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POINT-SET DOMINATION IN DIRECTED GRAPHS

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ABSTRACT

E.Sampathkumar and K.Pushpalatha introduced [8] the concept of point set domination number of graph. Let D be a connected digraph, a set $S \subseteq V(D)$ is a point set dominating set of D if for every set $R \subseteq V - S$ there exists a vertex $v \in S$ such that the subdigraph $< R \cup \{v\} >$induced by $R \cup \{v\}$ is weakly connected. The point set domination number $\gamma_{ps}(D)$ of D is minimum cardinality of point-set dominating set. In this paper we analyze the domination parameters corresponding to point-set domination in digraphs and obtain several results on these parameters.

Key words: *Dominating sets in digraphs, point-set dominating sets in digraphs, connected digraphs.*

AMS Subject Classification: *O5C69.*

1. INTRODUCTION

Throughout this paper $D=(V, A)$ is a finite directed graph with neither loops nor multiple arcs(but pairs of arcs are allowed) and $G=(V, E)$ is a undirected graph with neither loops nor multiple edges. For basic terminology on graphs and digraphs, we refer to Chartrand and Lesniak [2].

[6] Let $G=(V, E)$ be a graph.A subset *S*of *V* is called dominating set of *G* if every vertex in $V- S$ is adjacent to atleast one vertex in *S*.The minimum cardinality of dominating set of *G* is called domination number of *G* and is denoted by $\gamma(G)$.

[7] E.Sampthkumar and L.Pushpalatha has introduced the concept of point set domination in graphs. A set *D* of a vertices in a connected graph *G* is a psd-set if for every set $S \subseteq V - D$ there exists a vertex $v \in D$ such that subgraph $< S \cup \{v\} >$ induced by $S \cup \{v\}$ is connected. The point set domination number $\gamma_p(G)$ of *G* is the minimum cardinality of a psd-set.

Let $D=(V, A)$ be a digraph. A subset *S* of *V* is called a dominating set of *D* if for every vertex $v \in V- S$ there exists a vertex $u \in S$ such that $(u, v) \in A$. The domination number $\gamma(D)$ is the minimum cardinality of dominating set *D*. [5]

[1]Let $D=(V, A)$ be a digraph. For any vertex $u \in V$, the sets $O(u)=\{v/(u,v) \in A\}$ and $I(u)=\{v/(v, u) \in A\}$ are called outset and inset of *u*. The indegree and outdegree of *u* are defined by $id(u)=|I(u)|$ and $od(u)=|O(u)|$. The minimum indegree, the minimum outdegree, the maximum indegree and maximum outdegree of *D* is denoted by $\delta^-, \delta^+, \Delta^-$ and Δ^+ respectively.

[3, 5] An out- domination set of digraph D is a set S^+ of vertices of *D* such that every vertex of $V- S^+$ is adjacent from some vertex of *S*. The minimum cardinality of out-domination set for *D* is the out-domination number $\gamma^+(D)$.

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The in – domination number $\gamma^{-}(D)$ is defined as expected.

Although domination and other related concepts have extensively studied for undirected graphs, the respective analogues on digraphs have not received much attention.

The purpose of this paper is to introduce the concept of point-set domination in directed graphs

γ - set is the set of all vertices in dominating set with # $\gamma(D)$

γ^{+}- set is the set of all vertices in out dominating set with # $\gamma^{+}(D)$

γ^{-}- set is the set of all vertices in in-dominating set with # $\gamma^{-}(D)$

γ_{ps} - set is set of all vertices in point-set dominating set with # $\gamma_{ps}(D)$

$\gamma_{ps}{}^{+}$ - set is set of all vertices in point-set out dominating set with # $\gamma_{ps}{}^{+}(D)$

$\gamma_{ps}{}^{-}$ - set is set of all vertices in point-set in dominating set with # $\gamma_{ps}{}^{-}(D)$

Throughout this paper we are analyzing the results of out-domination number of point-set domination in directed graphs.

Definition: Let *D* be a connected digraph. A set $S^{+} \subseteq V(D)$ is a point set out-dominating set of *D* if for every set $R \subseteq V - S^{+}$, then there exists a vertex $v \in S^{+}$ such that the sub digraph<$R \cup \{v\}$> is induced by $R \cup \{v\}$ is weakly connected. The point set out-domination number $\gamma_{p}{}^{+}(D)$ of *D* is minimum cardinality of point set out-dominating set of digraphs.

For path P_n : $\gamma_{ps}{}^{+}(D) = n - 1$ or $n - \Delta^{+}$ for $n = 2, 3$

$\gamma_{ps}{}^{+}(D) = n - 2$ for $n \geq 4$

For Cycle: $\gamma_{ps}{}^{+}(D) = n - 1$ for $n=3$

$\gamma_{ps}{}^{+}(D) = n - 2$ for $n \geq 4$

For Tree: $\gamma_{ps}{}^{+}(T) = n - 1$ for $n=3$

$\gamma_{ps}{}^{+}(T) = n - 2$ for $n \geq 4$

Theorem 1: For any digraph *D*

$\gamma_{ps}{}^{+}(D) \leq n - \Delta^{+}$ where Δ^{+} is the maximum out degree of *D*.

Proof: Let $V - S^{+}$ be the set of vertices adjacent from and adjacent to the maximum degree in S^{+}.Then S^{+} is a point set dominating set.

Example: For any digraph *D*

Fig.-1

$S^{+} = \{V_{1,}V_3\}$, $V - S^{+} = \{V_2, V_4\}$

In this digraph V_2 and V_3 have maximum degree

$deg(V_2) = 2$, $deg(V_3) = 2$

V_2 is adjacent to maximum deg V_3

V_4 is adjacent from maximum degree V_3

Theorem 2: Let S^+ be a poin-set out –dominating set of digraph D and u, v$\in$ V-S^+ then d(u, v)$\le$2.

Proof: Let R= {u, v} then there exist a vertex x in S^+ such that the subdigraph < {u, v, x}> is unilaterally connected. This implies that d(u,v) $\le$2.

Example:
1. For path

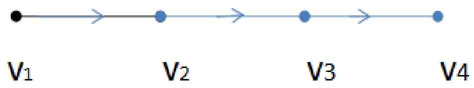

Fig.-2

S^+={V_1,V_3} V-S^+= { V_2,V_4}

By definition point-set out -dominating set < { V_2,V_3,V_4} > is unilaterally connected $d(V_2, V_4)$ =2

2. For complete digraph:

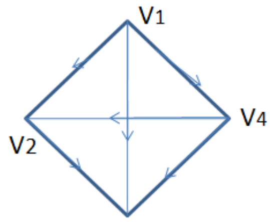

Fig.-3: S^+ = {V_1} V-S^+={V_2,V_3,V_4} R={V_2,V_3}

there exist $V_1 \in D$ such that {V_1,V_2,V_3} is unilaterally connected

Theorem 3: If a digraph D has an independent point-set out- dominating set then diam D$\le$ $n-1$

Proof: Suppose S^+ is an independent point-set out- dominating set of D. we consider 3 different cases.

Case-1: Let u, v $\in$V-S^+ .since S^+ is an independent point set out- dominating set both u and v are adjacent from a common vertex in S^+.

Case-2: Let u, v$\in$ S^+. Since D is unilaterally or strongly connected digraph and S^+ is an independent set Then there exist vertices $u_1 v_1 \in V - S^+$ such that $u_1\, u$ *and* $v_1\, v$ are edges.

Hence

Case-3: Let u $\in$ S^+ and v $\in$ V- S^+ then there exist $u_1 \in$ V- S^+ such that u_1 is adjacent from u and d(u, v) $\le$d(u,u_1) +d(u_1v)$\le$ $n-1$

Example: for cycle C_4

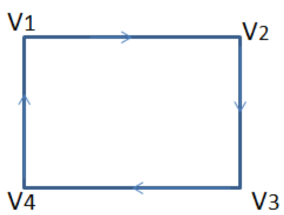

Fig.-4

Case-1: $\{v_1, v_3\} \in S^+$ $\{v_2, v_4\} \in V - S^+$, v_2 is adjacent to v_3 and v_4 is adjacent from v_3

Hence $d(v_2, v_4) \le 2$

Case-2: $u = v_1$ $v = v_3 \in S^+$ then there exists a vertices $\{v_2, v_4\} \in V - S^+$

Hence $d(v_1, v_3) \le 2 + d(v_4, v_1) \le n-1$

Case-3: Let $v_1 \in S^+$ and $v_4 \in V - S^+$ then there exists $v_2 \in V - S^+$
Then $d(v_1,v_4) \le d(v_1,v_2) + d(v_2,v_4) \le n-1$

Theorem 4: Let *D* be a digraph having cut vertices. Then there exist a γ_{ps}^+ -set of *D* containing all the cut vertices.

Proof: Let v be cutvertex and S^+ is a γ_{ps}^+ -set of D .If v $\not\in S^+$ then $D-V$ has exactly two components D_1 and D_2 such t1hat atleast one of digraphs $H_1 =< D_1 \cup \{v\} >$ or $H_1 =< D_1 \cup \{v\} >$ is a path.

Example:

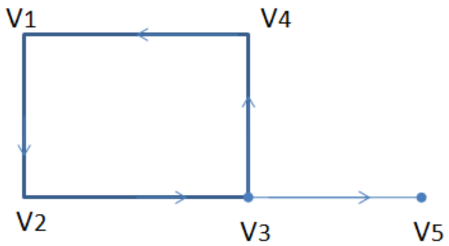

Fig.-5

$S^+ = \{v_1, v_3\}$ $V - S^+ = \{v_2, v_4, v_5\}$

v_3 is a cut vertex D_1 is a digraph with vertices { v_1, v_2, v_4 }

D_2 is digraph with isolated vertex v_5

Therefore $H_1 =< D_1 \cup \{v_3\} >$ *or* $H_1 =< D_1 \cup \{v\} >$ is a path.

Theorem 5: Let *D* be a strongly or unilaterally connected graph having cut vertices then $\gamma_{ps}^+(D) \le \gamma_{uc}^+(D)$.

Proof: By the theorem 4, there exists a γ_{ps}^+ -set of D containing all the cut vertices. If the subdigraph< S^+ > is unilaterally connected then S^+ is unilaterally connected dominating set and $\gamma_{uc}^+(D) = |S^+| = \gamma_{ps}^+(D)$

On the other hand suppose subdigraph < S^+ > is disconnected then $\gamma_{ps}^+(D) < \gamma_{uc}^+(D)$

Therefore $\gamma_{ps}^+(D) \le \gamma_{uc}^+(D)$

Example:

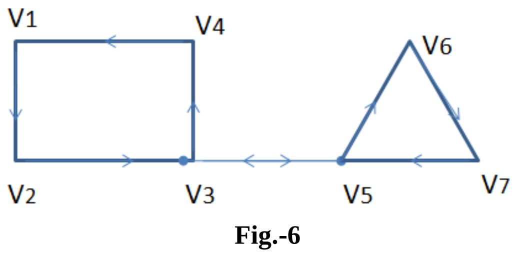

Fig.-6

$S^+ = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ $V - S^+ = \{v_4\}$

Therefore $\gamma_{uc}^+(D) = |S^+| = \gamma_{ps}^+(D)$

on the other hand suppose subdigraph S^+ is disconnected

$S^+ = \{v_1, v_3, v_4, v_5, v_6, v_7\}$ $V - S^+ = \{v_2, v_4\}$

then $\gamma_{ps}^+(D) < \gamma_{uc}^+(D)$

Theorem 6: Let T be a ditree for n ≥ 4 then $\gamma_{ps}^{+}(T) = n-2$.

Proof: Let S^{+} be a point set out dominating set of T then V-S^{+} contains two vertices which is adjacent from a cut vertex which is adjacent from root vertex.

Theorem7: Let T be a ditree such that any two adjacent cutvertices u and v with atleast one of u and v is adjacent to an end vertex then $\gamma_{uc}^{+}(T) \leq \gamma_{ps}^{+}(T)$

Proof: Let S^{+} be a $\gamma_{uc}^{+}(T)$ of T then we consider two cases

Case-1: suppose atleast one of u, v$\in S^{+}$ then $<V-S^{+}>$ is disconnected with atleast one vertex, Hence S^{+} is a $\gamma_{ps}^{+}(D)$ is a set of T. Thus the theorem is tree.

Case-2: Suppose u, v$\in S^{+}$ since there exists a root vertex which is adjacent to u or v which implies w$\in S^{+}$ therefore $\gamma_{uc}^{+}(T)$ is unilaterally connected.

From case 1 and 2 $\gamma_{uc}^{+}(T) \leq \gamma_{ps}^{+}(T)$

Example:

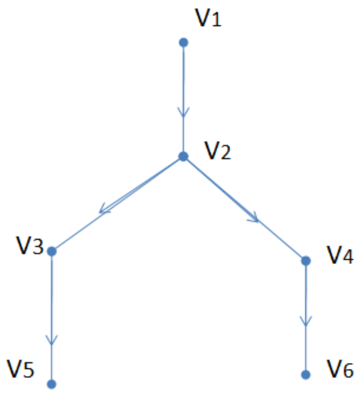

Fig.-7

$S^{+} = \{v_1, v_2, v_5, v_6\}$ $\qquad V - S^{+} = \{v_3, v_4\}$

Therefore $\gamma_{ps}^{+}(T) = |S^{+}| = 4$

On the otherhand

$S^{+} = \{v_1, v_2, v_3, v_4\}$ $\qquad V - S^{+} = \{v_5, v_6\}$

Therefore $\gamma_{uc}^{+}(T) = |S^{+}| = 4$

or

$S^{+} = \{v_1, v_2, v_3\}$ $\qquad V - S^{+} = \{v_4, v_5, v_6\}$

Therefore $\gamma_{uc}^{+}(T) = |S^{+}| = 3$

Hence $\gamma_{uc}^{+}(T) \leq \gamma_{ps}^{+}(T)$.

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