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STRONG NONSPLIT LINE SET DOMINATING NUMBER OF GRAPH

P. SOLAI RANI*[1], Mrs. R. POOVAZHAKI[2]

[1]Assistant Professor, Department of Mathematics,
R. V. S. College of Arts and Science, Sulur, India.

[2]Principal & Head, E. M. G. Yadava women's College, Madurai, India.

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ABSTRACT

I. H. Nagaraja Rao & B. Vijayalakmi introduced the concept of Line set domination number of a Graph and derived results parallel to those of E.Sampathkumar and L. Pushpalatha. Let G be a graph. A set F⊆E(G) is a line set dominating set (LSD-set) of G. if for each set L ⊆ E –F, there exists an edge e in F such that the subgraph {< L∪{e}>} induced by L∪{e} is connected. (ϑ'_l (G)). V.R. Kulli and B.Janakiram introduced the concept of non-split domination number of a graph and derived results parallel. A dominating set F of a graph of G is a strong nonsplit dominating set if the induced subgraph (E-F) is complete. The Stong nonsplit domination number (γsns(G)) of G is the minimum cardinality of strong nonsplit dominating set. Combining the two concepts Strong nonsplit line set dominating set is introduced as follows, A line set dominating set L⊆E(G) of a graph G=(V,E) is said to be a Strong nonsplit line set dominating set. if the induced subgraph <E-L> is complete. A Strong nonsplit line set dominating number $\vartheta'snsl$ (G) of G is the minimum cardinality of a Strong nonsplit line set dominating set. In this paper we analyse the dominating parameters corresponding to Strong nonsplit line set dominating set and obtain the some bounds and some exact value for ((ϑ' snsl (G).

Keywords: *Line set dominating set, Strong Non-split dominating set, Strong Non-split line set dominating number.*

AMS Subject Classification: *O5C69.*

INTRODUCTION

Domination is an active subject in graph theory. All the graphs considered here are assumed to be finite, undirected, and nontrivial and connected without loops or multiple edges.

Let G = (V, E) be a graph. A set D⊆V(*G*) of vertices in a graph G =(V, E) is a dominating set. if every vertex in V-D is adjacent to some vertex in D. The domination number γ(G) of G is the minimum cardinality of dominating set in G.

A dominating set S is called an independent set if no two vertices of *S* are adjacent. The independent domination number γi (*G*) (or *i* for short) of *G* is the minimum cardinality taken over all independent or dominating set of *G*.

A set F⊆E is an edge dominating set of G. iff every edge in E-F is adjacent to some edge in F. The edge domination number ϑ'(G) of G is the minimum of cardinalities of all edge dominating sets of G. (Hedetniemi & Laskar).

A line graph *L*(*G*) is the graph whose vertices corresponds to the edges of *G* and two vertices in *L*(*G*) are adjacent if and only if the corresponding edges in *G* are adjacent (that is, are incident with a common cut vertex). (V(L(G))=q).

The connectivity number λ(G) is defined as the minimum number of edges whose removal disconnects the remaining graph, that is G-e is a disconnected graph. If degv=1, then v is called an end vertex. An edge incident with an end vertex is called an end edge. The degree an edge e=uv of G is defined by deg(e)=deg(u)+deg(v)-2. The maximum and minimum degree among the edge of graph G is denote by Δ'(G) & δ'G) (The degree of an edge is the number of edges adjacent to it).

Corresponding Author: P. Solai Rani[1]*
[1]Assistant Professor, Department of Mathematics, R. V. S. College of Arts and Science, Sulur, India.

A dominating set D of a graph G= (V, E) is a nonsplit dominating set if the induced subgraph <V-D> is connected. The nonsplit domination number γ_{ns} (*G*) of G is the minimum cardinality of a nonsplit dominating set. A dominating set D of a graph G= (V, E) is a strong nonsplit dominating set if the induced subgraph (V-D) is complete. A dominating set D of a connected graph G is a global nonsplit dominating set if D is a nonsplit dominating are both G & $\bar{G}$. A dominating set D of a graph G is called a clique dominating set of G if <D> is complete.

E.Sampathkumar and L.Pushpalatha introduced the concept of Point set domination set as follows,

A set D of vertices is a Point set dominating set (psd-set) of G. if for each set S⊆V-D, there exists a vertex set v in D such that the subgraph$<S\cup\{v\}>$ induced by $S\cup\{v\}$ is connected. The point set domination number $\gamma_p(G)$ of G is defined as the minimum cardinality of a psd-set of G.

I.H.Nagaraja Rao & B.Vijayalakmi introduced the concept of Line set domination set and derived results parallel to those of E.Sampathkumar and L.Pushpalatha.

Let G be a graph. A set F⊆E is a line set dominating set (LSD-set) of G. if for each set L ⊆ E –F, there exists an edge e in F such that the subgraph $\{< L\cup \{e\}>\}$ induced by $L\cup \{e\}$ is connected . The line set domination number (LSD-number) $\boldsymbol{\vartheta}_l'$ (G) of G is the minimum cardinalities of all LSD-Set of G.

For any edge e, let N'(e) = {f∈E: e and f have a vertex in common}.

For a se F⊆E let N'(F) = ∪ N'(e) for all e∈F.

Dominating sets whose complements induce a complete subgraph have a great diversity of applications. One such application is the following, In setting up the communication links in a network one might want a strong core group that can communicate with each other member of the core group and so that everyone in the group receives the message from someone outside the group and communicate it to every other in the group.

Definition: A line set dominating set L⊆ E(*G)* of a graph G=(V,E) is said to be a Strong nonsplit line set dominating set. if the induced subgraph <E-L> is complete. A Strong nonsplit line set dominating number $\boldsymbol{\vartheta}'$snsl (G) of G is the minimum cardinality of a nonsplit line set dominating set.

RESULTS

Theorem 1: For any graph G

$$\boldsymbol{\vartheta}'(G)) \leq (\boldsymbol{\vartheta}'_l (G)) \leq \boldsymbol{\vartheta}'nsl (G) \leq \boldsymbol{\vartheta}'snsl (G)$$

Proof: Since every strong nonsplit line set dominating set of G is a nonsplit line set dominating set and every nonsplit line set dominating set is a line set dominating set and line set dominating set is an edge dominating set of G. Hence

$$(\boldsymbol{\vartheta}' (G)) \leq (\boldsymbol{\vartheta}_1'(G)) \leq \boldsymbol{\vartheta}'nsl (G) \leq \boldsymbol{\vartheta}'snsl (G).$$

Theorem 2: If H is a connected spanning subgraph of G, then

$$\boldsymbol{\vartheta}' snsl (H) \leq \boldsymbol{\vartheta}'snsl (G)$$

Proof: Since every strong nonsplit line set dominating set of G is a strong n.onsplit line set dominating set of H. that is

$$\boldsymbol{\vartheta}'snsl (H) \leq \vartheta'snsl (G).$$

In the next result, we list the exact values of $\boldsymbol{\vartheta}'$snsl (G) for some standard graphs.

Observation 3: For any path P_p

$$\vartheta_{snsl} (P_p) = p - 2, \text{ for any } p \geq 3.$$

Observation 4: For any cycle C_p

$$\boldsymbol{\vartheta}'_{snsl} (C_p) = 2, \quad \text{for } p = 3 .$$
$$\boldsymbol{\vartheta}'_{snsl} (C_p) = p-1 \quad \text{for any } p\geq 4 .$$

Observation 5: For any complete graph K_p

$$\boldsymbol{\vartheta}'snsl (K_p) = p-1 \text{ for any positive integer } p\geq 3.$$

Observation 6: For any star $K_{1,p}$

$$\boldsymbol{\vartheta}'_{snsl}(K_{1,p}) = p - 2 \text{ for any } p \geq 2.$$

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Observation 7: If $K_{n,m}$ is a complete bi-partite graph of G for any two positive integer $2 \le m \le n$ vertices, then

$$\vartheta'_{snsl}(K_{n,m}) = mn-1.$$

Observation 8: For any wheel graph W_p

$$\vartheta'_{snsl}(W_p) = 2p-5 \text{ for } p \ge 4.$$

In the next result, Strong non split line set dominating set in graph G is the Point set dominating number of the line graph L (G).

Observation 9: For any path P_n, for any positive integer n≥5 vertices.

$$\vartheta'_{snsl}(P_n) = \gamma_p(L(P_n)) = \gamma_p(P_{n-1}) = p-2.$$

Observation 10: For any cycle C_n, for any positive integer n≥5 vertices

$$\vartheta'_{snsl}(C_n) = \gamma_p(L(C_n)) = \gamma_p(C_n).$$

Observation 11: For any star $K_{1,n}$ with $q \ge 2$.

$$\vartheta'_{snsl}(K_1,n) = \gamma_p(L(K_{n-1})).$$

Theorem12: Let G be a graph. Then

$$\frac{q}{\Delta'(G)+1} \le \vartheta'_{snsl}(L(G) \le q-\delta'(G).$$

Proof: Let e in E(G) with deg (e)= δ'(G). Now without loss of generality by defn. of line graph e=u in V(L(G)) and Let J be a strong nonsplit line set dominationset of L(G). Such that $|J| = \vartheta'_{snsl}(L(G)$. If δ'(G) ≤ 2 Then

$$\vartheta'_{snsl}(L(G) \le q-2,$$

$$\vartheta''_{snsl}(L(G) \le q - \delta'(G). \qquad (A)$$

If δ'(G) > 2,Then for any edge f in N(e) and by defn. of L(G) f=w∈N(u), that is

$$J \subseteq (V(L(G)) - N(u)) \cup \{w\}.$$

Hence

$$\vartheta'_{snsl}(L(G) \le q - (\delta'(G)+1)+1$$

$$\le q - \delta'(G).$$

Now for the lower bound $|V(L(G)-D| \le \sum deg(e)$ (e∈J(G) and deg(e) ≤Δ'(G). We have

$$q - \vartheta'_{snsl}(L(G) = |V(L(G)-D| \le \sum deg(e)$$

$$\le \vartheta'_{snsl}(L(G)\,(\Delta'(G))$$

$$q \le \vartheta'_{snsl}(L(G)\,(\Delta'(G)) + \vartheta'_{snsl}(L(G)$$

$$\le \vartheta'_{snsl}(L(G)((\Delta'(G)+1)$$

$$\frac{q}{\Delta'(G)+1} \le \vartheta'_{snsl}(L(G) \qquad (B)$$

Hence

$$\frac{q}{\Delta'(G)+1} \le \vartheta'_{snsl}(L(G) \le q-\delta'(G).$$

BOUNDS ON ϑ'_{snsl}

Theorem 13: For any graph G, $q - w(G) \le \vartheta'_{snsl}(G)$, where w(G) is the clique number of G.

Proof: Let F be a $\vartheta'_{snsl}(G)$ –set of G. Since <E-F> is complete.

$$w(G) \ge |E-F|$$

$$w(G) \ge E - \vartheta'_{snsl}(G) \ge q - \vartheta'_{snsl}(G)$$

$$q - w(G) \le \vartheta'_{snsl}(G)$$

$$\vartheta'_{snsl}(G) \ge q-w(G)$$

Corollary13.1: Let G be a graph with $w(G) \geq \delta'(G)$. Then

$$q - \delta'(G) \leq \vartheta'_{snsl}(G)$$

where $\delta'(G)$ is the minimum degree of G. Further the bound is attained iff one of the following conditions is satisfied.

i) $w(G) = \delta'(G)$
ii) $w(G) = \delta'(G) + 1$ and every w-set M contains a edge not adjacent to any edge of E-M.

Proof: Suppose $w(G) \geq \delta'(G)+1$, Then from $q - w(G) \leq \vartheta'_{snsl}(G) \Rightarrow q - \delta'(G) \leq \vartheta_{snsl}(G))$.

Suppose $w(G) = \delta'(G)$, and let T be a w-set of G. Then E-T is a strong nonsplit line set dominating set of G and hence $\vartheta'_{snsl}(G) \leq q - \delta'(G)$. Now we prove the second part, Suppose one of the given condition is satisfied. Then from $q - w(G) \leq \vartheta'_{snsl}(G)$. It easy to see that $q - \delta'(G) \leq \vartheta'_{snsl}(G)$. Suppose the bound is attained. Then again from $q - w(G) \leq \vartheta'_{snsl}(G)$, $w(G) = \delta(G)$ or $w(G) = \delta'(G)+1$. Suppose there exists a w-set S with $|S| = \delta'(G)+1$ such that every edge in S is adjacent to some edge in E-T is a strong nonsplit

line set dominating set of G and hence $\vartheta'_{snsl}(G) \leq q - \delta'(G)-1$, Which is a contradiction. Hence one of the given condition is satisfied.

Theorem 14: For any graph G,

$$\lambda_0(G) \leq \vartheta'_{snsl}(G)$$

where $\lambda_0(G)$ is the independent number of G.

Proof: Let L be a $\vartheta'_{snsl}(G)$ –set of G and H be an independent set of edges in G .Then either $H \subseteq L$ or H contains at most one edge from E-L and at most $|H|$ - 1 edge from L. This implies $\lambda_0(G) \leq \vartheta'_{snsl}(G)$.

Theorem 15: If $diam(G) \leq 3$, Then

$$\vartheta'_{snsl}(G) \leq q - m + 1, \text{ where m is the number of cut edge of G.}$$

Proof: If G has no cut edges, then the result is trivial. Let S be the set of all cut edges with $|S| = m$. Let e, f in S, Suppose e and f are not adjacent. Since two edges e_1 and f_1 ince such that e_1 is adjacent to e and f_1 is adjacent to f. it implies $d(e, f) \geq 2$ a contradiction. Hence every two edges in S are not adjacent and every edge in S is adjacent to atleast one edge in $(E\text{-}S) \cup \{e\}$ is a strong nonsplit line set dominating set of G.

$$\vartheta'_{snsl}(G) \leq (E\text{-}S) \cup \{e\}$$
$$\vartheta'_{snsl}(G) \leq q - m + 1.$$

Kulli and Janakiram introduced the following concept.

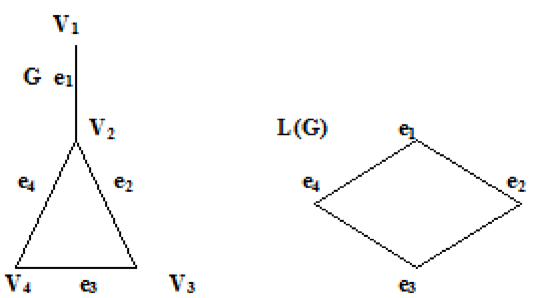
A dominating set D of a graph G = (V, E) is a regulator set dominating set if for any set $I \subseteq V\text{-}D$, there exists a set $S \subseteq D$ such that the induced subgraph $\langle I \cup D \rangle$ is regular. The regular set domination number $\gamma_{rs}(G)$ of G is the minimum cardinality of a regular set dominating set.

The Following Theorem Relates Regulator Set Dominating and Strong Nonsplit Line Set Domination Number in L(G)

Theorem 15: For any graph G

$$\gamma_{rs}(G) \leq \vartheta'_{snsl}(L(G)) + 1$$

Proof:



$R = \{e_2, e_3\}$, $(V(L(G) = q)$

$D = \{v_1, v_2\}$ $\vartheta'_{snsl}(L(G) = 2$

$\gamma_{rs}(G) = 2$

Let R be a ϑ'_{snsl} -set of L(G). Since <V(L(G) –R> is complete, for any e∈V(L(G) –R, R∪{e} is a regular set dominating set of G.

Hence $\gamma_{rs}(G) \le \vartheta'_{snsl}(L(G))+1$

The following definition is used to prove our next result.

A dominating set D of a graph G=(V, E) is an efficient dominating set if every vertex in V-D is adjacent to exactly one vertex in D. This concept was introduced by Cockayne *et.al.*

An edge subset E'⊆E is an efficient edge dominating set for G if each in E is dominated by exactly one edge in E'. Efficient edge dominating sets correspond to efficient dominating sets in the Line graph L(G). Thus, if G has an efficient dominating set, then its cardinality is $\gamma'_e(G) = \gamma_e(L(G),\ (V(L(G)) = q)$.

Theorem 16: Let G be a graph. If R is a efficient dominating set of L(G). Then V(L(G) – R is a strong nonsplit line set of L(G).

Proof: Since every vertex in V(L(G)) – R is adjacent to exactly one vertex in R. It implies that every two vertices in V(L(G)) – R is adjacent. As V(L(G)) – R is complete .Thus, V(L(G)) – R is a strong nonsplit line set dominating set.

The next relates to $\vartheta'_{nsl}(G)$ and $\vartheta'_{nsl}(\bar{G})$ where $\bar{G}$ is the complement of G.

Theorem 17: If $\Delta'(G) < \vartheta'_{sns}(G) + 1$, Then

$$v'snsl\left(\overline{G}\right) \le v'_{snsl}(G)+1$$

Where $\Delta'(G)$ and $\bar{G}$ *are* the maximum degree and the complement of G respectively and $\vartheta'_{snsl}(\bar{G})$ is an edge domination number.

Proof: Let F be a $\vartheta'_{snsl}(G)$ *of G*. Let H_1, H_2 be the subgraphs induced by F and E-F in G respectively. Then H_1 has no isolates and H_2 is complete. Thus

$$\vartheta'_{snsl}(\bar{G}) \le \vartheta_1'(H_1) + \vartheta'(H_2)$$
$$= \vartheta'_{snsl}(G) + 1$$

$$\vartheta' snsl\left(\overline{G}\right) \le \vartheta'_{snsl}(G) + 1$$

Theorem 18: Let G be a graph with $\Delta'(G) < \alpha_1(G)$. Then

$$\vartheta'_{snsl}(\bar{G}) \ge P- \lambda_0(G) + 1.$$

Proof: Let S be an edge cover of G with $|S| = \alpha_1(G)$. Since $\Delta'(G) < \alpha_1(G)$, $G \ne K_p$ and E-S is an independent set with atleast two edges. This proves that S∪{e} is a strong nonsplit line set dominating set of $\bar{G}$. Hence

$$F(\bar{G}) = |S\cup\{e\}|$$
$$= \alpha_1(G)+1, \qquad (\alpha_1(G) \ge (p- \lambda_0(G))$$
$$\ge (p- \lambda_0(G)) + 1$$

$$\vartheta'_{snsl}(\bar{G}) \ge p- \lambda_0(G) + 1.$$

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