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ACCURATE INDEPENDENT DOMINATION IN FUZZY GRAPHS

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ABSTRACT

A dominating set D of a fuzzy graph G= (V, E) is an independent dominating set if the induced subgraph <D> has no edges. An independent dominating set D of a fuzzy graph G is an accurate independent dominating set if V – D has no independent dominating set of cardinality /D/. The fuzzy accurate independent domination number $i_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set of G. In this paper we study a accurate independent domination in fuzzy graphs and investigate the relationship of $i_{fa}(G)$ with other known parameters.

***Index terms:** Fuzzy Graph, Fuzzy Independent Dominating set, Fuzzy Accurate Independent Dominating set, Fuzzy Accurate Independent Domination Number.*

INTRODUCTION

A fuzzy subset of a non empty set V is a mapping σ: V→[0,1].A fuzzy relation on V is a fuzzy subset of V×V. A *fuzzy graph* G = (σ, μ) is a pair of function σ :V→[0,1] and μ : V×V →[0, 1], where μ (u, v) ≤ σ(u)Λ σ (v) for all u, v ∈ V. The *order p* and *size q* of the fuzzy graph G = (σ, μ) are define by $p = \sum_{v\in V} \sigma(v)$ and $q = \sum_{u,v\in E} \mu(u,v)$. The *complement* of a fuzzy graph G = (σ, μ) is a fuzzy graph G = (σ, μ) where σ = σ and μ^c (u, v) = σ(u)Λσ (v) - μ (u, v) for all u, v in V. The fuzzy cardinality of a fuzzy subset D of V is $|D|_f = \sum_{v\in D} \sigma(v)$. An edge e = {u, v} of a fuzzy graph is called an effective edge if μ (u, v) = σ(u) Λ σ (v). The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by $d_E(u)$. The Minimum effective degree $\delta_E(G) = \min\{d_E(u) / u\in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{d_E(u) / u\in V(G)\}$. A set of fuzzy vertex which cover all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and is denoted by $\alpha_0(G)$. A set of fuzzy edge which cover all the fuzzy vertices is called a fuzzy edge cover of G and the minimum cardinality of a fuzzy edge cover is called a edge covering number of G and is denoted by $\alpha_1(G)$. The vertex independence number $\beta_0(G)$ of G is the maximum cardinality among the independent sets of vertices. The edge independence number $\beta_1(G)$ of G is the maximum cardinality among the independent sets of edges. For any graph G is a complete subgraph of G is called a Clique of G. The number of vertices in a largest Clique of G is called the Clique number ω(G) of G. If μ(u, v) = 0 for every v∈V then u is called isolated node. A set S ⊆V in a fuzzy graph G is said to be independent if μ (u, v) < σ(u)Λ σ (v) for all u, v ∈ S. A dominating set is called an independent dominating set if D is independent. An independent dominating set S of a fuzzy graph G is said to be a maximal independent dominating set if there is no independent dominating set S^1 of G such that $S^1 \subset S$. An independent dominating set S of a fuzzy graph G is said to be a maximum independent dominating set if there is no independent dominating set S^1 of G such that $|S^1| > |S|$. The minimum scalar cardinality of an maximum independent dominating set of G is called the independent domination number of G and is denoted by i(G). Let x, y ∈V. We say that x dominates y in G if μ (u, v) = σ(u) Λ σ (v). A subset S of V is called a dominating set in G if for every v ∉ S, there exists u∈S such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G). Let G = (σ, μ) be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every v∈V–D there exists u ∈D such that (u, v) is a strong arc. A dominating set D of a graph G is called minimal dominating set of G if for every node v∈ D, D–{v} is not a dominating set of the domination number γ(G) is the minimum cardinalities taken over all minimal dominating sets of G. A dominating set D of a graph G is an accurate dominating set, if V–D has no dominating set of cardinality |D|. The accurate domination number $\gamma_a(G)$ of G is the

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minimum cardinality of an accurate dominating set. A dominating set D of a fuzzy graph G is an fuzzy accurate dominating set, if V−D has no dominating set of cardinality $|D|$. The fuzzy accurate domination number $\gamma_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set. An independent dominating set D of a graph G is an accurate independent dominating set if V−D has no independent dominating set of cardinality $|D|$. The accurate independent domination number $i_a(G)$ of G is the minimum cardinality of an accurate dominating set of G. An independent dominating set D of a fuzzy graph G is an fuzzy accurate independent dominating set if V−D has no independent dominating set of cardinality $|D|$. The fuzzy accurate independent domination number $i_{fa}(G)$ of G is the minimum cardinality of an accurate independent dominating set of G.

1. ACCURATE DOMINATION IN FUZZY GRAPHS

Definition: 1.1 A dominating set D of G is an accurate dominating set if V−D has no dominating set of cardinality $|D|$. The accurate domination number $\gamma_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set of G.

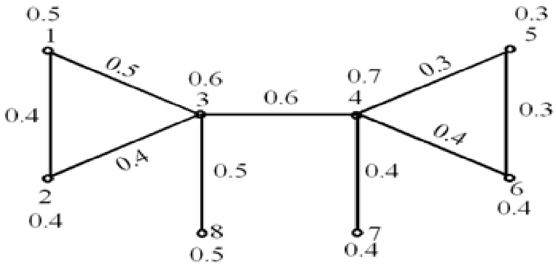
Theorem: 1.1 For any fuzzy graph $p - q \le \gamma_{fa} \le p - \delta_E$ where p, q and δ_E are the order, size and minimum effective incident degree of G respectively.

Proof: Let D be a accurate dominating set and γ_{fa} be the minimum fuzzy domination number in G. Then the scalar cardinality of V−D is less than or equal to the scalar cardinality of V×V. Hence $p-q \le \gamma_{fa}$. Now, let u be the node with minimum effective incident degree $\delta_{E,}$ clearly V−{u} is a accurate dominating set and hence $\gamma_{fa} \le p - \delta_E$. Hence $p - q \le \gamma_{fa} \le p - \delta_E$ is true for any fuzzy graph.

Theorem: 1.2 $\lceil \frac{p}{1+\Delta(G)} \rceil \le \gamma_{fa} \le p - \Delta(G)$.

Theorem: 1.3 If G is a fuzzy graph without isolated nodes then $\gamma_{fa}(G) \le \min\{\alpha_0(G), \alpha_1(G), \beta_0(G), \beta_1(G)\}$.

Example: 1.1



Here D = {3, 4}, $\gamma_{fa}(G) = 1.3$

$\alpha_0(G) = 1.3$, $\alpha_1(G) = 1.3$

$\beta_0(G) = 1.7$, $\beta_1(G) = 1.7$

Theorem: 1.4 For any fuzzy graph G and $\bar{G}$ are both connected then $\gamma_{fa}(G) + \gamma_{fa}(\bar{G}) \le p + 1$.

Proof: We know that $\gamma_{fa}(G) \le p - \Delta(G)$ and $\gamma_{fa}(\bar{G}) \le p - \Delta(\bar{G})$.

$$\begin{aligned}\text{Therefore } \gamma_{fa}(G) + \gamma_{fa}(\bar{G}) &\le p - \Delta(G) + p - \Delta(\bar{G}) \\ &= 2p - (\Delta(G) + \Delta(\bar{G})) \\ &= 2p - (\Delta(G) + p - 1 - \delta(G)) \\ &= p+1 + \delta(G) - \Delta(G) \text{ Since } \delta(G) - \Delta(G) \le 0 \\ &\le p+1.\end{aligned}$$

Theorem: 1.5 For any fuzzy graph G and $\bar{G}$ are both connected then $\gamma_{fa}(G) + \gamma_{fa}(\bar{G}) \le p\,(p - 3)$.

Theorem: 1.6 For any fuzzy graph G and $\bar{G}$ are both connected then

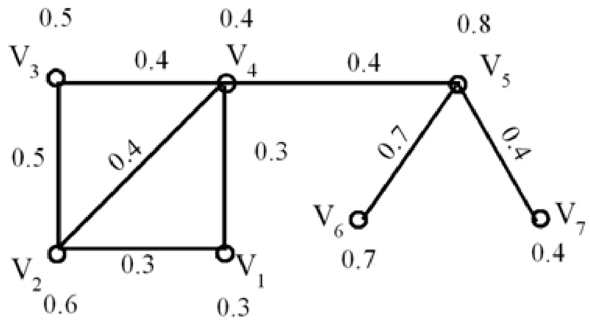
(i) $\gamma_{fa}(G) + \gamma_{fa}(\bar{G}) \le 2\,(p - 2)$

(ii) $\gamma_{fa}(G) + \gamma_{fa}(\bar{G}) \le (p - 2)^2$

2. ACCURATE INDEPENDENT DOMINATION IN FUZZY GRAPHS

Definition: An independent dominating set D of G is an accurate independent dominating set if V−D has no independent dominating set of cardinality $|D|$. The accurate independent domination number $i_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set of G

Example: 2.1



Here D = {v_2, v_5} and $i_{fa}(G) = 1.4$
P = 3.7, q = 3.4, $\Delta(G) = 1.5$
$\delta(G) = 0.4$, $\alpha_0(G) = 1.4$, $\beta_0(G) = 1.9$

Theorem: 2.1 For any fuzzy graph G , $i_{fa}(G) \leq \beta_0(G)$.

Proof: Let S be an Independent set of nodes in G such that $|S| = \beta_0(G)$. Then G contains no larger independent set. Then V – S has no independent dominating set of cardinality $|S|$. Therefore S is a accurate independent dominating set. Thus $i_{fa}(G) \leq |S|$, $\therefore$ $i_{fa}(G) \leq \beta_0(G)$.

Theorem: 2.2 For any fuzzy graph G, $i_{fa}(G) \leq p - \gamma_f(G) + 1$.

Theorem: 2.3 For any fuzzy graph G, $\frac{p}{\Delta+1} \leq i_{fa}(G) \leq \frac{p\Delta}{\Delta+1} + 1$.

Proof: We know that $\frac{p}{\Delta+1} \leq \gamma_f(G) \to$ (a) and since $\gamma_f(G) \leq i_{fa}(G) \to$ (b)

From equation (a) and (b) we get $\frac{p}{\Delta+1} \leq i_{fa}(G)$. So lower bound is attained.

Using the previous theorem, $i_{fa}(G) \leq p - \gamma_f(G) + 1$
$\leq p - \frac{p}{\Delta+1} + 1$
$\leq \frac{p\Delta}{\Delta+1} + 1 \to$ (c)

From equation (a), (b) & (c) we get
$\frac{p}{\Delta+1} \leq i_{fa}(G) \leq \frac{p\Delta}{\Delta+1} + 1$.

Theorem: 2.4 For any fuzzy graph G, $\lfloor \frac{p}{1+\Delta(G)} \rfloor \leq i_{fa}(G)$.

Theorem: 2.5 For any fuzzy graph G with $p \geq 2$ nodes, an independent dominating set with $\lfloor \frac{p}{2} \rfloor + 1$ nodes is an accurate independent dominating set.

Proof: Let D be an independent dominating set with $\lfloor \frac{p}{2} \rfloor + 1$ nodes. Then $|V-D| < \frac{p}{2}$. Hence D is an accurate Independent dominating set of G.

Theorem: 2.6 For any connected non trivial fuzzy graph G, $i_{fa}(G) + i_{fa}[L(G)] \leq p$. Where L(G) is a line graph.

Proof: Let G be a connected graph. For any Fuzzy graph $i_{fa}[L(G)] \leq \beta_1(G)$

Also $i_{fa}(G) \leq \alpha_1(G)$

Hence $i_{fa}(G) + i_{fa}[L(G)] \leq \alpha_1(G) + \beta_1(G)$
$= V(G)$
$= p$

Therefore $i_{fa}(G) + i_{fa}[L(G)] \leq p$.

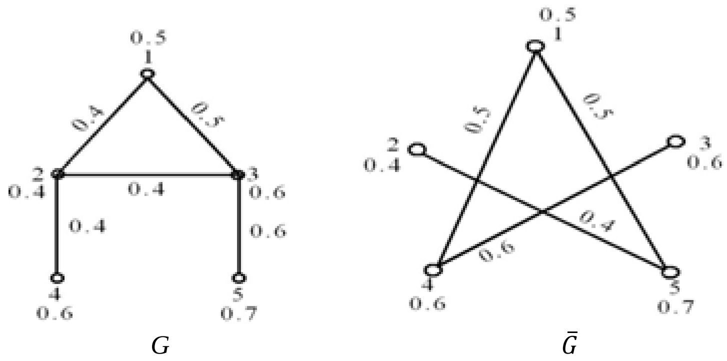
Theorem: 2.7 If G is a fuzzy graph without isolated nodes, then $i_{fa}(G) \le \alpha_0(G) + 1$.

Theorem: 2.8 Let G be a fuzzy graph such that both G and $\bar{G}$ have no isolated nodes then

$i_{fa}(G) + i_{fa}(\bar{G}) \le 2\lceil \frac{p}{2} \rceil$

$i_{fa}(G) \,.\, i_{fa}(\bar{G}) \le \lceil \frac{p}{2} \rceil^2$

Example: 2.2



Here D = {2,5} D = {4,5}

$i_{fa}(G) = 1.1$, p=2.8 $i_{fa}(\bar{G}) = 1.3$

Theorem: 2.9 For any fuzzy graph G and $\bar{G}$ have no isolated nodes then
$i_{fa}(G) + i_{fa}(\bar{G}) \le p + \alpha_0(G) - \omega(G) + 2$.

Proof: From Theorem 2.7, $i_{fa}(G) \le \alpha_0(G) + 1$

Also $i_{fa}(\bar{G}) \le \alpha_0(\bar{G}) + 1$
$\le p - \beta_0(\bar{G}) + 1$
$\le p - \omega(G) + 1$

Thus $i_{fa}(G) + i_{fa}(\bar{G}) \le p + \alpha_0(G) - \omega(G) + 2$.

3. RELATION BETWEEN ACCURATE DOMINATION AND ACCURATE INDEPENDENT DOMINATION IN FUZZY GRAPHS

Theorem: 3.1 For any fuzzy graph G, $\gamma_{fa}(G) \le i_{fa}(G) \to (1)$

Proof: Every independent accurate dominating set is an accurate dominating set. Thus (1) holds.

Theorem: 3.2 For any fuzzy graph G, $\gamma_f(G) \le \gamma_{fa}(G) \le i_{fa}(G)$

Theorem: 3.3 For any fuzzy graph G, $i_{fa}(G) \le p - \gamma_{fa}(G) + 1$.

Proof: Let D be a minimum independent accurate dominating set G. Then for any node $v \in D$, $(V - D) \cup \{v\}$ is an accurate independent dominating set of G.

Thus $i_{fa}(G) \le |(V - D) \cup \{v\}|$
$= p - \gamma_{fa}(G) + 1$.

Theorem: 3.4 For any non-trivial connected fuzzy graph G, $\gamma_{fa}(G) + i_{fa}(G) \le p + q$.

Proof: Since $i_{fa}(G) \le \beta_0(G)$ [From Theorem 2.1]

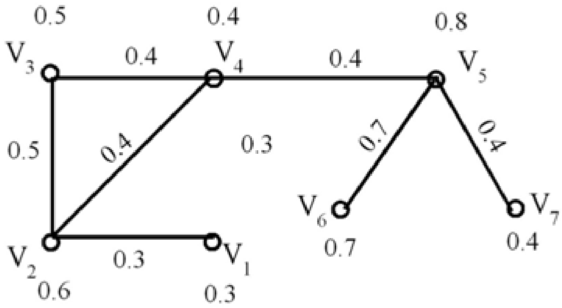
Also $\gamma_{fa}(G) \le \alpha_0(G)$

Further $\gamma_{fa}(G) + i_{fa}(G) \le \alpha_0(G) + \beta_0(G)$
$= \gamma(G) \cup E(G)$
$= p + q$

Hence $\gamma_{fa}(G) + i_{fa}(G) \leq p + q$.

Theorem: 3.5 For any fuzzy graph G, $i_{fa}(G) \leq \gamma_{fa}(G) + \delta(G)$.

Example: 3.1



D= { V_4, V_5} $\gamma_{fa}(G)=1.2$
D= { V_2, V_5 } $i_{fa}(G)=1.4$

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