \} \mid x \in X \}
\]

\( \frac{1}{n} A \equiv \{ \{ x, (1 - (1 - \mu_A(x))^n, 1 - (1 - \mu_A(x))^n) \} \mid x \in X \}
\]

Where \( n \geq 1 \) is natural number.

### 3. SOME NEW OPERATORS DEFINED OVER INTUITIONISTIC FUZZY SETS AND INTERVAL VALUED INTUITIONISTIC FUZZY SETS

We will introduce some new operator defined over IFS, that will be an analogous as of operations “extraction” as well as of operation “Multiplication of an IFS\( A^n \) with \( \frac{1}{n} \) and Multiplication of an IFS \( A^n \) within”. It has the form for every IFS and for every natural number \( n \geq 1 \):

\( A \cdot B = \{ x, (\mu_A(x) \cdot \mu_B(x), \mu_A(x) \cdot \mu_B(x)) \}
\]

\[ A^n = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

\[ \nu A = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

\[ \frac{1}{n} A = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

Also, we extended the same operators over IVIFS are proved.

\( A \cdot B = \{ x, (\mu_A^n(x), \mu_B^n(x), \mu_A^n(x)) \}
\]

\[ A^n = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

\[ \nu A = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

\[ \frac{1}{n} A = \{ x, (\mu_A^n(x), (1 - \mu_A^n(x)), (1 - \mu_A^n(x))) \mid x \in X \}
\]

First for (a), we must check that in a result of the operation we obtain an IFS. Really, for given IFS \( A^n \), for each \( x \in X \), and for each \( n \geq 1 \):

\( 1 - \frac{n}{n} - \frac{1}{n} - (\mu_A(x))^n \leq 1 \)

Because from \( \mu_A(x) \leq 1 - \gamma_A(x), 1 - (\mu_A(x))^n \leq 1 - (1 - \gamma_A(x))^n \)

it follows that \( \frac{1}{n} - (\mu_A(x))^n \leq \frac{1}{n} - (1 - \gamma_A(x))^n \).

Obviously, for every natural number \( n \geq 1 \): \( \frac{1}{n} (O^n) = O^*, \frac{1}{n} (U^n) = U^*, \frac{1}{n} (E^n) = E^* \). By similar to the above way for (b) also.

For (c), we must check that in a result of the operation we obtain an IVIFS. Really, for given IVIFS \( A^n \), for each \( x \in X \), and for each \( n \geq 1 \):

\( 1 - \frac{n}{n} - \frac{1}{n} - (\mu_A(x))^n \leq 1 \)

Because from \( \mu_A(x) \leq 1 - \gamma_A(x), 1 - (\mu_A(x))^n \leq 1 - (1 - \gamma_A(x))^n \).
It follows that
\[ n \left(1 - \left(\mu_A^n(x)\right)^n\right) \leq n \left(1 - \left(1 - \gamma_B^n(x)\right)^n\right) \]. Obviously, for every natural number \( n \geq 1 \):
\[ \frac{1}{n} (O^n) = O^* \]
\[ \frac{1}{n} (U^n) = U^*, \frac{1}{n} (E^n) = E^* \]. By similar to the above way for (d) also. Using the four operators, we can prove the following theorems.

**Theorem 3.1:** For any two IFSs A and B and for every natural number \( n \geq 1 \):

(a) \( \frac{1}{n} (A^n \cap B^n) = \frac{1}{n} A^n \cap \frac{1}{n} B^n \)

(b) \( \frac{1}{n} (A^n \cup B^n) = \frac{1}{n} A^n \cup \frac{1}{n} B^n \)

(c) \( \frac{1}{n} (A^n \cap B^n) = nA^n \cap nB^n \)

(d) \( \frac{1}{n} (A^n \cup B^n) = nA^n \cup nB^n \)

**Proof:**

\[ \frac{1}{n} (A^n \cap B^n) = \frac{1}{n} \left(\left\{x, \left(\mu_A(x)\right)^n, 1 - \left(1 - \gamma_B(x)\right)^n\right\} \cap \left\{x, \left(\mu_B(x)\right)^n, 1 - \left(1 - \gamma_B(x)\right)^n\right\} \mid x \in X\right\} \]

\[ = \left\{\frac{1}{n} \left(\left\{x, \left(\mu_A(x)\right)^n, 1 - \left(1 - \gamma_B(x)\right)^n\right\} \cap \left\{x, \left(\mu_B(x)\right)^n, 1 - \left(1 - \gamma_B(x)\right)^n\right\} \mid x \in X\right\} \right\} \]

\[ = \left\{x, 1 - \frac{1}{n} \left(\left(\mu_A(x)\right)^n, \left(\mu_B(x)\right)^n\right)\right\} \]

\[ = \left\{x, 1 - \frac{1}{n} \left(\left(\mu_A(x)\right)^n, \left(\mu_B(x)\right)^n\right) \mid x \in X\right\} \]

\[ = \left\{x, 1 - \frac{1}{n} \left(\left(\mu_A(x)\right)^n, \left(\mu_B(x)\right)^n\right) \mid x \in X\right\} \]

Hence (a) is proved and similarly (b), (c) & (d) is proved by analogy.

**Theorem 3.2:** For every IFS A and for every natural number \( n \geq 1 \):

(a) \( \square \frac{1}{n} A^n = \frac{1}{n} \square A^n (b) \)

(b) \( \diamond \frac{1}{n} A^n = \frac{1}{n} \diamond A^n \)

(c) \( \Box \frac{1}{n} A^n = \frac{1}{n} \Box A^n \)

**Proof:**

\[ \square \frac{1}{n} A^n = \frac{1}{n} \left\{\left\{x, \left(\mu_A(x)\right)^n, 1 - \left(1 - \gamma_B(x)\right)^n\right\} \mid x \in X\right\} \]

\[ = \left\{x, 1 - \frac{1}{n} \left(\left(\mu_A(x)\right)^n, \left(1 - \gamma_B(x)\right)^n\right) \mid x \in X\right\} \]

\[ = \left\{x, 1 - \frac{1}{n} \left(\left(\mu_A(x)\right)^n, \left(1 - \gamma_B(x)\right)^n\right) \mid x \in X\right\} \]

(3.1)
\[ \frac{1}{n} A^n = \frac{1}{n} \left[ \left\{ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \right\} | x \in X \right] \]

\[ = \frac{1}{n} \left[ \left\{ x, (\mu_A(x))^n, 1 - (\mu_A(x))^n \right\} | x \in X \right] \]

\[ = \frac{1}{n} \left\{ x, \left( 1 - \sqrt[n]{(1 - (\mu_A(x))^n)} \right)^n \left( 1 - \sqrt[n]{(1 - (\mu_A(x))^n)} \right)^n \right\} | x \in X \] \tag{3.2}

From (3.1) and (3.2), we get
\[ \frac{1}{n} A^n = \frac{1}{n} A^n \]

Hence (a) is proved and similarly (b), (c) & (d) is proved by analogy.

**Theorem 3.3:** For any two IFSs A and B and for every natural number \( n \geq 1 \):

(a) \( \frac{1}{n} (A^n + B^n) = \frac{1}{n} A^n + \frac{1}{n} B^n \),

(b) \( \frac{1}{n} (A^n \cdot B^n) = \frac{1}{n} A^n \cdot \frac{1}{n} B^n \)

(c) \( n \left( A^\frac{1}{n} + B^\frac{1}{n} \right) = nA^\frac{1}{n} + nB^\frac{1}{n} \),

(d) \( n \left( A^\frac{1}{n} \cdot B^\frac{1}{n} \right) = nA^\frac{1}{n} \cdot nB^\frac{1}{n} \)

**Proof:**

\[ \frac{1}{n} (A^n + B^n) = \frac{1}{n} \left( \left\{ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \right\} + \left\{ x, (\mu_B(x))^n, 1 - (1 - \gamma_B(x))^n \right\} | x \in X \right) \]

\[ = \frac{1}{n} \left\{ \left( x, (\mu_A(x))^n + (\mu_B(x))^n \right) - (\mu_A(x))^n (\mu_B(x))^n \right\} | x \in X \]

\[ = \left\{ \left( 1 - \sqrt[n]{(1 - (\mu_A(x))^n)} \right)^n (1 - \sqrt[n]{(1 - (\mu_B(x))^n)}) \right\} | x \in X \]

\[ = \left\{ x, 1 - \sqrt[n]{(1 - (\mu_A(x))^n)} \right\} \left( 1 - \sqrt[n]{(1 - (\mu_B(x))^n)} \right) | x \in X \]

\[ = \frac{1}{n} \left\{ \left[ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \right] + \left[ x, (\mu_B(x))^n, 1 - (1 - \gamma_B(x))^n \right] | x \in X \right\} \]

\[ = \frac{1}{n} A^n + \frac{1}{n} B^n \]

Hence (a) is proved and similarly (b), (c) & (d) is proved by analogy.

**Theorem 3.4:** For every IFS A and for every natural number \( n \geq 1 \):

(a) \( n \left( \frac{1}{n} A^n \right) = A^n \)

(b) \( \frac{1}{n} (n A^n) = A^n \)

**Proof:**

\[ n \left( \frac{1}{n} A^n \right) = n \left( \left\{ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \right\} | x \in X \right) \]

\[ = n \left\{ \left( x, 1 - \sqrt[n]{1 - (\mu_A(x))^n} \right) \left( 1 - \sqrt[n]{1 - (1 - \gamma_A(x))^n} \right) | x \in X \right\} \]

\[ = \left\{ \left( x, 1 - \left( 1 - \sqrt[n]{(1 - (\mu_A(x))^n)} \right)^n \right) \left( 1 - \sqrt[n]{1 - (1 - \gamma_A(x))^n} \right) | x \in X \right\} \]

\[ = \left\{ x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n \right\} | x \in X \]

\[ = A^n \]

Hence (a) is proved and similarly (b) is proved by analogy.
Theorem 3.5: For any two IVIFSs \(A\) and \(B\) and for every natural number \(n \geq 1\):

(a). \(\frac{1}{n}(A^n \cap B^n) = \frac{1}{n}A^n \cap \frac{1}{n}B^n\),

(b). \(\frac{1}{n}(A^n \cup B^n) = \frac{1}{n}A^n \cup \frac{1}{n}B^n\)

(c). \(n \left(\frac{1}{n}A^n \cap B^n\right) = nA^n \cap nB^n\),

(d). \(n \left(\frac{1}{n}A^n \cup B^n\right) = nA^n \cup nB^n\)

Proof: (a).

\[
\frac{1}{n}(A^n \cap B^n) = \frac{1}{n} \left\{ x, (\mu_A^1(x))^n, (\mu_B^1(x))^n, (1-(1-\gamma_A^1(x))^n, 1-(1-\gamma_B^1(x))^n) \right\} \cap \left\{ x, ((\mu_A^0(x))^n, (\mu_B^0(x))^n, (1-(1-\gamma_A^0(x))^n, 1-(1-\gamma_B^0(x))^n) \right\} \mid x \in X
\]

\[
= \frac{1}{n} \left\{ x, \left(1 - \frac{1}{\sqrt[n]{\max\left((\mu_A^1(x))^n, (\mu_B^1(x))^n\right)}} - \frac{1}{\sqrt[n]{\min\left((\mu_A^0(x))^n, (\mu_B^0(x))^n\right)}}\right), 1 - \frac{1}{\sqrt[n]{\max\left((\mu_A^0(x))^n, (\mu_B^0(x))^n\right)}} - \frac{1}{\sqrt[n]{\min\left((\mu_A^1(x))^n, (\mu_B^1(x))^n\right)}} \right\} \mid x \in X
\]

\[
= \left\{ x, \left(1 - \frac{\max\left(\sqrt[n]{(1-\gamma_A^1(x))^n}, \sqrt[n]{(1-\gamma_B^1(x))^n}\right)}{\sqrt[n]{\max\left((\mu_A^0(x))^n, (\mu_B^0(x))^n\right)}}\right), 1 - \frac{\max\left(\sqrt[n]{(1-\gamma_A^0(x))^n}, \sqrt[n]{(1-\gamma_B^0(x))^n}\right)}{\sqrt[n]{\max\left((\mu_A^1(x))^n, (\mu_B^1(x))^n\right)}} \right\} \mid x \in X
\]
From (3.3) and (3.4), we get
\[
\begin{align*}
\left\{ x, \left( 1 - \sqrt[n]{1 - (\mu_A^1(x))^n}, 1 - \sqrt[n]{1 - (\mu_A^0(x))^n} \right) \right\} \cap X \}
\end{align*}
\]
Hence (a) is proved and similarly (b), (c) & (d) is proved by analogy.

**Theorem 3.6:** For every IFS \( A \) and for every natural number \( n \geq 1 \):

(a). \( \frac{1}{n} A^n = \frac{1}{n} A^n(b) \). \( \frac{1}{n} A^n = \frac{1}{n} \cdot A^n \)

(c). \( A^n \cap \frac{1}{n} A^n = n \cdot A^n \cdot \frac{1}{n} \cdot A^n \)

**Proof:** (a).
\[
\begin{align*}
\frac{1}{n} A^n &= \frac{1}{n} \left\{ \left[ x, \left( \frac{1}{n} \left( \sqrt[n]{1 - (\mu_A^1(x))^n}, 1 - \sqrt[n]{1 - (\mu_A^0(x))^n} \right) \right) \right] \cap X \}
\end{align*}
\]
\[
\begin{align*}
\frac{1}{n} A^n &= \frac{1}{n} \left\{ \left[ x, \left( \frac{1}{n} \left( \sqrt[n]{1 - (\mu_A^1(x))^n}, 1 - \sqrt[n]{1 - (\mu_A^0(x))^n} \right) \right) \right] \cap X \}
\end{align*}
\]
From (3.3) and (3.4), we get
\[
\frac{1}{n} A^n = \frac{1}{n} \cdot A^n
\]
Hence (a) is proved and similarly (b), (c) & (d) is proved by analogy.

**Theorem 3.7:** For any two IVIFSs \( A \) and \( B \) and for every natural number \( n \geq 1 \):

(a). \( \frac{1}{n} (A^n + B^n) = \frac{1}{n} A^n + \frac{1}{n} B^n \), \( \frac{1}{n} (A^n \cdot B^n) = \frac{1}{n} A^n \cdot \frac{1}{n} B^n \)

(c). \( n (A^n \cap B^n) = n A^n \cap n B^n \), \( n (A^n \cdot B^n) = n A^n \cdot n B^n \)

**Proof:** (a).
\[
\begin{align*}
\frac{1}{n} (A^n + B^n) &= \frac{1}{n} \left\{ \left[ x, \left( \frac{1}{n} \left( \sqrt[n]{1 - (\mu_A^1(x))^n}, 1 - \sqrt[n]{1 - (\mu_A^0(x))^n} \right) \right) \right] \cap X \}
\end{align*}
\]
Hence (a) is proved and similarly (b), (c)&(d) is proved by analogy.

**Theorem 3.8:** For every IVIFS A and for every natural number \( n \geq 1 \):

\[(a) \quad n \left( \frac{1}{n} A^n \right) = A^n (b) \quad \frac{1}{n} (n A^n) = A^n \]

**Proof:** (a).

\[
n \left( \frac{1}{n} A^n \right) = n \left\{ \frac{1}{n} \left[ x, \left( \left( \mu_A^1(x) \right)^n, \left( \mu_A^2(x) \right)^n \right), \left( (1 - \gamma_A^1(x))^n, 1 - \left( 1 - \gamma_A^2(x) \right)^n \right) \right] | x \in X \right\} = n \left\{ x, \left( 1 - \frac{1}{n} \left( \mu_A^1(x) \right)^n, 1 - \frac{1}{n} \left( \mu_A^2(x) \right)^n \right), \left( \frac{1}{n} (1 - \gamma_A^1(x))^n, \frac{1}{n} (1 - \gamma_A^2(x))^n \right) \right| x \in X \right\} = n + \frac{n}{n} \left( \frac{1}{n} A^n \right) = A^n
\]
\[ L^2 S. Sudharsan*, D. Ezhilmaran / \\
Some Operators Defined Over Intuitionistic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets / IJMA- 6(10), Oct.-2015. \]

\[
\left\{ \begin{array}{c}
\{ x, \left( 1 - \left( \frac{1}{1 - \left( \mu^b_A(x) \right)^n} \right), \left( 1 - \left( \frac{1}{1 - \left( \mu_A^b(x) \right)^n} \right) \right) \right\} \\
\{ x, \left( 1 - \left( \frac{1}{1 - \left( \nu^b_A(x) \right)^n} \right), \left( 1 - \left( \frac{1}{1 - \left( \nu_A^b(x) \right)^n} \right) \right) \right\} \\
\{ x, \left( 1 - \left( \frac{1}{1 - \left( \gamma^b_A(x) \right)^n} \right), \left( 1 - \left( \frac{1}{1 - \left( \gamma_A^b(x) \right)^n} \right) \right) \right\}
\end{array} \right. \\
\text{for } x \in X
\]

Hence (a) is proved and similarly (b) is proved by analogy.

REFERENCES

19. Vasiliev,T, Four equalities connected with intuitionistic fuzzy sets, Notes on Intuitionistic Fuzzy Sets, 14 (3) (2008), 1-4

Source of support: Nil, Conflict of interest: None Declared

[Copyright © 2015. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]