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A CLASSIFICATION OF NEARLY LORENTZIAN SASAKIAN MANIFOLD

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ABSTRACT

In 2011, R. Nivas and A. Bajpai [4] studied on generalized Lorentzian Para-Sasakian manifolds. Hayden [1] introduced the idea of metric connection with torsion tensor in a Riemannian manifold. In 1970, K.Yano [6] studied on semi-symmetric metric connections and their curvature tensors. In 1980, R. S. Mishra and S. N. Pandey [2] discussed on quarter-symmetric metric F-connection. In 1992, Nirmala S. Agashe and Mangala R. Chafle [3] studied semi-symmetric non-metric connection in a Riemannian manifold. In this paper, generalized nearly Sasakian and generalized nearly special Sasakian manifolds have been discussed and some of their properties have been established with generalized Co-symplectic manifolds. Semi-symmetric metric F-connection in a generalized special Sasakian manifold has also been studied.

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Keywords: *Generalized nearly Lorentzian Sasakian manifold, generalized nearly Lorentzian Special Sasakian manifold, generalized Lorentzian Co-symplectic manifolds and semi-symmetric metric F-connection.*

1. INTRODUCTION

An n *(=2m+1)* dimensional differentiable manifold M_n, on which there are defined covariant vector fields A_i, where $i = 3,4,5, \dots .. (n-1)$, the associated contravariant vector fields T_i, where $i = 3,4,5, \dots .. (n-1)$, a tensor field $F$of type (1, 1) and a Lorentzian metric g, satisfying

(1.1) $F^2 = -I_n - \sum_{i=3}^{n-1} A_i \otimes T_i, \quad FT_i = 0, \quad A_i(T_i) = -1, \quad A_i(FX) = 0,$ Rank $F = n - i$

(1.2) $g\,(FX, FY) = g\,(X, Y) + \sum_{i=3}^{n-1} A_i(X)\, A_i(Y),$ where $A_i(X) = g(X, T_i), i = 3,4,5, \dots .. (n-1),$

$`F(X, Y) \stackrel{\text{def}}{=} g(\,FX, Y) = -`F(Y, X),$

Then M_n is called a generalized Lorentzian contact manifold (a generalized L-contact manifold) and the structure $(F,\ T_i,\ A_i,\ g\ )$ is known as generalized L-contact structure [5]

Let D be a Riemannian connection on M_n, then we have

(1.3) (a) $(D_X `F)(FY, Z) - (D_X `F)(Y, FZ) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(Z) + \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(Y) = 0$

(b) $(D_X `F)(\,FY, F^2 Z) = (D_X `F)(F^2 Y\,, FZ)$

A generalized L-contact manifold is called a generalized Lorentzian Sasakian manifold, if

(1.4) (a) $i(D_X F)(Y) - F^2 X \sum_{i=3}^{n-1} A_i(Y) \ - g(\,FX, FY) \sum_{i=3}^{n-1} T_i \ = 0 \Leftrightarrow$

(b) $i(D_X `F)(Y, Z) + g(\,FX, FZ) \sum_{i=3}^{n-1} A_i(Y) \ - g(\,FX, FY) \sum_{i=3}^{n-1} A_i(Z) \ = 0 \Leftrightarrow$

(c) $iD_X T_i = FX + T_i - \sum_{i=3}^{n-1} T_i,$

From which, we get

(1.5) (a) $i(D_X A_i)(FY) = g(\,FX, FY) \Leftrightarrow$

(b) $i(D_X A_i)(\,Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = `F(X, Y)$

A generalized L-contact manifold is called a generalized Lorentzian Special Sasakian manifold (a generalized LS-Sasakian manifold), if

(1.6) (a) $i(D_X F)(Y) + FX \sum_{i=3}^{n-1} \ A_i(Y) \ - `F(X, Y) \sum_{i=3}^{n-1} T_i \ = 0 \Leftrightarrow$

(b) $i(D_X `F)(Y, Z) + \ `F(X, Z) \sum_{i=3}^{n-1} A_i(Y) \ - \ `F(X, Y) \sum_{i=3}^{n-1} A_i(Z) \ = 0 \Leftrightarrow$

(c) $iD_X T_i = F^2 X + T_i - \sum_{i=3}^{n-1} T_i$

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From which, we get

(1.7) (a) $i(D_X A_i)(FY) = `F(X,Y) \Leftrightarrow$

(b) $i(D_X A_i)(Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = -g(FX,FY)$

A generalized L-contact manifold is called a generalized Lorentzian Co-symplectic manifold, if

(1.8) (a) $(D_X F)Y - \sum_{i=3}^{n-1} A_i(Y)FD_X T_i - \sum_{i=3}^{n-1}(D_X A_i)(FY)T_i = 0 \Leftrightarrow$

(b) $(D_X `F)(Y,Z) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(FZ) - \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(FY) = 0$

Therefore, a generalized Lorentzian Co-symplectic manifold will be a generalized Lorentzian Sasakian manifold, if

(1.9) (a) $i(D_X A_i)(FY) = g(FX,FY) \Leftrightarrow$ (b) $i(D_X A_i)(Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = `F(X,Y) \Leftrightarrow$

(c) $iD_X T_i = FX + T_i - \sum_{i=3}^{n-1} T_i$

And a generalized Lorentzian Co-symplectic manifold will be a generalized LS-Sasakian manifold, if

(1.10) (a) $i(D_X A_i)(FY) = `F(X,Y) \Leftrightarrow$

(b) $i(D_X A_i)(Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = -g(FX,FY) \Leftrightarrow$ (c) $iD_X T_i = F^2X + T_i - \sum_{i=3}^{n-1} T_i$

Nijenhuis tensor in a generalized almost contact metric manifold is given by

(1.11) $`N(X,Y,Z) = (D_{FX} `F)(Y,Z) - (D_{FY} `F)(X,Z) + (D_X `F)(Y,FZ) - (D_Y `F)(X,FZ)$

Where $`N(X,Y,Z) \stackrel{\text{def}}{=} g(N(X,Y),Z)$

2. GENERALIZED NEARLY LORENTZIAN SASAKIAN MANIFOLD

A generalized L-contact manifold is called a generalized nearly Lorentzian Sasakian manifold, if

(2.1) $i(D_X `F)(Y,Z) + \sum_{i=3}^{n-1} A_i(Y)g(FZ,FX) - \sum_{i=3}^{n-1} A_i(Z)g(FX,FY)$

$= i(D_Y `F)(Z,X) + \sum_{i=3}^{n-1} A_i(Z)g(FX,FY) - \sum_{i=3}^{n-1} A_i(X)g(FY,FZ)$

$= i(D_Z `F)(X,Y) + \sum_{i=3}^{n-1} A_i(X)g(FY,FZ) - \sum_{i=3}^{n-1} A_i(Y)g(FZ,FX)$

From which, we get

(2.2) (a) $i(D_X F)Y + i(D_Y F)X - \sum_{i=3}^{n-1} A_i(Y)F^2X - \sum_{i=3}^{n-1} A_i(X)F^2Y - 2\sum_{i=3}^{n-1} T_i\, g(FX,FY) = 0 \Leftrightarrow$

(b) $i(D_X `F)(Y,Z) + i(D_Y `F)(X,Z) + \sum_{i=3}^{n-1} A_i(Y)g(FX,FZ) + \sum_{i=3}^{n-1} A_i(X)g(FY,FZ) -$
$2\sum_{i=3}^{n-1} A_i(Z)g(FX,FY) = 0$

From which, we get

(2.3) (a) $i(D_X F)FY + i(D_{FY} F)X + \sum_{i=3}^{n-1} A_i(X)FY + 2\sum_{i=3}^{n-1} T_i\, `F(X,Y) = 0 \Leftrightarrow$

(b) $i(D_X `F)(FY,Z) - i(D_{FY} `F)(Z,X) + \sum_{i=3}^{n-1} A_i(X)\, `F(Y,Z) + 2\sum_{i=3}^{n-1} A_i(Z)\, `F(X,Y) = 0$

(2.4) (a) $i(D_X F)F^2Y + i(D_{F^2Y} F)X + \sum_{i=3}^{n-1} A_i(X)F^2Y + 2\sum_{i=3}^{n-1} T_i\, g(FX,FY) = 0 \Leftrightarrow$

(b) $i(D_X `F)(F^2Y,Z) - i(D_{F^2Y} `F)(Z,X) - \sum_{i=3}^{n-1} A_i(X)g(FY,FZ) + 2\sum_{i=3}^{n-1} A_i(Z)g(FX,FY) = 0$

(2.5) (a) $(D_X F)Y + (D_Y F)X - \sum_{i=3}^{n-1} A_i(Y)\{FD_X T_i - (D_{T_i} F)X\} - \sum_{i=3}^{n-1} A_i(X)\{FD_Y T_i - (D_{T_i} F)Y\}$
$- \sum_{i=3}^{n-1} T_i\{(D_X A_i)(FY) + (D_Y A_i)(FX)\} = 0 \Leftrightarrow$

(b) $(D_X `F)(Y,Z) + (D_Y `F)(X,Z) + \sum_{i=3}^{n-1} A_i(Y)\{(D_X A_i)(FZ) - (D_{T_i} `F)(Z,X)\}$
$+ \sum_{i=3}^{n-1} A_i(X)\{(D_Y A_i)(FZ) + (D_{T_i} `F)(Y,Z)\} - \sum_{i=3}^{n-1} A_i(Z)\{(D_X A_i)(FY) + (D_Y A_i)(FX)\} = 0$

Pre-multiplying X, Y, Z by F in (1.11) and using equations (2.1), (1.3) (b), we see that a generalized nearly Lorentzian Sasakian manifold will be completely integrable, if

(2.6) $(D_{\overline{X}} `F)\left(\overline{Y}, \overline{\overline{Z}}\right) = (D_{\overline{Y}} `F)(\overline{X}, \overline{\overline{Z}})$

3. GENERALIZED NEARLY LORENTZIAN SPECIAL SASAKIAN MANIFOLD

A generalized L-contact manifold is called a generalized nearly Lorentzian Special Sasakian manifold (a generalized nearly LS-Sasakian manifold), if

(3.1) $i(D_X `F)(Y,Z) - \sum_{i=3}^{n-1} A_i(Y) `F(Z,X) - \sum_{i=3}^{n-1} A_i(Z)\, `F(X,Y)$

$= i(D_Y `F)(Z,X) - \sum_{i=3}^{n-1} A_i(Z) `F(X,Y) - \sum_{i=3}^{n-1} A_i(X)\, `F(Y,Z)$

$= i(D_Z `F)(X,Y) - \sum_{i=3}^{n-1} A_i(X) `F(Y,Z) - \sum_{i=3}^{n-1} A_i(Y)\, `F(Z,X)$

From which, we obtain

(3.2) (a) $i(D_X F)Y + i(D_Y F)X + \sum_{i=3}^{n-1} A_i(Y)FX + \sum_{i=3}^{n-1} A_i(X)FY = 0 \Leftrightarrow$

(b) $i(D_X `F)(Y,Z) + i(D_Y `F)(X,Z) - \sum_{i=3}^{n-1} A_i(Y)\, `F(Z,X) + \sum_{i=3}^{n-1} A_i(X)\, `F(Y,Z) = 0$

This gives

(3.3) (a) $i(D_X F)FY + i(D_{FY} F)X + \sum_{i=3}^{n-1} A_i(X)F^2Y = 0 \Leftrightarrow$

(b) $i(D_X `F)(FY, Z) - i(D_{FY} `F)(Z, X) - \sum_{i=3}^{n-1} A_i(X)g(FY, FZ) = 0$

(3.4) (a) $i(D_X F)F^2Y + i(D_{F^2Y} F)X - \sum_{i=3}^{n-1} A_i(X)FY = 0 \Leftrightarrow$

(b) $i(D_X `F)(F^2Y, Z) - i(D_{F^2Y} `F)(Z, X) - \sum_{i=3}^{n-1} A_i(X) `F(Y, Z) = 0$

(3.5) (a) $(D_X F)Y + (D_Y F)X - \sum_{i=3}^{n-1} A_i(Y)\{FD_X T_i - (D_{T_i} F)X\} - \sum_{i=3}^{n-1} A_i(X)\{FD_Y T_i - (D_{T_i} F)Y\} = 0 \Leftrightarrow$

(b) $(D_X `F)(Y, Z) + (D_Y `F)(X, Z) + \sum_{i=3}^{n-1} A_i(Y)\{(D_X A_i)(FZ) - (D_{T_i} `F)(Z, X)\}$
$+ \sum_{i=3}^{n-1} A_i(X)\{(D_Y A_i)(FZ) + (D_{T_i} `F)(Y, Z)\} = 0$

Pre-multiplying X, Y, Z by F in (1.11) and using equations (3.1), (1.3) (b), we see that a generalized nearly LS-Sasakian manifold will be completely integrable, if

(3.6) $(D_{\overline{X}} `F)\left(\overline{\overline{Y}}, \overline{Z}\right) + (D_{\overline{Y}} `F)\left(\overline{Z}, \overline{\overline{X}}\right)$

4. GENERALIZED NEARLY LORENTZIAN CO-SYMPLECTIC MANIFOLD

A generalized L-contact manifold is called a generalized nearly Lorentzian Co-symplectic manifold, if

(4.1) $(D_X `F)(Y, Z) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(FZ) - \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(FY)$
$= (D_Y `F)(Z, X) + \sum_{i=3}^{n-1} A_i(Z)(D_Y A_i)(FX) - \sum_{i=3}^{n-1} A_i(X)(D_Y A_i)(FZ)$
$= (D_Z `F)(X, Y) + \sum_{i=3}^{n-1} A_i(X)(D_Z A_i)(FY) - \sum_{i=3}^{n-1} A_i(Y)(D_Z A_i)(FX)$

Therefore, a generalized nearly Lorentzian Sasakian manifold will be a generalized nearly Lorentzian Co-symplectic manifold, if

(4.2) (a) $i(D_X A_i)(FY) = g(FX, FY) \Leftrightarrow$

(b) $i(D_X A_i)(Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = `F(X, Y) \Leftrightarrow$ (c) $iD_X T_i = FX + T_i - \sum_{i=3}^{n-1} T_i$

And a generalized nearly LS-Sasakian manifold will be a generalized nearly Lorentzian Co-symplectic manifold, if

(4.3) (a) $i(D_X A_i)(FY) = `F(X, Y) \Leftrightarrow$

(b) $i(D_X A_i)(Y) - A_i(Y) + \sum_{i=3}^{n-1} A_i(Y) = -g(FX, FY) \Leftrightarrow$ (c) $iD_X T_i = F^2X + T_i - \sum_{i=3}^{n-1} T_i$

5. SEMI-SYMMETRIC METRIC F-CONNECTION IN A GENERALIZED LS-SASAKIAN MANIFOLD

Let M_{2m-1} be submanifold of M_{2m+1} and let $c : M_{2m-1} \to M_{2m+1}$ be the inclusion map such that

$$d \in M_{2m-1} \to cd \in M_{2m+1},$$

Where c induces a Jacobian map (linear transformation) $J : T'_{2m-1} \to T'_{2m+1}$.

T'_{2m-1} is tangent space to M_{2m-1} at point d and T'_{2m+1} is tangent space to M_{2m+1} at point cd such that

$$\hat{X} \text{ in } M_{2m-1} \text{ at } d \to J\hat{X} \text{ in } M_{2m+1} \text{ at } cd$$

Let $\tilde{g}$ be the induced Lorentzian metric in M_{2m-1}, then

(5.1) $\tilde{g}(\hat{X}, \hat{Y}) = ((g(J\hat{X}, J\hat{Y}))b$

Semi- symmetric metric F-connection B in generalized Lorentzian special Sasakian manifold M_n is given by

(5.2) $iB_X Y = iD_X Y - \sum_{i=3}^{n-1} A_i(Y)X + \sum_{i=3}^{n-1} g(X, Y)T_i - 2\sum_{i=3}^{n-1} A_i(X)Y$

Where X and Y are arbitrary vector fields of M_{2m+1}. Let

(5.3) $T_i = Jt_i + \rho_i M + \sigma_i N$, where $i = 3, 4, 5, \ldots\ldots (n-1)$.

Where t_i, $i = 3, 4, 5, \ldots\ldots (n-1)$, are C^∞ vector fields in M_{2m-1}. M, N are unit normal vectors to M_{2m-1}.

Denoting by $\grave{D}$ the connection induced on the submanifold from D. Gauss equation is

(5.4) $D_{JX} J\hat{Y} = J(\grave{D}_X \hat{Y}) + p(\hat{X}, \hat{Y})M + q(\hat{X}, \hat{Y})N$

Where p and q are symmetric bilinear functions in M_{2m-1}. Also

(5.5) $B_{JX} J\hat{Y} = J(\grave{B}_X \hat{Y}) + h(\hat{X}, \hat{Y})M + k(\hat{X}, \hat{Y})N$,

Where $\grave{B}$ is the connection induced on the submanifold from B and h, k are symmetric bilinear functions in M_{2m-1}.

Inconsequence of (5.2), we have

(5.6) $iB_{JX} J\hat{Y} = iD_{JX} J\hat{Y} - \sum_{i=3}^{n-1} A_i(J\hat{Y})J\hat{X} + \sum_{i=3}^{n-1} g(J\hat{X}, J\hat{Y})T_i - 2\sum_{i=3}^{n-1} A_i(J\hat{X})J\hat{Y}$

Using (5.4), (5.5) and (5.6), we have

(5.7) $iJ(\grave{B}_X\hat{Y}) + ih(\hat{X},\hat{Y})M + ik(\hat{X},\hat{Y})N = iJ(\grave{D}_X\hat{Y}) + ip(\hat{X},\hat{Y})M + iq(\hat{X},\hat{Y})N - \sum_{i=3}^{n-1} A_i(J\hat{Y})J\hat{X} + \sum_{i=3}^{n-1} g(J\hat{X},J\hat{Y})T_i - 2\sum_{i=3}^{n-1} A_i(J\hat{X})J\hat{Y}$

Using (5.3), we get

(5.8) $iJ(\grave{B}_X\hat{Y}) + ih(\hat{X},\hat{Y})M + ik(\hat{X},\hat{Y})N = iJ(\grave{D}_X\hat{Y}) + ip(\hat{X},\hat{Y})M + iq(\hat{X},\hat{Y})N - \sum_{i=3}^{n-1} a_i(\hat{Y})J\hat{X} + \sum_{i=3}^{n-1}(Jt_i + \rho_i M + \sigma_i N)\,\tilde{g}(\hat{X},\hat{Y}) - 2\sum_{i=3}^{n-1} a_i(\hat{X})J\hat{Y}$

Where $\tilde{g}(\hat{Y},t_i) \overset{\text{def}}{=} a_i(\hat{Y})$

This implies

(5.9) $i\grave{B}_X\hat{Y} = i\grave{D}_X\hat{Y} - \sum_{i=3}^{n-1} a_i(\hat{Y})\hat{X} + \sum_{i=3}^{n-1} \tilde{g}(\hat{X},\hat{Y})t_i - 2\sum_{i=3}^{n-1} a_i(\hat{X})\hat{Y}$

Iff

(5.10) (a) $ih(\hat{X},\hat{Y}) = ip(\hat{X},\hat{Y}) + \sum_{i=3}^{n-1} \rho_i\tilde{g}(\hat{X},\hat{Y})$

(b) $ik(\hat{X},\hat{Y}) = iq(\hat{X},\hat{Y}) + \sum_{i=3}^{n-1} \sigma_i\tilde{g}(\hat{X},\hat{Y})$

Thus, we have

Theorem 5.1: The connection induced on a submanifold of a generalized Lorentzian special Sasakian manifold with a Semi-symmetric metric F-connection with respect to unit normal vectors M and N is also Semi- symmetric metric F-connection iff (5.10) holds.

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