International Journal of Mathematical Archive-6(10), 2015, 187-196
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SELECTION OF CRITICAL PATH IN A PROJECT NETWORK USING TOPSIS METHOD

B. PARDHA SARADHI*[1], N. RAVI SHANKAR[2], CH. SURYANARAYANA[3]

[1]Dr. L. B. college, Visakhapatnam, (A.P.), India.

[2]GITAM University, Visakhapatnam, (A.P.), India.

[3]Department of Applied Mathematics, Andhra University, Visakhapatnam, (A.P.), India.

(Received On: 01-10-15; Revised & Accepted On: 24-10-15)

ABSTRACT

In this paper, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is applied to project management to determine the critical path in a fuzzy project network. Trapezoidal fuzzy numbers are added to determine the final evaluation value of fuzzy activity times to determine the final evaluation value of fuzzy activity for each path in the network. A numerical example related to real life problem is provided to explain the procedure of proposed TOPSIS method in determining critical path with different criteria.

Key words: *Trapezoidal fuzzy numbers, TOPSIS, Project network, Critical path.*

1. INTRODUCTION

Fuzzy TOPSIS, for the first time, was initiated by Hwang and Yoon in [7]. In order to tackle Multi Criteria Decision Making (MCDM) issue, Fuzzy TOPSIS is considered as a standout in the midst of most established techniques. It focuses on the hypothesis that elective picked up should at the longest distance from the negative perfect outcome, therefore the result that extends the cost criteria and abates the benefit criteria and the momentary division from the positive faultless outcome, where the result that broadens the benefit criteria and reduces the expense criteria. The weights and investigations of the criteria are known precisely in settled TOPSIS. On the other hand, Hwang and Yoon, in [7], stated that, even under genuine state of affairs, crisp information is inadequate to demonstrate regular circumstance since human adjudications are uncertain and they cannot be evaluated with accurate numeric attributes. In order to envisage the ambivalence that stems out customarily in information from human judgments, fuzzy set speculation has been amalgamated in innumerable MCDM procedures that embrace TOPSIS. In fuzzy TOPSIS, all the weights and evaluations are depicted by the technique for semantic variables. Numbers of fuzzy TOPSIS methods and procurements have been engendered of late. Chen and Hwang, in their work of [27] for the first time connected fuzzy numbers to create TOPSIS. In [31], Triantaphyllou and Lin initiated a technique in fuzzy TOPSIS in which the relative proximity of every substitute is weighed and concentrated around fuzzy number juggling operations. Liang in [27], advocated fuzzy MCDM focused around ideal and anti-ideal perceptions. In [5], Chen conceded triangular fuzzy numbers and characterized crisp Euclidean separation between two fuzzy numbers to expand the TOPSIS strategy to fuzzy GDM circumstances. Chu [12] and Chu and Lin [13] once again cultivated the technique put forth by Chen in [5]. Chen and Tsao endeavoured to augment the strategy focused around interval-esteemed fuzzy sets in decision analysis. Jahanshahloo and others, in [18], as well as Chu and Lin in [14] gave a boost to the fuzzy TOPSIS technique that has been focused around alpha level sets with interim number-crunching. In [9], Chen and Lee broadened the theory focusing on sort-2 fuzzy TOPSIS system, keeping in mind the end goal to offer extra level of opportunity to advocate for the vulnerabilities and fuzziness of this reality of the present day. Fuzzy TOPSIS has been presented for various multi-characteristic issues to production of choices. In [34], Yong had put into use fuzzy TOPSIS for determining the plant area and in the same year, Chen and others used it for determining the suppliers. In [19], Kahraman and others had utilized fuzzy TOPSIS for choosing the mechanical automated framework. The very same fuzzy TOPSIS was connected to assist the Air Force Academy in Taiwan to pick ideal introductory criteria for determining the site of TPP airplane in a fuzzy environment. In order to assess alterably, the administration nature of three inns, by means of overviews, fuzzy TOPSIS approach was used in a very vital organization in Gran Canaria Island by Benitez and others in [3]. A fuzzy progressive TOPSIS model was put forth by Kahraman and others in [19]

Corresponding Author: B. Pardha Saradhi[1]*
[1]Dr. L. B. college, Visakhapatnam, (A.P.), India.

for the assessment of the multi-criteria of the modern automated frameworks. In [2], Ashtiani and others had put to use an interim-esteemed fuzzy TOPSIS strategy that points at looking after MCDM issues in which the weights of the criteria are unequal, using ideas of interim esteemed fuzzy sets. Ekmekcioglu and others in [15], used an altered fuzzy TOPSIS for selecting a strong waste transfer strategy as well as the site. Kutlu and Ekmekcioglu [26], had coordinated fuzzy TOPSIS and fuzzy AHP to propose another FMEA disappointment modes and Impacts dissection that allows the conquering of the deficiencies of classical FMEA. Again in [22], Kaya and Kahraman have come forward with a proposal of an altered fuzzy TOPSIS for choosing the best vitality elective of engineering. The customers' item reception process has been displayed using fuzzy TOPSIS by Kim and others in [23].

So in this we proposed an algorithm based on fuzzy TOPSIS method, projected by authors in another paper, to prefer the critical path under the four criteria. In section-2 we discussed some fundamental definition of trapezoidal fuzzy number and its Arithmetic operations like addition, subtraction, multiplication and division, also discussed on linguistic variable and finally considered the distance between the trapezoidal fuzzy numbers. In section 3, we discussed on fuzzy TOPSIS method, section4, we presented an algorithm to deal with the critical path selection problem in the project network method. In section 5 we exemplify our projected algorithmic method with real life problem like civil construction.

2. TRAPEZOIDAL FUZZY NUMBERS AND LINGUISTIC VARIABLES

In this segment, some basic definitions of fuzzy sets, fuzzy numbers, and linguistic variables are reviewed

Fuzzy set

Let X is the space of positive real values associated with variable and X is a generic element of X. A fuzzy set $\tilde{A}$ in X defined as the set of ordered pairs $\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) / x \in X\right\}$ such that $\mu_{\tilde{A}} : X \to [0,1]$

Fuzzy number

A Fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics

(i) $\tilde{A}$ is convex i.e., $\tilde{A}\left(\lambda x_1 + (1-\lambda) x_2\right) \geq \min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$

(ii) $\tilde{A}$ is normal i.e., $\exists\, x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continous A fuzzy number $\tilde{A}$ is called non negative number if $\mu_{\tilde{A}}(x) = 0 \ \forall x < 0$

Trapezoidal fuzzy number

A fuzzy number $\tilde{A}=(a,b,c,d;w)$ is said to be Trapezoidal fuzzy number, if it is a convex set which is defined as

$$\tilde{A}=\left(x, \mu_{\tilde{A}}(x)\right) \text{ where } \mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a, \\ \dfrac{w(x-a)}{b-a}, & a < x \leq b, \\ w & b < x \leq c, \\ \dfrac{w(x-d)}{c-d}, & c < x \leq d, \\ 0, & \text{otherwise}, \quad \text{where } 0 \leq w \leq 1 \end{cases}$$

Generalized trapezoidal fuzzy number

A fuzzy set A defined on the universal set of real number R, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : X \to [0, w]$ is continuous

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a] \cup [d, \infty]$

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$

(iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [b,c]$, $0 \leq w \leq 1$. .

Normal trapezoidal fuzzy numbers

If $w=1$ then $\tilde{A}=(a,b,c,d;1)$ is a normalized fuzzy number.

Arithmetic operation between trapezoidal fuzzy numbers

Addition and subtraction of any two trapezoidal fuzzy numbers is a trapezoidal fuzzy numbers but the multiplication of any two Trapezoidal fuzzy number is only an approximate trapezoidal fuzzy number. Two, positive trapezoidal fuzzy numbers, $\tilde{A}=(a_1,b_1,c_1,d_1)$ and $\tilde{B}=(a_2,b_2,c_2,d_2)$,and a positive real number k ,the operation between the trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be as follows:

$$\tilde{A}+\tilde{B}=(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2) \quad (1)$$

$$-\tilde{B}=(-d_2,-c_2,-b_2,-a_2) \quad (2)$$

$$\tilde{A}-\tilde{B}=(a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2) \quad (3)$$

$$k(.)\tilde{A}=(ka_1, kb_1, kc_1, kd_1) \quad (4)$$

$$\tilde{A}\times\tilde{B}=(a_1a_2, b_1b_2, c_1c_2, d_1d_2) \quad (5)$$

$$\frac{\tilde{A}}{\tilde{B}}=\left(\frac{a_1}{d_2},\frac{b_1}{c_2},\frac{c_1}{b_2},\frac{d_1}{a_2}\right) \quad (6)$$

Fuzzy matrix

A matrix $\tilde{D}$ is a fuzzy matrix if at least one element in a matrix is a fuzzy number.

Linguistic variables

A linguistic variable is a variable values of which are expressed in linguistic terms. The concept of using a linguistic variable comes very handy in dealing with situations that are very complex or ill-defined to be reasonable described in conventional quantitative. For example, “weight” is a linguistic variable whose values are very low, low, medium, high, very high, and also usually values of some linguistic variable such as “rating” are usually presented as very poor, poor, medium poor, fair, medium good, good, and very good presented in the Table-1 and Table-2 respectively.

Table-1: Linguistic variables for importance weight of each criterion

Linguistic variables	Trapezoidal Fuzzy Number
Very low	$(0,1,1,1)$
Low	$(0,1,3,5)$
Medium low	$(1,3,5,7)$
Medium	$(3,5,7,9)$
Medium high	$(5,7,9,11)$
High	$(7,9,10,12)$
Very high	$(9,9,10,10)$

Table-2: Linguistic variables for rating

Linguistic variables	Trapezoidal Fuzzy Number
Very low	$(0,1,1,1)$
Low	$(0,1,3,5)$
Medium low	$(1,3,5,7)$
Medium	$(3,5,7,9)$
Medium high	$(5,7,9,11)$
High	$(7,9,10,12)$
Very high	$(9,9,10,10)$

Table-3: Importance weight of criteria from decision makers

Criteria	Decision makers
Time	Very high
Cost	Very high
Risk	High
Quality	High

Distance between trapezoidal fuzzy numbers

Two Trapezoidal fuzzy numbers $\tilde{A}=(a_1, b_1, c_1, d_1)$ and $\tilde{B}=(a_2, b_2, c_2, d_2)$, the distance between them can be determined by using the vertex method

$$f_d = \sqrt{\frac{1}{6}\left((a_1-a_2)^2 + 2(b_1-b_2)^2 + 2(c_1-c_2)^2 + (d_1-d_2)^2\right)} \quad (7)$$

3. FUZZY TOPSIS METHOD

TOPSIS method is offered by Chen and Hwang [8], in reference to Hwang and Yoon [17]. TOPSIS is a method of multiple criteria so that solutions can be identified from a finite set of alternatives which have been widely applied to numerous sciences in order to take different decisions. The basic principle of the method is that the alternative picked up should be at the shortest distance from the positive ideal solution PIS A^+, and at the farthest distance from the negative ideal solution, NIS A^-. An extension version of TOPSIS has been proposed by Chen et al [6]. This fuzzy TOPSIS method can deal with the ratings of both quantitative as well as qualitative criteria and can effectively select a suitable alternative. As a matter of fact, the fuzzy TOPSIS method is very supple. As per the coefficient of closeness, not only the ranking order but also the assessment status of all alternatives can be determined. And therefore, basing on these advantages and the fuzzy data which are put to use in this paper, we apply the proposed fuzzy TOPSIS method [6] to find out the best alternative. This method is given as the following steps

Step-1: Construct the normalized fuzzy decision matrix:
The linear-scale method is used here so as to avoid complexity of mathematical operations and to transform the various criteria scales into comparable scales. The set of criteria can be classified into benefit criteria (the larger the rating, the greater the preference) and cost criteria (the smaller the rating, the greater the preference). And thus, the normalized fuzzy decision matrix can be represented as $\tilde{R} = \left[\tilde{r}_{ij}\right]_{m\times n}$

where

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}\right), j \in B \quad (8)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) j \in C \quad (9)$$

where B in Eq.8 and C in Eq.9 are the sets of benefit criteria and cost criteria, respectively.

$$d_j^+ = \max_i \left(c_{ij}\right) \quad (10)$$

$$a_j^- = \min_i \left(a_{ij}\right) \quad (11)$$

The normalization method mentioned above is designed to preserve the property in which the elements $\tilde{r}_{ij}$ are standardized (normalized) trapezoidal fuzzy numbers.

Step-2: Construct weighted normalized fuzzy-decision matrix:
Considering the different importance of each criterion, the weighted normalized fuzzy decision matrix is constructed as

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m\times n} \; i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n \quad (12)$$

where $\tilde{v}_{ij} = w_j(.)\tilde{r}_{ij}$ and also the fuzzy weight of each criterion.

Step-3: Determine FPIS and FNIS: According to the weighted normalized fuzzy-decision matrix, normalized positive triangular fuzzy numbers can also approximate the elements $\tilde{v}_{ij}$. Then, the fuzzy positive ideal solution, FPIS $\left(A^{+}\right)$ and fuzzy negative ideal Solution, FNIS $\left(A^{-}\right)$ can be defined as

$$A^{+} = \left(\tilde{v}_1^{+}, \tilde{v}_2^{+}, ..., \tilde{v}_n^{+}\right) \tag{13}$$

$$A^{-} = \left(\tilde{v}_1^{-}, \tilde{v}_2^{-}, ..., \tilde{v}_n^{-}\right) \tag{14}$$

where $\tilde{v}_j^{-} = \min_i \left(v_{ij1}\right), \tilde{v}_j^{+} = \max_i \left(v_{ij4}\right)$, $i = 1, 2, 3, ..., m$ & $j = 1, 2, 3, ..., n$.

The index v_{ij1} and v_{ij4} ,1 and 4 determine the first and fourth elements in a Trapezoidal fuzzy number respectively.

Step-4: Calculate the distance of each alternative from FPIS and FNIS respectively: The distance of each alternative from A^{+} and A^{-} can be currently calculated as

$$d_i^{+} = \sum_{j=1}^{n} f_{d_v}\left(\tilde{v}_{ij}, \tilde{v}_j^{+}\right), i = 1, 2, 3, ..., m \tag{15}$$

$$d_i^{-} = \sum_{j=1}^{n} f_{d_v}\left(\tilde{v}_{ij}, \tilde{v}_j^{-}\right) \; i = 1, 2, 3, ..., m \tag{16}$$

Step-5: Calculate the closeness coefficient of each alternative:

A closeness coefficient is defined to demarcate the ranking order of all possible alternatives once d_i^{+} and d_i^{-} wherein for each alternative, A_i has been calculated. The coefficient of closeness represents the distances to the fuzzy positive ideal solution A^{+} and the fuzzy negative ideal solution A^{-} simultaneously by taking the relative closeness to the fuzzy positive ideal solution. The closeness coefficient cc_i of each alternative is calculated as

$$cc_i = \frac{d_i^{-}}{d_i^{-} + d_i^{+}} \tag{17}$$

Step-6: According to the closeness coefficient, we can understand the assessment status of each alternative and determine the ranking order of all alternatives (each criterion has a larger closeness coefficient has a higher level in the ranking order of all alternatives).

In next section, we propose our method to select the critical path in fuzzy environment.

4. PROPOSED METHOD FOR THE CRITICAL PATH SELECTION

In this section a methodical approach to find the critical path problem under fuzzy atmosphere. In this paper, various criteria like weights and qualitative are supposed as linguistic variables which are represented as positive Trapezoidal fuzzy numbers. Now detailed explanation is given for the proposed method. Huge project can be divided in to many activities. Determine the duration and preference relations of these activities. The preference relationship of these activities may be visualized in the Fuzzy project network. So drawing the precedence project network which arc denote activities also determining all criteria which are important to select the critical path in the project network under these criteria. A path is one of the route from starting node to the ending node. Identify all the paths in the fuzzy project network which start with starting event and end with the ending event. Under the consideration of the each path we choose one path as a critical path. Choose the suitable linguistic variables for qualitative criteria and trapezoidal fuzzy number for quantitative criteria, to obtain the fuzzy evaluation of activity under each criterion. Then, all linguistic evaluations are converted into suitable trapezoidal fuzzy numbers. The length of a path is the sum of durations of activities on the path, so add up trapezoidal fuzzy number to establish the final assessment value of each criterion for paths. The length of longest path of the entire project network is the project duration. The longest path of the project network is also called as critical path. So, build the fuzzy-decision matrix in which its alternative is the paths that start with the starting event and end with the ending event. To calculate the completion time of the project, we require concluding the critical path in the project network under different criterion. To prefer suitable alternative, critical path, under different criteria, apply fuzzy TOPSIS method can deal with the ratings of both qualitative as well as quantitative criteria.

5. APPLICATION OF PROPOSED METHOD TO CIVIL CONSTRUCTION

The recommended method is at present employed to real life problem, wherein the process of computing it is summarized as follows. The first and foremost activity for constructing a building is to obtain material for making the beams stand up. This is followed by the excavation of foundations in which the beams are erected. When this job of excavation is done with, the next thing is to procure bricks for in order the make the walls and then wood is secure wood for the basic furnishings such as cupboards and shelves. The process of acquisition does not stop just here as it is essential to obtain sanitary fittings like commodes and faucets and then the process furthers to procure electrical equipment such as bulb-holders, switches and sockets. After acquiring all the requisite materials, it is to further proceed to the beginning of the construction of the building. When the pits for are ready, foundations are laid for the edifice to stand strong. Once the foundation job is done, brick work is initiated so as to bring a basic look to the structure. A perfect building would not complete without a good drainage system that is to follow the brick work. Once the drainage and brick work is completed, the activity proceeds to the placing of roof timbers, which is very necessary for the construction to look complete. Then roof covers are erected so that the roofing is finished. Then comes the turn of fixing the exterior doors such that the house is completely secure and impregnable. Completing the plumbing is a must for the water to run to and fro. Electric wiring is another crucial stage in making a building illuminated since no one can live in darkness. The electric work is very promptly followed by plaster work to give a fine look to the structure. Getting done the job of carpentry is another task in building a group house which is followed by placing all the sanitary fitting in the respective places. Finally it comes to the fixing the doors and point brick work after finishing which, it can be quoted that the construction of a group house is fully completed in all respects. Just the same way, an activity is divided into many a particle, so that a conclusion could be reached through the longest way in the shortest while possible. The required activities of the civil construction is collected and represented in Table -4. Draw the priority of the network. It is displayed in Fig.2. Decision makers use the linguistic weighing variables as depicted in the Table-1 to measure the significance of the criteria. The important weights of the criteria as stipulated by the decision makers are exhibited in the Table-3. Regulate all activities in the paths which commence with the starting event and conclude with the ending event. At this point decision makers use the linguistic rating variables as shown in Table-2 and the trapezoidal fuzzy numbers to appraise the ratings of these activities pertaining to each criterion. The ratings of the activities by the decision makers under various criteria are disclosed in Table-4. Then the linguistic calculations as shown in the Tables 3 and 5 are converted into trapezoidal fuzzy numbers as displayed in Table-6. Sum up trapezoidal fuzzy numbers to ascertain the values of final evaluation of each criterion for paths that get under way with the starting event and end up with the ending event. Subsequently create a fuzzy decision matrix as depicted in Table-7. The standardized fuzzy decision matrix is constructed as shown in Table-8. The weighed up and regularized fuzzy decision matrix is built up as described in the Table-9. At this point finalize FPIS and FNIS as

$$A^{+}=\begin{Bmatrix}(9,9,9,9),(9,9,9,9),\\(11,11,11,11),(11,11,11,11)\end{Bmatrix}$$

$$A^{-}=\begin{Bmatrix}(1.12,1.12,1.12,1.12),\\(1.68,1.68,1.68,1.68),\\(0.9,0.9,0.9,0.9),(1.3,1.3,1.3,1.3)\end{Bmatrix}$$

Assess the distance of each path from FPIS and FNIS in regard to each criterion as exhibited in the Tables 10 and 11 respectively. Then workout and of five conceivable paths and then calculate the closeness of coefficient of each path as cited in the Table-12. According to the closeness coefficient of five paths, we know that the second path (1-3-6-10) is the critical path under the time, cost, risk, and quality criteria. A notable is that this example is solved only with time being the yardstick. In this case the trapezoidal fuzzy activity times to determine the final evaluation have been added up so that the value of fuzzy activity times of each path could be worked out that commence with the starting event and conclude with the ending event. As per this, we possessed five alternatives (paths) with trapezoidal fuzzy numbers assigned. Thus Yager's fuzzy ranking method [29] is applied to classify these five fuzzy numbers and pick out the alternative which is the critical path as per the criteria of time. Hence, according to this method of ranking, the first path has first rank, according to the largest amount of ranking function among other paths and therefore it in the critical path. Therefore, it is very essential to take into account different criteria in ascertaining a critical path.

Table- 4: Activities with description of Building construction

Activity	Description
1-2	Obtain material for beams ,Excavate foundations, Obtain bricks, Obtain wood
1-3	Obtain sanitary fittings, etc
1-4	Obtain electric equipment
2-5	Lay foundations, Brick work, Place roof timbers
3-5	Lay drains

3-6	Plumbing
6-10	Place sanitary fittings
4-6	Plaster
4-8	Electric wiring
8-9	Board fitting
9-10	main connection
5-7	Complete roofing, carpentry
7-10	Fit enterer doors, etc

Table-5: Rating of the activity by decision makers under various criteria

Activity	Time	Cost	Risk	Quality
1 – 2	$(4, 7, 10, 12)$	$(1500, 2000, 2500, 3000)$	Medium poor	$(1, 2, 3, 4)$
1 – 3	$(3, 6, 9, 12)$	$(3500, 1000, 1500, 2000)$	Medium poor	$(3, 4, 5, 6)$
1 – 4	$(2, 4, 6, 8)$	$(200, 700, 1200, 1700)$	Fair	$(2, 3, 4, 5)$
2 – 5	$(3, 5, 7, 9)$	$(200, 700, 1200, 1700)$	Fair	$(3, 4, 5, 6)$
3 – 5	$(3, 4, 5, 6)$	$(1500, 2000, 2500, 3000)$	Fair	$(3, 4, 5, 6)$
4 – 6	$(2, 3, 4, 5)$	$(5500, 6000, 6500, 7000)$	Medium poor	$(2, 3, 4, 5)$
8 – 9	$(8, 10, 12, 14)$	$(1500, 2000, 2500, 3000)$	Medium poor	$(2, 3, 4, 5)$
3 – 6	$(2, 4, 6, 8)$	$(1500, 2000, 2500, 3000)$	Medium poor	$(4, 3, 5, 7)$
5 – 7	$(5, 8, 11, 14)$	$(700, 1200, 1700, 2200)$	Medium poor	$(2, 3, 4, 5)$
4 – 8	$(4, 5, 6, 7)$	$(1000, 1500, 2000, 2500)$	Fair	$(3, 4, 5, 6)$
6 – 10	$(3, 6, 9, 12)$	$(900, 2000, 2500, 5000)$	Medium poor	$(2, 3, 4, 5)$
7 – 10	$(3, 5, 7, 9)$	$(3500, 4000, 4500, 5000)$	Medium poor	$(3, 4, 5, 6)$
9 – 10	$(4, 5, 6, 7)$	$(2500, 3000, 3500, 4000)$	Medium poor	$(1, 2, 3, 4)$

Table-6: Converted linguistic evaluation in to Trapezoidal fuzzy numbers.

Activity	Time	Cost	Risk	Quality
1 – 2	$(4, 7, 10, 12)$	$(1500, 2000, 2500, 3000)$	$(5, 7, 9, 11)$	$(1, 2, 3, 4)$
1 – 3	$(3, 6, 9, 12)$	$(3500, 1000, 1500, 2000)$	$(1, 3, 5, 7)$	$(3, 4, 5, 6)$
1 – 4	$(2, 4, 6, 8)$	$(200, 700, 1200, 1700)$	$(1, 3, 5, 7)$	$(2, 3, 4, 5)$
2 – 5	$(3, 5, 7, 9)$	$(200, 700, 1200, 1700)$	$(3, 5, 7, 9)$	$(3, 4, 5, 6)$
3 – 5	$(3, 4, 5, 6)$	$(1500, 2000, 2500, 3000)$	$(3, 5, 7, 9)$	$(3, 4, 5, 6)$
4 – 6	$(2, 3, 4, 5)$	$(5500, 6000, 6500, 7000)$	$(3, 5, 7, 9)$	$(2, 3, 4, 5)$
8 – 9	$(8, 10, 12, 14)$	$(1500, 2000, 2500, 3000)$	$(1, 3, 5, 7)$	$(2, 3, 4, 5)$
3 – 6	$(2, 4, 6, 8)$	$(1500, 2000, 2500, 3000)$	$(1, 3, 5, 7)$	$(4, 3, 5, 7)$
5 – 7	$(5, 8, 11, 14)$	$(700, 1200, 1700, 2200)$	$(1, 3, 5, 7)$	$(2, 3, 4, 5)$
4 – 8	$(4, 5, 6, 7)$	$(1000, 1500, 2000, 2500)$	$(5, 7, 9, 11)$	$(3, 4, 5, 6)$
6 – 10	$(3, 6, 9, 12)$	$(900, 2000, 2500, 5000)$	$(3, 5, 7, 9)$	$(2, 3, 4, 5)$
7 – 10	$(3, 5, 7, 9)$	$(3500, 4000, 4500, 5000)$	$(5, 7, 9, 11)$	$(3, 4, 5, 6)$
9 – 10	$(4, 5, 6, 7)$	$(2500, 3000, 3500, 4000)$	$(5, 7, 9, 11)$	$(1, 2, 3, 4)$

Table-7: Fuzzy –decision matrix, Fuzzy weight of criteria

Criteria Activity	Time $(7,9,9,9)$	Cost $(7,9,9,9)$	Risk $(5,7,9,11)$	Quality $(5,7,9,11)$
1 – 2 – 5 – 7 – 9	$(15,25,35,44)$	$(5900,7900,9900,11900)$	$(14,22,30,38)$	$(9,13,17,21)$
1 – 3 – 6 – 10	$(8,16,24,32)$	$(2900,5000,6500,8000)$	$(7,13,19,25)$	$(8,10,14,18)$
1 – 3 – 5 – 7 – 10	$(14,23,32,41)$	$(6200,8200,10200,12200)$	$(14,22,30,38)$	$(11,15,19,23)$
1 – 4 – 6 – 10	$(7,13,19,25)$	$(6600,8700,10200,11700)$	$(9,15,21,27)$	$(6,9,12,15)$
1 – 4 – 8 – 9 – 10	$(18,24,30,36)$	$(5200,7200,9200,11200)$	$(12,20,28,36)$	$(8,12,16,20)$

Table-8: Normalized Fuzzy –decision matrix

Criteria Activity	Time	Cost	Risk	Quality
1 – 2 – 5 – 7 – 9	$(0.16,0.2,0.28,0.47)$	$(0.24,0.29,0.37,0.49)$	$(0.18,0.23,0.32,0.5)$	$(0.39,0.57,0.74,0.91)$
1 – 3 – 6 – 10	$(0.22,0.29,0.44,0.88)$	$(0.36,0.45,0.58,1)$	$(0.28,0.37,0.54,1)$	$(0.35,0.43,0.61,0.78)$
1 – 3 – 5 – 7 – 10	$(0.17,0.22,0.3,0.5)$	$(0.24,0.28,0.35,0.47)$	$(0.18,0.23,0.32,0.5)$	$(0.48,0.65,0.83,1)$
1 – 4 – 6 – 10	$(0.28,0.37,0.54,1)$	$(0.25,0.28,0.33,0.44)$	$(0.26,0.33,0.47,0.78)$	$(0.26,0.39,0.52,0.65)$
1 – 4 – 8 – 9 – 10	$(0.19,0.23,0.29,0.39)$	$(0.26,0.32,0.4,0.56)$	$(0.19,0.25,0.35,0.58)$	$(0.34,0.52,0.69,0.87)$

Table-9: Normalized Fuzzy –decision matrix

Criteria Activity	Time	Cost	Risk	Quality
1 – 2 – 5 – 7 – 9	$(1.12,1.8,2.52,4.23)$	$(1.68,2.61,3.33,4.41)$	$(0.9,1.61,2.8,5.5)$	$(1.95,3.99,6.66,10.01)$
1 – 3 – 6 – 10	$(1.54,2.61,3.96,7.92)$	$(2.52,4.05,5.22,9)$	$(1.4,2.59,4.86,11)$	$(1.75,3.01,5.49,8.58)$
1 – 3 – 5 – 7 – 10	$(1.19,1.98,2.7,4.5)$	$(1.68,2.52,3.15,4.23)$	$(0.9,1.61,2.8,5.5)$	$(2.4,4.55,7.47,11)$
1 – 4 – 6 – 10	$(1.96,3.33,4.86,9)$	$(1.75,2.52,2.97,3.96)$	$(1.3,2.31,4.23,8.58)$	$(1.3,2.73,4.68,7.15)$
1 – 4 – 8 – 9 – 10	$(1.33,2.07,2.61,3.51)$	$(1.82,2.88,3.6,5.04)$	$(0.95,1.75,3.15,6.38)$	$(1.7,3.64,6.21,9.57)$

Table-10: Distance between paths and A^+ with respect to each criterion

Criteria Activity	Time	Cost	Risk	Quality
$f_d(A_1, A^+)$	6.74	6.06	8.59	6.04
$f_d(A_2, A^+)$	5.62	4.46	7.17	6.83
$f_d(A_3, A^+)$	6.58	6.17	8.59	5.51
$f_d(A_4, A^+)$	4.97	6.25	7.55	7.37
$f_d(A_5, A^+)$	6.67	5.78	8.33	4.59

Table-11: Distance between paths and A^- with respect to each criterion

Criteria Activity	Time	Cost	Risk	Quality
$f_d\left(A_1, A^-\right)$	1.56	1.56	2.21	4.97
$f_d\left(A_2, A^-\right)$	3.34	3.89	4.82	3.96
$f_d\left(A_3, A^-\right)$	1.73	1.43	2.21	5.67
$f_d\left(A_4, A^-\right)$	4.09	1.29	3.77	3.19
$f_d\left(A_5, A^-\right)$	1.41	1.89	2.63	4.61

Table-12: Distances d_i^+ , d_i^- and cc_i

Criteria Activity	d_i^+	d_i^-	$d_i^+ + d_i^-$	$cc_i = \frac{d_i^-}{d_i^- + d_i^+}$
1 – 2 – 5 – 7 – 9	27.43	10.30	37.73	0.273
1 – 3 – 6 – 10	24.05	16.01	40.06	0.40
1 – 3 – 5 – 7 – 10	26.85	11.03	37.89	0.30
1 – 4 – 6 – 10	26.13	12.35	38.48	0.32
1 – 4 – 8 – 9 – 10	25.38	10.57	35.95	0.130

Critical path of the fuzzy project network as per criteria using TOPSIS method is $1-3-6-10$

6. CONCLUSIONS

In this paper TOPSIS method has applied to fuzzy project network to determine the critical path using several criteria. Trapezoidal fuzzy numbers have been used as fuzzy activity times, fuzzy cost, fuzzy quality etc. to find criticality using linguistic terms. To rank the critical paths in fuzzy project network, several new methods are introduced in TOPSIS algorithm. A numerical example related to real life problem has provided to explain the procedure of proposed TOPSIS method in determining critical path with different criteria. For future work, this method is applicable to developing a group decision support system and solving time cost trade-off problems in fuzzy environment.

REFERENCES

1. Ashrafzadeh, M., Rafiei, F. M., Isfahani, N. M., Zare, Z. (2012), Application of fuzzy TOPSIS method for the selection of Warehouse Location: A Case Study, Interdisciplinary Journal of Contemporary Research in Business;Jan2012, Vol. 3, Issue 9, pp 655
2. Ashtiani, B., Haghighirad, F., Makui, A. and Montazer, G. A. (2008), Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets, Applied Soft Computing, Vol. 9, No. 2, pp. 457-461.
3. Benitez, J. M., Martin, J. C. and Roman, C. (2007), Using fuzzy number for measuring quality of service in the hotel industry, Tourism Management, Vol. 28, No. 2, pp. 544-555.
4. Brown, P. A. and Gibson, D. F. (1972), A quantified model for facility site selection: An application to a multi-facility location problem, AIIE Transactions, Vol. 4, pp. 1–10.
5. Chen, C. T. (2000), Extension of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems, Vol. 114, No. 1, pp. 1-9.
6. Chen, C. T., Lin, C. T. and Huang, S. F. (2006), A fuzzy approach for supplier evaluation and selection in supply chain management, International Journal of Production Economics, Vol. 102, No. 2, pp. 289-301.
7. Chen, C.T. (2001), A fuzzy approach to select the location of the distribution center, Fuzzy Sets and Systems, Vol. 118, pp 65–73.
8. Chen, S. J. and Hwang, C. L. (1992), Fuzzy multi attribute decision making, lecture notes in economics and mathematical system series, Vol. 375. Springer-Verlag, New York.
9. Chen, S.M. and Lee, L.W. (2010), Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method, Expert Systems with Applications, Vol. 37, No. 4, pp. 2790-2798.

10. Chen, T.Y. and Tsao, C.Y. (2008), The interval-valued fuzzy TOPSIS method and experimental analysis, Fuzzy Sets and Systems, Vol. 159, No. 11, pp. 1410- 1428.
11. Chou, S.Y., Chang, Y.H. and Shen, C.Y.(2008), A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes, European Journal of Operational Research, Vol. 189, pp 132–145.
12. Chu, T. (2002), Selecting plant location via a fuzzy TOPSIS approach, International Journal of Advanced Manufacturing Technology, Vol. 20, No. 11, pp. 859-864.
13. Chu, T. and Lin, Y. C. (2002), Improved extensions of the TOPSIS for group decision making under fuzzy environment, Journal of Information and Optimization Sciences, Vol. 23, pp. 273-286.
14. Chu, T. C. and Lin, Y. C. (2009), An interval arithmetic based fuzzy TOPSIS model, Expert Systems with Applications, Vol. 36, No. 8, pp. 10870-10876.
15. Ekmekcioglu, M., Kaya, T. and Kahraman, C. (2010), Fuzzy multi-criteria disposal method and site selection for municipal solid waste,Waste Management, Vol. 30, No. 8-9, pp. 1729-1736.
16. Ghose, S. (2008), West Coast Regional Carbon Sequestration Partnership Annual Business Meeting, Anchorage AK, slide. 1-18.
17. Hwang C. L. and Yoon K. (1981), Multiple Attribute Decision Making Methods and Applications, A State-of-the-Art Survey, Springer Verlag, New York.
18. Jahanshahloo, G. R. Hosseinzadeh Lotfi, F. and Izadikhah, M. (2006), Extension of the TOPSIS method for decision-making problems with fuzzy data, Applied Mathematics and Computation, Vol. 181, No. 2, pp. 1544–1551.
19. Kahraman, C., Cevik, S., Ates, N. Y. and Gulbay, M. (2007), Fuzzy multi-criteria evaluation of industrial robotic systems, Computers & Industrial Engineering, Vol. 52, No. 4, pp. 414-433.
20. Kahraman, C., Ruan, D. and Dogan, I. (2003), Fuzzy group decision-making forfacility location selection, Information Sciences, Vol. 157, pp. 135–153.
21. Kannan G., Shaligram P., P. Sasi Kumar (2009), A hybrid approach using ISM and fuzzy TOPSIS for the selection of reverse logistics provider, Resour. Conserv. Recycl, 54, pp. 28–36.
22. Kaya, T. and Kahraman, C. (2011), Multi-criteria decision making in energy planning using a modified fuzzy TOPSIS methodology, Expert Systems with Applications, Vol. 38, No. 6, pp. 6577-6585 .
23. Kim, S., Lee, K., Cho, J. K. and Kim, C.O. (2011), Agent-based diffusion model for an automobile market with fuzzy TOPSIS-based product adoption process, Expert Systems with Applications, Vol. 38, No. 6, pp. 7270-7276.
24. Kuo, R.J., Chi, S.C. and Kao, S.S. (1999), A decision support system for locating convenience store through fuzzy AHP, Computers and Industrial Engineering, Vol. 37, pp 323–326. National Institute of Technology, Rourkela Page 37
25. Kuo, R.J., Chi, S.C. and Kao, S.S. (2002), A decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network, Computers in Industry, Vol. 47, pp 199–214.
26. Kutlu, A. C. and Ekmekcioglu, M. (2011), Fuzzy failure modes and effects analysis by using fuzzy TOPSIS integrated with fuzzy AHP, Expert Systems with Applications, Vol. 39, No. 1, 61-67.
27. Liang, G. S. (1999), Fuzzy MCDM based on ideal and anti-ideal concepts, European Journal of Operational Research, Vol. 112, No. 3, pp. 682-691.
28. Liang, G.S. and Wang, M.J.J. (1991), A fuzzy multi-criteria decision-making method for facility site selection, International Journal of Production Research, Vol. 29, pp. 2313–2330.
29. Rao, P.P.B. and Ravi Shankar, N. (2012), Ranking Generalized Fuzzy Numbers using Area, Mode, Spreads and Weights, International Journal of Applied Science and Engineering, Vol. 10, No.1, pp. 41-57.
30. Tavakkoli-Moghaddam, R. and Mousavi, S.M. (2011), An integrated AHP-VIKOR methodology for plant location selection, IJE Transactions B: Applications, Vol. 24, No. 2, pp. 127-137.
31. Triantaphyllou, E. and Lin, C.L. (1996), Development and evaluation of five fuzzy multi attribute decision making methods, International Journal of Approximate Reasoning, Vol. 14, No. 4, pp. 281–310.
32. Wang, T. C. and Chang, T. H. (2007), Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment, Expert Systems with Applications, Vol. 33, No. 4, pp. 870-880.
33. Yager RR (1981) A procedure for ordering fuzzy subsets of the unit interval. Inf Sci 24:143–161.
34. Yong, D. (2006), Plant location selection based on fuzzy TOPSIS, International Journal of Advanced Manufacturing Technologies, Vol. 28, No. 7-8, pp. 323-326. National Institute of Technology, Rourkela Page 38.
35. Zangeneh, A., Jadid, S. and Rahimi-Kian, A. (2011), A fuzzy environmental-technical economic model for distributed generation planning, Energy, Vol.36, No.2, pp.3437-3445.

Source of support: Nil, Conflict of interest: None Declared