

## ON $sb\hat{g}$ - CLOSED SETS IN TOPOLOGICAL SPACES

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(Received On: 15-09-15; Revised & Accepted On: 14-10-15)

### ABSTRACT

*In this paper, we introduce a new class of sets called  $sb\hat{g}$ -closed sets in topological spaces. A subset  $A$  of  $X$  is said to be  $sb\hat{g}$ -closed if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b\hat{g}$ -open in  $X$ . Also we study some of its basic properties and investigate the relationship with other existing closed sets in topological space. As an application, we introduce two new spaces namely,  $T_{sb\hat{g}}$  and  $T_{sb\hat{g}}^\alpha$ .*

**Keywords:**  $b\hat{g}$ -open sets, semi-closure, semi-closed sets,  $sb\hat{g}$ -closed sets,  $T_{sb\hat{g}}$ -space and  $T_{sb\hat{g}}^\alpha$ -space.

**AMS Mathematics Subject Classification (2010):** 54A05.

### 1. INTRODUCTION

N. Levine [6] introduced semi-open sets in Topology and studied its properties in 1963. In 1970, N. Levine [7] introduced generalized closed (briefly  $g$ -closed) sets and studied their basic properties.  $b$ -open sets have been introduced and investigated by Andrijevic [2] in 1996. M.K.R.S. Veerakumar [14] defined  $\hat{g}$ -closed sets in Topological Spaces and studied their properties. Also, R. Subasree and M. Maria Singam [13] introduced  $b\hat{g}$ -closed sets and studied its properties in 2013.

Now, we introduce the concept of  $sb\hat{g}$ -closed sets and  $sb\hat{g}$ -open sets in Topological space and study some of their properties. Applying these sets, we obtain two new spaces namely  $T_{sb\hat{g}}$ -space and  $T_{sb\hat{g}}^\alpha$ -space.

### 2. PRELIMINARIES

Throughout this paper  $(X, \tau)$  (or simply  $X$ ) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset  $A$  of  $(X, \tau)$ ,  $Cl(A)$ ,  $Int(A)$  and  $A^c$  denote the closure of  $A$ , interior of  $A$  and the complement of  $A$  respectively. We are giving some definitions.

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called

1. a semi-open set [6] if  $A \subseteq Cl(Int(A))$ .
2. an  $\alpha$ -open set [10] if  $A \subseteq Int(Cl(Int(A)))$ .
3. a  $b$ -open set [2] if  $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$ .
4. a regular open [12] set if  $A = Int(Cl(A))$ .

The complement of a semi-open (resp.  $\alpha$ -open,  $b$ -open, regular-open) set is called semi-closed (resp.  $\alpha$ -closed,  $b$ -closed, regular-closed) set.

The intersection of all semi-closed (resp.  $\alpha$ -closed,  $b$ -closed, regular-closed) sets of  $X$  containing  $A$  is called the semi-closure (resp.  $\alpha$ -closure,  $b$ -closure, regular closure) of  $A$  and is denoted by  $sCl(A)$  (resp.  $\alpha Cl(A)$ ,  $bCl(A)$ ,  $rCl(A)$ ). The family of all semi-open (resp.  $\alpha$ -open,  $b$ -open, regular-open) subsets of a space  $X$  is denoted by  $SO(X)$  (resp.  $\alpha O(X)$ ,  $bO(X)$ ,  $rO(X)$ ).

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**Definition 2.2:** A subset  $A$  of a topological space  $(X, \tau)$  is called

- 1) a generalized closed set (briefly  $g$ -closed)[7] if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- 2)  $asg$ -closed set[4] if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $X$ .
- 3)  $ags$ -closed set[3] if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- 4)  $agb$ -closed set[1] if  $bCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- 5)  $arb$ -closed set[9] if  $rCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b$ -open in  $X$ .
- 6) a  $g^*b$ -closed set[16] if  $bCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $X$ .
- 7)  $ag\hat{g}$ -closed set[14] if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $X$ .
- 8)  $ab\hat{g}$ -closed set[13] if  $bCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $X$ .
- 9) a  $\alpha b\hat{g}$ -closed set[11] if  $\alpha Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b\hat{g}$ -open in  $X$ .

The complement of a  $g$ -closed (resp.  $sg$ -closed,  $gs$ -closed,  $gb$ -closed,  $rb$ -closed,  $g^*b$ -closed,  $\hat{g}$ -closed,  $b\hat{g}$ -closed and  $\alpha b\hat{g}$ -closed) set is called  $g$ -open (resp.  $sg$ -open,  $gs$ -open,  $gb$ -open,  $rb$ -open,  $g^*b$ -open,  $\hat{g}$ -open,  $b\hat{g}$ -open and  $\alpha b\hat{g}$ -open) set.

**Definition 2.3:**  $sCl(A)$  is defined as the intersection of all semi-closed sets containing  $A$ .

**Definition 2.4:** A space  $(X, \tau)$  is called a

- (i) a  $T_b$ -space[5] if every  $gs$ -closed set in  $X$  is closed.
- (ii) a  $T_{gs}$ -space[1] if every  $gb$ -closed set in  $X$  is  $b$ -closed.
- (iii) a  $T_{b\hat{g}}$ -space[13] if every  $b\hat{g}$ -closed set in  $X$  is  $b$ -closed.
- (iv) a  $T_{b\hat{g}}^*$ -space[13] if every  $b\hat{g}$ -closed set in  $X$  is closed.
- (v) a  $T_{ab\hat{g}}^c$ -space[11] if every  $\alpha b\hat{g}$ -closed set in  $X$  is closed.

### 3. $sb\hat{g}$ -CLOSED SETS

We introduce the following definition.

**Definition 3.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called a  $sb\hat{g}$ -closed set if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b\hat{g}$ -open in  $X$ . The family of all  $sb\hat{g}$ -closed sets of  $X$  are denoted by  $sb\hat{g}-C(X)$ .

**Definition 3.2:** The complement of a  $sb\hat{g}$ -closed set is called  $sb\hat{g}$ -open set. The family of all  $sb\hat{g}$ -open sets of  $X$  are denoted by  $sb\hat{g}-O(X)$ .

**Example 3.3:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$  then  $\{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are  $sb\hat{g}$ -closed sets and  $\{X, \phi, \{b\}, \{a\}, \{a, c\}, \{a, b\}\}$  are  $sb\hat{g}$ -open sets in  $X$ .

**Proposition 3.4:** Every closed set is  $sb\hat{g}$ -closed set

**Proof:** Let  $A$  be any closed set in  $X$  and  $U$  be any  $b\hat{g}$ -open set in  $X$  such that  $A \subseteq U$ . Since  $A$  is closed,  $Cl(A) = A$  for every subset  $A$  of  $X$ . Therefore,  $sCl(A) \subseteq Cl(A) = A \subseteq U$ . Hence,  $A$  is  $sb\hat{g}$ -closed set.

The following example shows that the converse of the above proposition need not be true.

**Example 3.5:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}\}$ .  $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ . Here,  $\{b\}, \{c\}$  are  $sb\hat{g}$ -closed sets but not closed sets in  $X$ .

**Proposition 3.6:** A subset  $A$  of  $(X, \tau)$  is semi-closed set in  $X$  iff  $A$  is  $sb\hat{g}$ -closed set in  $X$ .

**Proposition 3.7:** Every  $\alpha$ -closed set is  $sb\hat{g}$ -closed set.

**Proof:** Let  $A$  be any  $\alpha$ -closed set in  $X$  such that  $A \subseteq U$  where  $U$  is  $b\hat{g}$ -open. Since  $A$  is  $\alpha$ -closed set,  $sCl(A) \subseteq \alpha Cl(A) \subseteq U$ . Therefore,  $sCl(A) \subseteq U$ . Hence,  $A$  is  $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

**Example 3.8:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ .  $\alpha-C(X) = \{X, \phi, \{a\}, \{a, b\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ . Here,  $\{b\}$  is  $sb\hat{g}$ -closed set but not  $\alpha$ -closed set in  $X$ .

**Proposition 3.9:** Every regular closed set is  $sb\hat{g}$ -closed set.

**Proof:** Let  $A$  be any regular closed set in  $X$  such that  $A \subseteq U$  where  $U$  is  $b\hat{g}$ -open. Since  $A$  is regular closed set,  $sCl(A) \subseteq rCl(A) \subseteq U$ . Therefore,  $sCl(A) \subseteq U$ . Hence,  $A$  is  $sb\hat{g}$ -closed set.

The reverse implication does not hold as shown in the following example.

**Example 3.10:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{b\}\}$ .  $r-C(X) = \{X, \phi\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ . Here,  $\{a\}, \{c\}, \{a, c\}$  are  $sb\hat{g}$ -closed sets but not regular closed sets in  $X$ .

**Proposition 3.11:** Every  $sb\hat{g}$ -closed set is  $b$ -closed set.

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set in  $X$  such that  $A \subseteq U$  where  $U$  is  $b\hat{g}$ -open. Since  $A$  is  $sb\hat{g}$ -closed set,  $bCl(A) \subseteq sCl(A) \subseteq U$ . Therefore,  $bCl(A) \subseteq U$ . Hence,  $A$  is  $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

**Example 3.12:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{c\}, \{a, b\}\}$ .  $bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{c\}, \{a, b\}\}$ . Here,  $\{a\}, \{b\}, \{a, c\}, \{b, c\}$  are  $sb\hat{g}$ -closed sets but not  $b$ -closed sets.

**Proposition 3.13:** Every  $sb\hat{g}$ -closed set is  $sg$ -closed set

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set in  $X$  and  $U$  be any semi-open set in  $X$  such that  $A \subseteq U$ . Since “Every semi-open set is  $b\hat{g}$ -open set”, we have  $sCl(A) \subseteq U$  where  $U$  is semi-open. Hence,  $A$  is  $sg$ -closed.

Every  $sg$ -closed set need not be  $sb\hat{g}$ -closed set as shown in the following example.

**Example 3.14:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a, c\}\}$ .  $sg-C(X) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{b\}\}$ . Here,  $\{a, b\}, \{b, c\}$  are  $sg$ -closed sets but not  $sb\hat{g}$ -closed sets.

**Proposition 3.15:** Every  $sb\hat{g}$ -closed set is  $gs$ -closed set

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set in  $X$  and  $U$  be any open set in  $X$  such that  $A \subseteq U$ . Since “Every open set is  $b\hat{g}$ -open set”, we have  $sCl(A) \subseteq U$  where  $U$  is open. Hence,  $A$  is  $gs$ -closed.

The following example shows that the converse of the above proposition need not be true.

**Example 3.16:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ .  $gs-C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{b, c\}\}$ . Here,  $\{b\}, \{c\}, \{a, c\}, \{a, b\}$  are  $gs$ -closed sets but not  $sb\hat{g}$ -closed sets.

**Proposition 3.17:** Every  $sb\hat{g}$ -closed set is  $gb$ -closed set.

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set in  $X$ . Let  $U$  be open set such that  $A \subseteq U$ . Since, “Every open set is  $b\hat{g}$ -open”, we have  $bCl(A) \subseteq sCl(A) \subseteq U$ . Therefore,  $bCl(A) \subseteq U$  where  $U$  is open in  $X$ . Hence,  $A$  is  $gb$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

**Example 3.18:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ .  $gb-C(X) = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ . Here,  $\{a, c\}$  is  $gb$ -closed set but not  $sb\hat{g}$ -closed set.

**Proposition 3.19:** Every  $rb$ -closed set is  $sb\hat{g}$ -closed set.

**Proof:** Let  $A$  be any  $rb$ -closed set in  $X$ . Let  $U$  be any  $b$ -open set in  $X$  such that  $A \subseteq U$ . Since “Every  $b$ -open set is  $b\hat{g}$ -open set”, we have  $sCl(A) \subseteq rCl(A) \subseteq U$  where  $U$  is  $b\hat{g}$ -open. Therefore,  $sCl(A) \subseteq U$ . Hence,  $A$  is  $sb\hat{g}$ -closed set.

The reverse implication does not hold as shown in the following example.

**Example 3.20:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{a, c\}, \{a, b, d\}\}$ .  $rb-C(X) = \{X, \phi, \{b, c, d\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$ . Here,  $\{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{b, d\}$  are  $sb\hat{g}$ -closed sets but not  $rb$ -closed sets.

**Proposition 3.21:** Every  $sb\hat{g}$ -closed set is  $g^*b$ -closed set.

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set in  $X$ . Let  $U$  be  $g$ -open set such that  $A \subseteq U$ . Since, “Every  $g$ -open set is  $b\hat{g}$ -open set”, we have  $bCl(A) \subseteq sCl(A) \subseteq U$ . Therefore,  $bCl(A) \subseteq U$  where  $U$  is  $g$ -open in  $X$ . Hence,  $A$  is  $g^*b$ -closed set.

The following example shows that the converse of the above proposition need not be true

**Example 3.22:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{c\}, \{a, b\}\}$ .  $g^*b-C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{c\}, \{a, b\}\}$ . Here,  $\{a\}, \{b\}, \{a, c\}, \{b, c\}$  are  $g^*b$ -closed sets but not  $sb\hat{g}$ -closed sets.

**Proposition 3.23:** Every  $sb\hat{g}$ -closed set is  $b\hat{g}$ -closed set.

**Proof:** Let  $A$  be any  $sb\hat{g}$ -closed set. By proposition 3.11,  $A$  is  $b$ -closed set in  $X$ . By proposition 3.3 in [13],  $A$  is  $b\hat{g}$ -closed set in  $X$ .

The converse of the above proposition need not be true as shown in the following example.

**Example 3.24:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c, d\}\}$ .  $b\hat{g}-C(X) = \{X, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ . Here  $\{a, b, c\}, \{a, b, d\}$  are  $b\hat{g}$ -closed sets but not  $sb\hat{g}$ -closed sets.

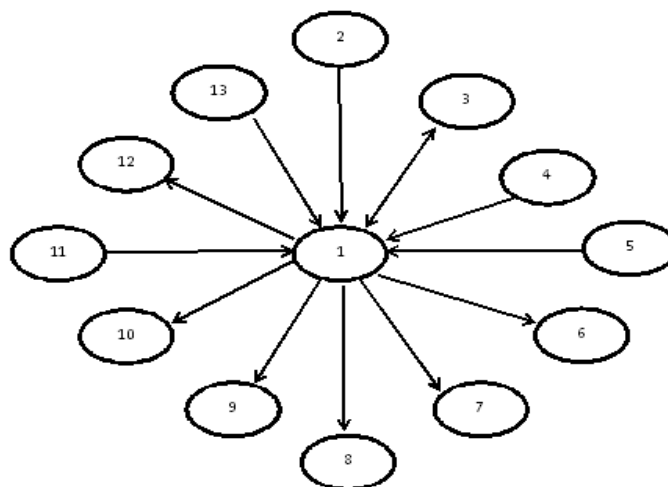
**Proposition 3.25:** Every  $ab\hat{g}$ -closed set is  $sb\hat{g}$ -closed set.

**Proof:** Let  $A$  be any  $ab\hat{g}$ -closed set. Let  $U$  be any  $b\hat{g}$ -open set in  $X$  such that  $A \subseteq U$ . To prove that,  $A$  is  $sb\hat{g}$ -closed set. Now,  $sCl(A) \subseteq \alpha Cl(A) \subseteq U$  where  $U$  is  $b\hat{g}$ -open set. Therefore,  $A$  is  $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

**Example 3.26:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ .  $ab\hat{g}-C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$  and  $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ . Here,  $\{a\}, \{b\}$  are  $sb\hat{g}$ -closed sets but not  $ab\hat{g}$ -closed sets.

**Remark 3.27:** The following diagram shows the relationship of  $sb\hat{g}$ -closed sets with other known existing sets.  $A \rightarrow B$  represents  $A$  implies  $B$  but not conversely.



- |                          |                    |                  |                        |
|--------------------------|--------------------|------------------|------------------------|
| 1. $sb\hat{g}$ -closed   | 2. closed          | 3. semi-closed   | 4. $\alpha$ -closed    |
| 5. regular-closed        | 6. $b$ -closed     | 7. $gs$ -closed  | 8. $sg$ -closed        |
| 9. $gb$ -closed          | 10. $g^*b$ -closed | 11. $rb$ -closed | 12. $b\hat{g}$ -closed |
| 13. $ab\hat{g}$ -closed. |                    |                  |                        |

#### 4. CHARACTERIZATION

**Lemma 4.1:** The finite union of  $sb\hat{g}$ -closed sets is  $sb\hat{g}$ -closed set.

**Lemma 4.2:** The finite intersection of  $sb\hat{g}$ -closed sets is  $sb\hat{g}$ -closed set.

**Proposition 4.3:** Let  $A$  be a  $sb\hat{g}$ -closed set of  $X$ . Then  $sCl(A) - A$  does not contain a non-empty  $b\hat{g}$ -closed set.

**Proof:** Suppose  $A$  is a  $sb\hat{g}$ -closed set. Let  $F$  be a  $b\hat{g}$ -closed set contained in  $sCl(A) - A$ . Now  $F^c$  is a  $b\hat{g}$ -open set of  $X$  such that  $A \subseteq F^c$ . Since  $A$  is  $sb\hat{g}$ -closed, we have  $sCl(A) \subseteq F^c$ . Hence,  $F \subseteq (sCl(A))^c$ . Also,  $F \subseteq sCl(A) - A$ . Therefore,  $F \subseteq sCl(A) \cap (sCl(A))^c = \phi$ . Hence,  $F$  must be  $\phi$ .

**Proposition 4.5:** If  $A$  is  $\hat{b}g$ -open and  $sb\hat{g}$ -closed set of  $X$ , then  $A$  is semi-closed.

**Proof:** Since  $A$  is  $\hat{b}g$ -open and  $sb\hat{g}$ -closed, we have  $sCl(A) \subseteq A$ . Hence,  $A$  is semi-closed.

**Proposition 4.6:** The intersection of a  $sb\hat{g}$ -closed set and a semi-closed set of  $X$  is always  $sb\hat{g}$ -closed set.

**Proof:** Let  $A$  be a  $sb\hat{g}$ -closed set and  $B$  be a semi-closed set. Since  $A$  is  $sb\hat{g}$ -closed,  $sCl(A) \subseteq U$  whenever  $U$  is  $\hat{b}g$ -open. Let  $B$  be such that  $A \cap B \subseteq U$  where  $U$  is  $\hat{b}g$ -open. Now,  $sCl(A \cap B) \subseteq sCl(A) \cap sCl(B) \subseteq U \cap B \subseteq U$ . Hence,  $A \cap B$  is  $sb\hat{g}$ -closed set. Therefore, intersection of any  $sb\hat{g}$ -closed set and a semi-closed set of  $X$  is always  $sb\hat{g}$ -closed set.

## 5. APPLICATIONS

As an applications of  $sb\hat{g}$ -closed sets, we introduce two new spaces namely,  $T_{sb\hat{g}}$ -space and  $T_{sb\hat{g}}^\alpha$ -space.

**Definition 5.1:** A Space  $(X, \tau)$  is called a  $T_{sb\hat{g}}$ -space if every  $sb\hat{g}$ -closed set in  $X$  is closed.

**Definition 5.2:** A Space  $(X, \tau)$  is called a  $T_{sb\hat{g}}^\alpha$ -space if every  $sb\hat{g}$ -closed set in  $X$  is  $\alpha$ -closed.

**Proposition 5.3:** Every  $T_b$ -space is  $T_{sb\hat{g}}$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_b$ -space. Let  $A$  be  $sb\hat{g}$ -closed set in  $(X, \tau)$ . By proposition 3.15,  $A$  is  $gs$ -closed. Since  $(X, \tau)$  is  $T_b$ -space,  $A$  is closed. Hence,  $(X, \tau)$  is  $T_{sb\hat{g}}$ -space.

The converse of the above proposition need not be true as shown in the following example.

**Example 5.4:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{c\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, b\}\}$

$gs\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, b\}\}$

Here,  $(X, \tau)$  is  $T_{sb\hat{g}}$  space but not  $T_b$ -space.

**Proposition 5.5:** Every  $T_{sb\hat{g}}$ -space is  $T_{gs}$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{sb\hat{g}}$ -space. Let  $A$  be  $s\hat{g}$ -closed set in  $(X, \tau)$ . By Proposition 3.17,  $A$  is  $gb$ -closed. Since every closed set is  $b$ -closed set,  $A$  is  $b$ -closed set in  $X$ . Therefore,  $(X, \tau)$  is  $T_{gs}$ -space.

The converse of the above proposition need not be true and is explained in the following example.

**Example 5.6:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$gb\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$

Here  $(X, \tau)$  is  $T_{gs}$ -space but not  $T_{sb\hat{g}}$ -space.

**Proposition 5.7:** Every  $T_{sb\hat{g}}$ -space is  $T_{b\hat{g}}$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{sb\hat{g}}$ -space. Let  $A$  be  $s\hat{g}$ -closed set in  $(X, \tau)$ . By Proposition 3.23,  $A$  is  $\hat{g}$ -closed. Since every closed set is  $b$ -closed set,  $A$  is  $b$ -closed set in  $X$ . Therefore,  $(X, \tau)$  is  $T_{b\hat{g}}$ -space.

The reverse implication does not hold as shown in the following example.

**Example 5.8:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$b\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$

Here,  $(X, \tau)$  is  $T_{b\hat{g}}$ -space but not  $T_{sb\hat{g}}$ -space.

**Proposition 5.9:** Every  $T_{b\hat{g}}^*$ -space is  $T_{sb\hat{g}}$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{b\hat{g}}^*$ -space. Let  $A$  be  $sb\hat{g}$ -closed set in  $(X, \tau)$ . By Proposition 3.23,  $A$  is  $\hat{g}$ -closed. Since  $(X, \tau)$  is  $T_{b\hat{g}}^*$ -space,  $A$  is closed set in  $X$ . Therefore,  $(X, \tau)$  is  $T_{sb\hat{g}}$ -space.

The reverse implication need not be true as shown in the following example.

**Example 5.10:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$   
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b, c\}\}$   
 $b\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$   
 $C(X) = \{X, \phi, \{a\}, \{b, c\}\}$   
 Here,  $(X, \tau)$  is  $T_{sb\hat{g}}$ -space but not  $T_{b\hat{g}}^*$ -space.

**Proposition 5.11:** Every  $T_{b\hat{g}}^*$ -space is  $T_{sb\hat{g}}^\alpha$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{b\hat{g}}^*$ -space. Let  $A$  be  $sb\hat{g}$ -closed set in  $(X, \tau)$ . By Proposition 3.23,  $A$  is  $b\hat{g}$ -closed. Since  $(X, \tau)$  is  $T_{b\hat{g}}^*$ -space,  $A$  is closed set in  $X$ . Since every closed set is  $\alpha$ -closed set,  $A$  is  $\alpha$ -closed in  $X$ . Therefore,  $(X, \tau)$  is  $T_{sb\hat{g}}^\alpha$ -space.

The following example shows that the converse of the above proposition need not be true.

**Example 5.12:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}\}$   
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$   
 $b\hat{g}\text{-}C(X) = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$   
 $\alpha C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$   
 $C(X) = \{X, \phi, \{b, c\}\}$   
 Here,  $(X, \tau)$  is  $T_{sb\hat{g}}^\alpha$ -space but not  $T_{b\hat{g}}^*$ -space.

**Proposition 5.13:** Every  $T_{sb\hat{g}}$ -space is  $T_{ab\hat{g}}^c$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{sb\hat{g}}$ -space. Let  $A$  be  $ab\hat{g}$ -closed set in  $(X, \tau)$ . By Proposition 3.25,  $A$  is  $sb\hat{g}$ -closed. Since  $(X, \tau)$  is  $T_{sb\hat{g}}$ -space,  $A$  is closed. Therefore,  $(X, \tau)$  is  $T_{ab\hat{g}}^c$ -space.

The converse of the above proposition need not be true and is explained in the following example.

**Example 5.14:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$   
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$   
 $ab\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$   
 $C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$   
 Here,  $(X, \tau)$  is  $T_{ab\hat{g}}^c$ -space but not  $T_{sb\hat{g}}$ -space.

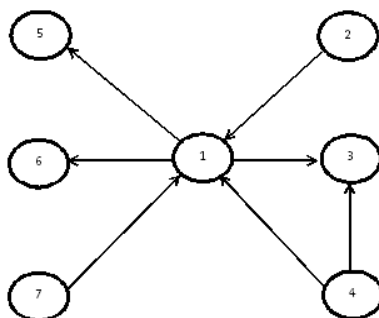
**Proposition 5.15:** Every  $T_{sb\hat{g}}$ -space is  $T_{sb\hat{g}}^\alpha$ -space.

**Proof:** Let  $(X, \tau)$  be  $T_{sb\hat{g}}$ -space. Let  $A$  be  $sb\hat{g}$ -closed set in  $(X, \tau)$ . Since  $(X, \tau)$  is  $T_{sb\hat{g}}$ -space,  $A$  is closed. Since every closed set is  $\alpha$ -closed set,  $A$  is  $\alpha$ -closed set in  $X$ . Therefore,  $(X, \tau)$  is  $T_{sb\hat{g}}^\alpha$ -space.

The following example shows that the converse of the above proposition need not be true.

**Example 5.16:** Let  $X = \{a, b, c\}$  and  $\tau = \{X, \phi, \{b\}\}$   
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$   
 $\alpha C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$   
 $C(X) = \{X, \phi, \{a, c\}\}$   
 Here,  $(X, \tau)$  is  $T_{sb\hat{g}}^\alpha$ -space but not  $T_{sb\hat{g}}$ -space.

**Remark 5.17:** The following diagram shows the relationship about  $T_{sb\hat{g}}$ -space and  $T_{sb\hat{g}}^\alpha$ -space with other known existing spaces.



- |                            |                             |                                   |
|----------------------------|-----------------------------|-----------------------------------|
| 1. $T_{sb\hat{g}}$ -space  | 2. $T_{ab\hat{g}}^c$ -space | 3. $T_{sb\hat{g}}^\alpha$ - space |
| 4. $T_{b\hat{g}}^*$ -space | 5. $T_{b\hat{g}}$ -space    | 6. $T_{gs}$ -space                |
| 7. $T_b$ -space.           |                             |                                   |

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**Source of support: Nil, Conflict of interest: None Declared**

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