International Journal of Mathematical Archive-6(10), 2015, 143-149
IJMA Available online through www.ijma.info ISSN 2229 – 5046

A PERSPECTIVE ON $\pi\beta$ –NORMAL TOPOLOGICAL SPACES

THAKUR C. K. RAMAN*

Associate Professor & Head, Dept. of Mathematics,
Jamshedpur Workers' College, Jamshedpur, Jharkhand. India.

(Received On: 24-09-15; Revised & Accepted On: 29-10-15)

ABSTRACT

The framing of this paper bears the main aim to introduce and study a weaker version of β- normality called $\pi\beta$- normality, which surely lies between β- normality and almost β- normality. It contains the fact that $\pi\beta$- normality is a topological property as well as hereditary property with respect to regularly closed subspaces. The characterization & preservation theorems in the context are presented which strengthen the evidence of the existence of such spaces. In fact, there are many $\pi\beta$- normal spaces which are not β- normal.

This paper also includes β- normality in terms of disjoint dense subsets and some basic properties. The relationships among πs- normal spaces, πp- normal spaces & $\pi\beta$- normal spaces are, here, investigated.

Last but not the least, the purpose of introducing this paper is to continue the study of the class of normal spaces, namely $\pi\beta$- normal spaces, which is a generalization of the class of πp- normal spaces & πs- normal spaces.

The effort of coining this paper is nothing but a humble dedication to the eminent mathematician Professor M.E. Abd. El Monsef who breathed his last breathing on 13th August, 2014.

1. INTRODUCTION & PRELIMINARY

D.Andrijevic introduced a new class of generalized open sets in a topological space, the so called β-open sets (i.e. semi-pre-open sets) [1]. The class of semi-pre-open sets contains all semi-open sets and pre-open sets. Professor M.E.Abd El- Monsef *et al.* projected the fundamental properties of β-open sets & β-open continuous mappings [2] along with the study of β-closure and β-interior operators [3]. We, however, know that a set in a topological space is said to be regular open set or open domain [4] if it is the interior of its closure. And the finite union of regular open sets is said to be π-open [5]. With the help of these two notions of β-open set & π-open set, the concept of a πβ-normal topological space is, here, introduced. Obviously, πβ-normality lies in between β-normality & almost β-normality and it is a weaker version of β-normality.

In the present paper, spaces (X, T) and (Y, σ) always mean topological spaces which are not assumed to satisfy any separation axioms are assumed unless explicitly mentioned.

Also, f: (X, T)→ (Y, σ) denotes a single valued function f of a space (X, T) into another space (Y, σ). And for a subset A of a space (X,T), X/A = A^c, cl(A) & int(A) denote the complement, the closure & the interior of A in (X,T) respectively.

If (M, T_M) is a subspace of (X, T) and A ⊆ M, then $cl_X(A)$, $cl_M(A)$ & $int_X(A)$, $int_M(A)$ denote the closure & interior of A in (X, T) and in (M, T_M) respectively.

We also need to recall the following definitions:

Definition 1.1: A subset A of a topological space (X, T) is called
- (i) regular open or open domain[4] if A = int(cl(A)).
- (ii) an α-open[9] set if A⊆ int(cl(int(A)))
- (iii) pre-open [6] or nearly open[7] set if A ⊆ int(cl (A))
- (iv) semi-open [8] set if A⊆ cl(int(A))
- (v) β-open [2] or semi-pre open [1] set if A⊆ cl(int(cl(A)).

*Corresponding Author: Thakur C. K. Raman**

(vi) Π-open [5] if A = $\bigcup_{n=1}^{p} B_n$ where B_n is a regular open set for n= 1, 2, 3……p.

The compliments of the above mentioned open sets are their respective closed sets. The smallest $\mathcal{K}$-closed set containing A is called $\mathcal{K}$cl(A) where $\mathcal{K}$ = regular, α, p, s,β & π. The largest $\mathcal{K}$-open set contained in A is called $\mathcal{K}$ int(A) where$\mathcal{K}$ = regular, α, p, s, β & π.

The family of all $\mathcal{K}$-open (resp. $\mathcal{K}$ -closed) sets of a space (X,T) is denoted by $\mathcal{K}$O(X)(resp. $\mathcal{K}$C(X)); here and above $\mathcal{K}$ = regular, α, p, s, β & π.

Clearly,

$$\begin{tikzcd}
 & & & & \text{p-closed} \arrow[dr] & \\
\text{regular closed} \arrow[r] & \pi\text{-closed} \arrow[r] & \text{closed} \arrow[r] & \alpha\text{-closed} \arrow[ur] \arrow[dr] \arrow[rr] & & \beta\text{-closed} \\
 & & & & \text{s-closed} \arrow[ur] &
\end{tikzcd}$$

None of the above implications is reversible.

Any other notion and symbol, not defined in this paper, may be found in the appropriate reference.

Definition 1.2[10]: Two sets A & B of a space (X, T) are said to be separated if there exist two disjoint open sets U & V in (X, T) such that A⊆U and B ⊆ V.

Definition 1.3:

(a) [10] A space (X, T) is called a normal space if any two disjoint closed sets can be separated.
(b) [11] A space (X, T) is called an almost normal space if any two disjoint closed subsets, one of which is regular closed, can be separated.
(c) [12] A space (X, T) is called a π- normal space if any two disjoint closed subsets, one of which is π-closed, can be separated.
(d) [13] A space (X, T) is called a mildly normal space if any two disjoint regular closed sets can be separated.

Definition 1.4 [14, 15, 16, 17, and 18]:

(a) A space (X,T) is said to be pre-normal or *p*-normal (resp. s-normal, β-normal) if for each pair of disjoint closed sets *A* and *B* of *X* there exist pre-open(resp. semi-open, semi-pre-open) sets U & V for which *A* ⊆U and *B*⊆V such that U ∩ V = φ.
(b) A space (X, T) is said to be almost p-normal(resp. almost s-normal, almost β-normal) if for each closed set *A* and each regular closed set B such that U ∩ V = φ,
there exist disjoint pre-open(resp. semi-open, semi-preopen) sets U&V such that *A* ⊆U and *B*⊆V.
(c) A space (X, T) is said to be mildly p-normal (resp. mildly s-normal, mildly β-normal) if for each pair of disjoint regular closed sets *A* and *B* of *X* there exist pre-open (resp. semi-open, semi-pre open)sets U&V in the manner *A* ⊆ U and *B* ⊆ V such that U ∩ V = φ.
(d) A space (X, T) is said to be π p-normal (resp. π s-normal) if for each pair of disjoint closed sets *A* and *B* one of which is π-closed ,there exist disjoint pre-open (resp. semi-open)sets U & V in the manner *A* ⊆U and *B*⊆V.

2. Π β-NORMAL SPACE

This section begins with the definition of Πβ-normality being motivated by the concept of π-normality.

Definition 2.1: A space (X, T) is said to be πβ-normal if for each pair of disjoint closed sets *A* and *B* on of which is π-closed ,there exist β-open sets U&V such that *A* ⊆U and *B*⊆V. The following is the implications diagram connecting the sorts of normal spaces indicated in definitions (1.3) & (1.4) & (2.1):

$$\begin{tikzcd}
\text{Normal} \arrow[r] \arrow[d] & \pi\text{-Normal} \arrow[r] \arrow[d] & \text{Almost Normal} \arrow[r] \arrow[d] & \text{Mildly Normal} \arrow[d] \\
\mathcal{K}\text{-Normal} \arrow[r] & \pi\mathcal{K}\text{-Normal} \arrow[r] & \text{Almost } \mathcal{K}\text{-Normal} \arrow[r] & \text{Mildly } \mathcal{K}\text{-Normal.}
\end{tikzcd}$$

where $\mathcal{K}$ = regular, α, p, s, β & π.

And,

$$\begin{tikzcd}
A \arrow[r] \arrow[d] & B \arrow[d] \\
C \arrow[r] & D
\end{tikzcd}$$

where, A = normal, π- normal, Almost normal, mildly normal
B = s-normal, πs- normal, Almost s-normal, mildly s-normal
C = p-normal, πp- normal, Almost p- normal, Mildly p- normal
D = β-normal, πβ- normal, Almost β- normal, Mildly β- normal

None of the above implications is reversible,

Example 2.2:

(1) If X = {a, b, c, d} and T = {φ, {c}, {d}, {b, c}, {c, d},{a, c, d},{b, c, d}, X}. Then the space (X, T) is β-normal but not p-normal.
(2) If X = {a, b, c, d, e} and T = {φ, {c, d}, {a, c, d}, {b, c, d}, {c, d, e}, {a, b, c, d}, {a, c, d, e}, {b, c, d, e}, X}. Then the space (X, T) is β-normal but not s-normal.
(3) If X = {a,b,c,d} and T = {φ, {b,d},{a,b,d},{b,c,d},X}, then T^c = the family of closed sets=φ,{a},{c},{a, c}, X} and PO(X) = {φ,{b},{d},{a,b},{a,d},{b,c},{b,d},{c,d},{a,b,c},{b,c,d},{a,b,d},{a,c,d},X}.

Now, (X, T) is p-normal because for the pair of disjoint closed sets {a} & {c} there exist p-open sets {a, b}&{c, d} such that {a} ⊆ {a, b} & {c} ⊆ {c, d} & {a, b} ∩ {c, d} = φ.

But (X, T) is not normal since, the pair of disjoint closed sets {a} & {c} have no disjoint neighbourhoods.

(4) If X = {a, b, c}, T = {φ, X, {a, b},{a, c}}, then T^c = the family of closed sets = {φ,{b},{c}, b, c}, X} and βO(X)={φ,{a},{a, b},{a, c}, X}.

Thus, (X, T) is πβ-normal space because the only π- closed sets in X are φ & X. But (X, T) is not β-normal since, the pair of disjoint closed sets {b} & {c} have no disjoint β-open sets containing them.

The following lemmas are enunciated as they are essential parts for the counterexamples about the other implications:

Lemma 2.3: If D be a dense subset of a space (X, T), then D is β-open.

Proof: Let D be a dense set in a space (X,T), then cl(D) = X. Thus, cl(int(cl(D))) = X. So, D ⊂cl(int(cl(D))) and consequently D is β-open.

Corollary: If D & E are disjoint dense subsets of a space (X, T), then D & E are naturally disjoint β-open sets.

Lemma 2.4: If D be a dense set & A is a closed set in a space (X, T), then D∪A is β-open set.

Proof: suppose that D & A are respectively a dense set and a closed set in a space (X, T). Then cl(D) = X& cl(A) = A.

Now, cl(D∪A) = cl(D) ∪ cl(A) = X ∪ A = X & int (cl D∪A)) = int X = X. Also, cl(int (cl D∪A)) = cl (X) = X. Hence, D∪A ⊂cl(int(cl(D∪ A))). i.e. D∪ A is β-open set.

Lemma 2.5: If D be a dense set & A is a closed set in a space (X, T), then D\A is a β-open set.

Proof: Suppose that D &A are respectively a dense set and a closed set in a space (X, T). Then D is β-open set by lemma (2.3). Also A^C is an open set.

Now, D\A = D ∩ A^C = intersection of a β-open set & an open set = A β-open set.

Lemma 2.6: For any two disjoint closed sets A & B in a space (X, T), the sets U = (D∩ A^c)∪B & V = (D∩ B^c)∪A are β-open sets where D is a dense set in X.

Proof: Let **D** be a dense set and A, B are disjoint closed sets in a space (X, T); then cl(D) = X; cl(A) = A; cl(B) = B; A∩B = φ.

Now, cl(D∪B) = cl D ∪ cl B = X ⋃ B = X & Int(cl(D∪B)) = X ⇒ cl{int (cl(D∪B)} = X.

This means that D∪B ⊆cl{int (cl (D∪ B)} and consequently, D∪B is β-open set.

Again, U = (D∩A^c)∪ B = (D∪B)∩ (A^c∪ B) = (D∪B) ∩A^c;

Since A ∩ B = φ ⇒ B ⊂ A^c
= intersection of a β-open set & an open set.
= a β-open set.

Similarly, V = (D∩B^c) ∪ A is also β-open set.

Theorem 2.7: If D & E are disjoint dense subsets in a space (X, T), then (X, T) is β-normal and so πβ- normal.

Proof: suppose that D&E are disjoint dense sets in a space (X, T) then $D\cap E = \varphi$.

Let A and B be any pair of disjoint closed set in (X, T) so that $A\cap B = \varphi$.

Let $U = (D\cap A^C) \cup B$ & $V = (E \cap B^C) \cup A$.

Then U & V are β-open sets by lemma (2.6). Also, $A \subseteq V$ and $B \subseteq U$.

Again,
$$\begin{aligned} U \cap V &= [(D\cap A^C) \cup B]\cap [(E \cap B^C) \cup A] \\ &= [(D\cup B) \cap (A^C\cup B)] \cap [(E\cup A) \cap (B^C\cup A)] \\ &= (D \cup B) \cap A^C\cap (E\cup A) \cap B^C \quad [A\cap B = \varphi \Rightarrow A \subseteq B^C \,\&\, B \subseteq A^C] \\ &= [(D \cup B) \cap B^C] \cap [(E\cup A) \cap A^C] \\ &= [(D \cap B^C)\cup (B\cap B^C)] \cap [(E \cap A^C)\cup (A\cap A^C)] \\ &= [(D \cap B^C)\cup \varphi] \cap [(E \cap A^C)\cup \varphi] \\ &= (D \cap B^C)\cap (E \cap A^C) = (D \cap E)\cap (A^C\cap B^C) = \varphi\cap (A^C\cap B^C) = \varphi. \end{aligned}$$

i.e. U & V are disjoint β-open sets containing disjoint closed set B&A respectively.

Consequently, a pair of disjoint closed set is separated by disjoint β-open sets i.e. (X, T) is β-normal space & hence, a πβ-normal space.

Example 2.8: (i) The co-finite topology on the set R of real numbers is a πβ-normal space but not normal.

Let R stand for the set of real numbers and CF= {A: $A\subseteq R$ and $A = \varphi$ or A^C is finite. Then (R, CF) is the co-finite topological space.

Let P & Q be the sets of irrational numbers & rational numbers respectively. Then $P\cup Q = R$, $P \cap Q = \varphi$. Again, cl(P) = R = cl(Q) so that P & Q are disjoint dense subsets of (R,CF). Hence, using theorem (2.7) (R, CF) is β-normal. Since, every β-normal space is a πβ-normal space. Hence, (R, CF) is also aa πβ-normal space.

We, however, know that (R, CF) is not a normal space. Therefore, (R,CF) is a πβ-normal space but not normal.
(ii) If R stand for the set of real numbers & $T_{\sqrt{2}}$ ={A: $A \subseteq R$ and $A = \varphi$ or $\sqrt{2} \in A$}, then (R, $T_{\sqrt{2}}$) is the particular point topological space which is πβ- normal space but not β-normal.

Now, let $A\subseteq R$, then cl(A) = R if $\sqrt{2} \in A$ & cl(A) = A if $\sqrt{2} \notin A$.
$\Rightarrow$ int (cl(A)) = R if $\sqrt{2} \in A$ & int(cl(A)) = A if $\sqrt{2} \notin A$.
$\Rightarrow$ cl{int (cl(A))}= R if $\sqrt{2} \in A$ & cl{int(cl(A))} = A if $\sqrt{2} \notin A$.

Therefore, the only β-open sets in the space are those which are open. Consequently, any two disjoint closed subsets in (R, $T_{\sqrt{2}}$) cannot be separated by two disjoint β-open sets i.e. (R, $T_{\sqrt{2}}$) is not β-normal space. Again, the only π-closed subset in the space are R & φ, which are disjoint. So that any two disjoint closed subsets in (R, $T_{\sqrt{2}}$), one of which is π-closed, can be separated. i.e. (R, $T_{\sqrt{2}}$) is a π- normal space and ultimately a πβ-normal space.

Characterization of πβ-normality: Some characterizations of πβ-normality have been enunciated through the following theorem.

Theorem 2.9: For a space (X, T) the following are equivalent:
(a) (X, T) is πβ-normal space.
(b) If U is an open set U and V is π-open set whose union is X, there exist β- closed sets A and B such that $A \subseteq U$, $B \subseteq V$ & $A\cup B = X$.
(c) For every closed set A and every π-open set B such that $A \subseteq B$, there exists a β-open set V such that $A\subseteq V\subseteq$ β-cl(V) $\subseteq B$.

Proof:
(a)⇒ (b): Let U and V be a π-open sets in a πβ-normal space (X,T) such that $X = U\cup V$. Then $U^C$is a closed set &V^C is a π- closed sets. i.e. $U^C\cap V^C = \varphi$. Since (X, T) is πβ-normal there exist disjoint β-open sets U_1 and V_1 such that $U^C\subseteq U_1$ and $V^C\subseteq V_1$.

Let A =U_1^c and B =V_1^c. Then A and B are β-closed sets such that A ⊆ U, B⊆V and A ∪ B = X.

(b) ⇒(c): Let A be a closed set and B, a π- open set in a space (X, T) in the manner that A ⊆ B.

Clearly, A∩ B^c = φ⇒A^c ∪H = X, where A^c is an open sets.

Then by (b), there exist β-closed sets G and H such that G⊆A^c and H⊆B along with G ∪ H = X. This implies that A ⊆ G^c & G^c ⊆ H.

Let V = G^c, we observe that V is a β-open set. Thus, all the above facts conclude that and V A⊆ V⊆β-cl(V)⊆ B.

(c)⇒ (a): Let A and B be any two pair of disjoint closed sets in a space (X, T) such that B is π-closed. Since A ∩ B = φ, hence, A ⊆B^c and B^c is π- open. Thus using the prescribed condition (c), there exist a β –open set V such that A⊆ V ⊆β-cl(V)⊆ B^c. Taking G = V and H = $[\beta\text{cl }(V)]^c$, we observe that G & H are disjoint β-open sets such that A⊆ G & B ⊆H. Consequently, (X, T) is a πβ- normal space.

Topological property: In order to establish the topological property of πβ-normality, we first prove the following theorem.

Theorem 2.10: If f: (X, T)→ (Y, σ) is an open & continuous function, then the image of a β-open set is β-open.

Proof: let f: (X, T)→ (Y, σ) be an injective, open & continuous function from a space (X, T) to another space (Y, σ).

Let A be a β-open set in (X, T), then A ⊆ {int (cl (A)}.

Now, f(A) ⊆ f ({int (cl (A)}) = f(cl B) where B = int (cl (A)).
⊆cl f(B), as f is a continuous mapping.

i.e. f(A)⊆ cl f (int(cl (A))) = cl f (int C) ,where C = cl(A)
⊆cl (int(f(C))) as f is an open mapping.

i.e. f(A) ⊆ cl(int(f(cl (A)))⊆ cl{int(cl(f(A))
⇒ f(A) is also β-open.

Theorem 2.11: πβ-normality is a topological property.

Proof: In order to show that πβ-normality is a topological property, one has to prove that the homeomorphic image of a πβ-normal space is a πβ-normal space.

Let f: (X,T)→ (Y,σ) be a one-one onto, an open & continuous function from a πβ- normal space (X, T) to another space (Y, σ). We need to show that f(X) = Y is also a πβ- normal space. Let A & B be a pair of disjoint closed sets in (Y, σ) such that A is π-closed. Obviously, the continuity of f provides that $f^{-1}(A)$ is π-closed & $f^{-1}(B)$ is closed in X such that $f^{-1}(A) \cap f^{-1}(B)$ = φ.

Now, the πβ – normality of (X, T), there exist β-open sets U & V of X in the manner that $f^{-1}(A)$⊆U, $f^{-1}(A)$ ⊆ V and U ∩ V = φ.

Since, f is an open, continuous one –to one function hence, A ⊆ f(U), B ⊆ f(V) and (U) ∩ f(V) = φ. Using the theorem (2.10), we observe that f(U) & f(V) are β –open sets as U & V are β-open sets and f is an open, continuous function.

Thus, for a pair of disjoint closed sets A & B of (Y, σ) where A is π-closed, there exist disjoint β-open sets f(U) & f(V) in (Y, σ) such that A⊆ f(U), B ⊆ f(V) . This provides that (Y, σ) is a πβ-normal space.

Hereditary property: The following lemmas are useful and necessary for the analysis of the hereditary property of a πβ-normal space.

Lemma 2.12: If M be a closed domain (i.e. regular closed) subspace of a space X and A is β-closed in X, then A∩M is a β-closed set in M.

Proof: Let A be a β-open set in (X, T). Let M be a closed domain in (X, T) i.e. a regular closed subset of X, then (M, T_M) is a closed domain subspace of (X, T).

Now, $int_X\{cl_X(int_X (A))\} \subseteq A$. It is required to show that $A \cap M$ is a β-closed set in (M, T_M).

We have, $cl_M\{int_M (A \cap M)\} = cl_M\{ int_M (A \cap M) \cap int_X (M)\}$
$= cl_M \{int_X (A\cap M)\}$
$= cl_X\{ int_X (A\cap M)\} \cap M \subseteq \{cl_X(int_X (A)\} \cap M$

i.e. $int_M\{cl_M(int_M (A\cap M))\} \subseteq int_M[\{cl_X(int_X (A)\} \cap M]$
$\subseteq int_X [\{cl_X(int_X (A)\} \cap M] \cap M$
$= int_X [\{cl_X(int_X (A)\} \cap int_X(M)\cap M \subseteq A\cap int_X(M) \subseteq A \cap M$

$\Rightarrow A\cap M$ is a β-closed set in (M, T_M).

Lemma 2.13: If (M, T_M) is a closed domain subspace of a space (X, T), then $A\cap M$ is a β-open set in (M, T_M) whenever A is a β-open set in (X, T).

Proof: Let A be a β-open set in (X, T). Let M be a closed domain in (X, T) i.e. a regular closed subset of X, then (M, T_M) is a closed domain subspace of (X, T). Now, A^C is β-closed set in (X, T), so with the help of the Lemma (2.12), the set $G = A^C \cap M$ is a β-closed set in (M, T_M). Therefore, $M \setminus G$ is a β-open set in $((M, T_M)$.

But $M\setminus G = M \cap G^C = M\cap(A \cup M^C) = (M\cap A)\cup (M\cap M^C) =(M\cap A)\cup \varphi = M \cap A$.

Consequently, $M\cap A$ is a β-open set in (M, T_M).

Theorem 2.14: πβ –Normality is a hereditary property with respect to closed domain subspaces.

Proof: Let (M, T_M) be a closed domain subspace of a πβ-normal space (X, T). Let A & B be any disjoint closed sets in (M, T_M) such that B is π-closed. Then A& B are disjoint closed sets in (X, T) such that B is π-closed in (X, T).

Now, πβ –Normality of (X, T), there exist β-open sets U & V of X such that $A \subseteq U$ & $B \subseteq V$ where $U \cap V = \varphi$. By lemma (2.13), $U \cap M$ & $V \cap M$ are disjoint β-open sets in (M, T_M) such that $A \subseteq U \cap M$ & $B \subseteq V \cap M$ so that (M, T_M) is a πβ-normal space.

Corollary 2.15: Since, every closed and open (clopen) set in a space is a regular closed set i.e. a closed domain, hence, every clopen subspace of a πβ-normal space is a πβ-normal space.

CONCLUSION

πβ-normality, being a weaker version of β-normality, has been introduced. It has been shown that πβ-normality is a topological property as well as hereditary property with regard to closed domain spaces. Characterization as well as preservation theorem for πβ-normality has been established. Some counter examples and the criteria for the space to bear πβ-normality in terms of disjoint dense subset have been derived.

Surly the literature content for the πβ-normality is a motivation to analyse $\pi\gamma$ -normality with fundamental properties which creates the future scope of the study.

REFERENCES

1. D.Andrijevic, Semi-Pre Open Sets, Mat. Vesnik, 38, No.1, 24-32(1986).
2. El-Deeb, S.N; Ei-Monsef, M.E.Abd & Mahmoud, R.A: B-Open Sets And B-Continuous Mappings, Bull. Fac. Sci. Assiut. Univ., 12, 77-90(1983).
3. M.E. Abd El – Monsef, R.A. Mahmoud & E.R.Lashim, B-Closure and B-Interior, J. Fac.Ed.Ain.Shams. Univ., 10 (1986), 235-245.
4. C.Kuratowski, Topology I, 4th Ed., In French, Hafner, New York, (1958).
5. V. Zaitsev, On Certain Classes Of Topological Spaces And Their Bicompactifications, Dokl. Akad. NaukSssr 178 (1968) 778–779.
6. A. S. Mashhour, M.E. Abd- El- MonsefAnd S.N. El- Deeb, “On Pre- Continuous And Weak Pre- Continuous Mappings”, Proc. Math. And Phys. Soc. Egypt 53(1982), 47-53.
7. M. Ganster And I. L. Reilly, Locally Closed Sets And Lc-Continuous Functions, Int. J. Math.Math. Sci., 3 (1989), 417–424.
8. N.Levine, Semi-Open Sets and Semi-Continuity In Topological Spaces, Amer. Math. Monthly, 70(1963), 36-41.
9. O.Njastad; On Some Class Of Nearly Open Sets, Pacific J.Math., (15)(1965), 961-970.
10. C.Patty, Foundation ofTopology,Pws-Kent Publishing Company, 1993.

11. M.K.Singal And S.P.Arya, On Almost Normal And Almost Completely Regular Spaces, Glasnik Mat., 5(5) (1970), 141-152.
12. L.N. Kalantan, Π-Normal Topological Spaces, Filomat 22:1 (2008) 173–181.
13. M. K. Singal, And A. R. Singal, (1973), Mildly Normal Spaces, Kyungpook Math.J., Vol.13, No. 1, Pp.27-31, (June).
14. Govindappa Navalagi, (2008), P-Normal, Almost P-Normal and Mildly P-Normal Spaces, International Journal of Mathematics, Computer Scineces And Information Technology, Vol. 1, No. 1, Pp. 23-31.
15. S.N. Maheshwari & R.Prasad, On S-Normal Spaces, Bull. Math. Soc. Sci. Math. R.S. Rounmanie(N.S.), 22(68) (1978),27-29.
16. O.Ravi, I.Rajasekaran, S.Murugesan And A.Pandi, On β-Normal Spaces, International Journal Of Mathematics And Its Applications, Volume 3, Issue 2 (2015), P.P 35-44.
17. S.A. SaadThabit & H.Kamarulhaili, Πp-Normal Topological Spaces, Int. Journ. Of Math. Analysis, Vol. 6, (2012), No. 21, 1023-1033.
18. R.L.Prasad&B.L.Prabhakar; Pre- Separation Axioms; ActaCienciaIndica, Vol Xxxiv M No. 1.191-196 (2008),

Source of support: Nil, Conflict of interest: None Declared