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NEW CLASS OF LOCALLY CLOSED SETS IN TOPOLOGICAL SPACE

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ABSTRACT

The aim of this paper is to introduce and study the new classes of generalized closed set namely $g^{\wedge\wedge}p$ -closed sets. Furthermore the relations with other notions connected with the forms of closed sets are investigated. Also we define the space namely $Tp^{\wedge}$ space using this definition.

***Keywords and Phrases**: $g^{\wedge\wedge}p$ -closed set, $Tp^{\wedge}$ space, $^{\wedge}Tp$ space, $Tp^{\wedge\wedge}$ space, α $Tp^{\wedge}$ space.*

1. INTRODUCTION

The study of generalized closed sets in topological space was initiated by Levine [10] in 1970 and concept of T1/2 -space was introduced. Manoj Garg and Shikha Agarwal, C.K.Goel [14] introduced the concept of $g^{\wedge\wedge}$-closed sets in topological space.

In this paper we first introduce a new class of closed sets namely $g^{\wedge\wedge}p$ –closed sets which is placed in between the class of closed sets and the class of g-closed sets and then investigate some of its properties . We also introduce new class of spaces namely $Tp^{\wedge}$ space, $^{\wedge}$ Tp space, $Tp^{\wedge\wedge}$ space, α $Tp^{\wedge}$ space.

2. PRELIMINARIES

Throughout this paper (X, τ) (or X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, interior of A and complement of A respectively in X.

We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X, τ) is called
(1) a pre-open set [15] if $A \subseteq int(cl(A)) \subseteq A$.
(2) a semi-open set [9] if $A \subseteq cl(int(A))$.
(3) an α-open set if [16] $A \subseteq int(cl(int(A)))$.
(4) a semi-preopen set [1] (= β -open) if $A \subseteq cl(int(cl(A)))$.

The class of all closed subsets of a space (X, τ) is denoted by $C(X, \tau)$. The intersection of all semi closed (resp. pre-closed, semi-preclosed, α-closed) sets containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure, semi-pre-closure and α-closure) of A and is denoted by scl(A)(resp. pcl(A), spcl(A) and αcl(A)).

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Definition 2.2: A subset A of a space (X, τ) is called

(1) a g-closed set [10] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(2) a g^*- closed set [19] if cl (A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
(3) a semi-generalized closed set [4] (briefly sg-closed) if scl (A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
(4) a generalized semi-closed set [2] (briefly gs-closed) if scl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(5) a generalized α-closed set [11] (briefly g α-closed) if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
(6) an α-generalized closed set [12] (briefly αg-closed) if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(7) a generalized semi-preclosed set [5] (briefly gsp-closed) if spcl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ) .
(8) a gene ralized pre closed set [13] (gp-closed) if pcl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(9) a generalized preregular closed set [8] (briefly gpr-closed) if pcl (A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).
(10) g^* p -closed set [21] if pcl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(11) $g^\#$ -closed set [20] if cl (A) ⊆ U whenever A ⊆ U and U is αg-open in (X, τ).
(12) g^* s -closed set [17] if scl (A) ⊆ U whenever A ⊆ U and U is gs-open in (X, τ).
(13) $g^{\wedge\wedge}$-closed set [14] if cl(A) ⊆ U whenever A ⊆ U and U is sg-open in (X, τ).

Definition 2.3: A subset A of a space (X, τ) is called

(1) locally closed (briefly lc) set [7] if A = U ∩ F, where U is open and F is closed in (X, τ).
(2) generalized locally closed (briefly glc) set [3] if U = F ∩F, where U is g-open and F is g-closed in (X, τ).
(3) $g^\wedge$ -locally closed (briefly $g^\wedge$lc) set [22] if A = U ∩ F, where U is $g^\wedge$ -open and F is g^-closed in (X, τ).
(4) $g^\#$ -locally closed (briefly $g^\#$ lc) set [23] if A = U ∩ F , where U is $g^\#$ -open and F is $g^\#$ -closed in (X, τ).
(5) g _ -locally closed (briefly g _lc) set [24] if A = U ∩ F, where U is g _ -open and F is g _ -closed in (X, τ).
(6) g^* s -locally closed (briefly g^* slc) set [18] if A = U ∩F, where U is g^* s –open and F is g^* s -closed in (X, τ).

Definition 2.4: A topological space (X, τ) is called

(1) Sub maximal space [6] if every dense subset of (X, τ) is open in (X, τ).
(2) Semi-pre-T 1/2 space [4] if every gsp-closed set is semi-preclosed.

Proposition 2.5:

(1) [14] Every open set is $g^{\wedge\wedge}$-open.
(2) [14] Every $g^{\wedge\wedge}$ -open set is g -open.

3. BASIC PROPERTIES OF $g^{\wedge\wedge}$ p -CLOSED SET

In this section we introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called a $g^{\wedge\wedge}$p -closed set if pcl(A) ⊆ U whenever A ⊆ U and U is $g^{\wedge\wedge}$ -open in (X, τ)

Theorem 3.2: Every closed set is $^{\wedge\wedge}$gp -closed.

Proof: Let A be a closed set. Then cl(A) = A. Let U be any $g^{\wedge\wedge}$ -open set containing A.

Since pcl(A) ⊆ cl(A) = A ⊆ U . Then A is $g^{\wedge\wedge}$ p -closed.

Remark 3.3: The following example supports that a $g^{\wedge\wedge}$ p -closed set need not be closed.

Example 3.4: Let X = {a, b, c} and τ = {φ, {a}, X}. $g^{\wedge\wedge}$ pC(X) = {φ, {b}, {c}, {a, b}, {b, c}, {a, c}, X}. Here {a, b} is $g^{\wedge\wedge}$ p -closed but not a closed set of (X, τ).

Theorem 3.5: Every g^* -closed set is $g^{\wedge\wedge}$ p -closed set.

Proof: Let A be a g^* -closed set. Let U be an $g^{\wedge\wedge}$ -open set containing A. Since pcl(A) ⊆ cl(A) ⊆ U, A is $^{\wedge\wedge}$gp -closed.

Remark 3.6: The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let X = {a, b, c} with τ = {φ, {c}, {b, c}, X}. $g^{\wedge\wedge}$ pC(X) = {φ, {a}, {b}, {a, b}, {a, c}, X}.

Here B = {b} is $g^{\wedge\wedge}$ p -closed but not a g^* -closed set.

Theorem 3.8: Every g^* p -closed set is g ^^ p -closed set.

Proof: Since every g^^ -open set is g -open, the theorem follows.

Remark 3.9: The converse of the above theorem need not be true as seen from the following example.

Example 3.10: Let (X, τ) be as in example 3.4, the set {a, b} is g^^ p –closed but not a g^* p -closed set.

Theorem 3.11: Every gα-closed set is g^^ p -closed set in (X, τ).

Proof: Let A be a gα-closed set. Let U be an g^^ -open set containing A. Since pcl(A) ⊆ αcl(A) ⊆ U, A is g^^ p -closed.

Remark 3.12: The following example shows that the converse of the above theorem is not necessarily true as seen from the following example.

Example 3.13: Let X = {a, b, c} with τ = {φ, {a}, {a, b}, X}. Then g^^ pC(X) = {φ, {b}, {c}, {b, c}, {a, c}, X}. The set {a, c} is g^^ p -closed set but not a gα-closed set.

Theorem 3.14: Every g^^ p -closed set is gsp-closed set in (X, τ).

Proof: It follows from the fact that every open is ^^g -open and spcl(A) ⊆ pcl(A) ⊆ cl(A) ⊆ U for any subset A of (X, τ).

Remark 3.15: The converse of the above theorem need not be true as seen from the following example.

Example 3.16: Let X = {a, b, c} with τ = {φ, {a}, {b}, {a, b}, X}. Then g^^ pC(X) = {φ, {c}, {b, c}, {a, c}, X}. The set {a} is gsp-closed but not a g^^ p -closed set.

Theorem 3.17: Every g^^ -closed set is g^^ p -closed in (X, τ).

Proof: Let A be a g^^ -closed set. Let U be an g^^ -open set containing A. Since pcl(A) ⊆ cl(A) ⊆ U, A is g^^ p -closed set.

Example 3.18: Let X and τ be as in example 3.13, the set {a, c} is g^^ p –closed but not a g^^ -closed set in (X, τ).

Theorem 3.19: Every pre-closed set is g^^ p -closed.

Proof: Obvious

Remark 3.20: The converse need not be true as seen from the following example.

Example 3.21: Let X = {a, b, c} with τ = {φ, {c}, {a, c}, X}. Then g^^ pC(X) = {φ, {a}, {b}, {a, b}, {b, c}, X}. The set {b, c} is g^^ p -closed but not a pre-closed in (X, τ) .

Theorem 3.22: Every g# -closed set is g^^ p -closed set in (X, τ).

Proof: Let A be $g^\#$ -closed set . Let U be an g^^ -open set containing A. Since pcl(A) ⊆ cl(A) ⊆ U, A is g^^ p -closed.

Remark 3.23: The converse need not be true as seen from the following example.

Example 3.24: Let X and τ be as in example 3.13, the set {a, c} is g^^ p –closed but not a g# -closed set.

Theorem 3.25: Every g^^ p -closed set is gpr-closed.

Proof: Since every g^^-open set is regular open, the theorem follows.

Remark 3.26: The converse need not be true as seen from the following example.

Example 3.27: Let (X, τ) be as in example 3.16, the set {a, b} is gpr-closed but not a g^^ p -closed.

Remark 3.28: Thus the class of g^^ p -closed sets properly contains the closed sets, g^* -closed sets, g^* p -closed sets, g α-closed sets, g^^ -closed sets, g# -closed sets and is properly contained in the classes of gsp-closed sets and gpr-closed sets.

Remark 3.29: $g^{\wedge\wedge} p$ -closed sets are independent of semi-closed set, semi-preclosed set, $g^{*} s$ -closed set, gs-closed set, sg-closed set as it can be seen from the following examples.

Example 3.30: Let (X, τ) be as in example 3.4, the set $\{a, b\}$ is $g^{\wedge\wedge} p$ –closed but neither semi-closed nor semi-preclosed. In example 3.16, the set $\{b\}$ is both semi-closed and semi-preclosed but not a $g^{\wedge\wedge} p$ -closed.

Example 3.31: Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{b, c\}, X\}$. Then the set $\{c\}$ is $g^{\wedge\wedge} p$ -closed but it is neither sg-closed set nor gs-closed in (X, τ).

Example 3.32: In example 3.16, the set $\{b\}$ is both gs-closed and sg-closed but not a $g^{\wedge\wedge} p$ -closed set.

Example 3.33: Let X and τ be as in example 3.31, the set $\{c\}$ is $g^{\wedge\wedge} p$ –closed but not $g^{*} s$ -closed set in (X, τ).

Example 3.34: In example 3.16, the set $\{a\}$ is $g^{*} s$ -closed set but not a $g^{\wedge\wedge} p$ -closed in (X, τ).

Remark 3.35: Union of two $g^{\wedge\wedge} p$ -closed sets need not be $g^{\wedge\wedge} p$ -closed set as can be verified from the following example.

Example 3.36: Let X and τ be as in example 3.31, $g^{\wedge\wedge} pC(X) = \{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$.

Here $A = \{b\}$, $B = \{c\}$ are $g^{\wedge\wedge} p$ -closed set but $A \cup B = \{b, c\}$ is not $g^{\wedge\wedge} p$ -closed.

Remark 3.37: Intersection of two $g^{\wedge\wedge} p$ -closed sets need not be $g^{\wedge\wedge} p$ -closed set as can be verified from the following example.

Example 3.38: In example 3.4, the sets $\{a, b\}$ and $\{a, c\}$ are $g^{\wedge\wedge} p$ -closed sets but $\{a, b\} \cap \{a, c\} = \{a\}$ is not $g^{\wedge\wedge} p$ -closed set in (X, τ).

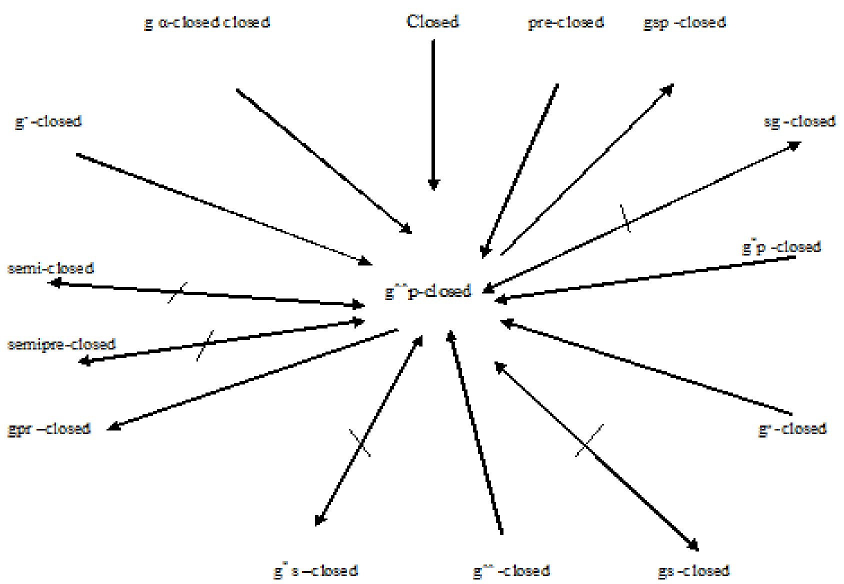
Theorem 3.39: A is a $g^{\wedge\wedge} p$ -closed set of (X, τ). Then $pcl(A) \subseteq A$ does not contain any non-empty $g^{\wedge\wedge}$ -closed set.

Proof: Let F be $g^{\wedge\wedge}$ -closed set of (X, τ) such that $F \subseteq pcl(A) \subseteq A$. Then $A \subseteq X - F$. Since $X - F$ is $g^{\wedge\wedge}$ -open, $A \subseteq X - F$ and A is $g^{\wedge\wedge} p$ -closed, $pcl(A) \subseteq X - F$, and thus $F \subseteq X - pcl(A)$. This implies that
$F \subseteq (X - pcl(A)) \cap (pcl(A) - A) \subseteq (X - pcl(A)) \cap pcl(A) = \varphi$ and hence $F = \varphi$.

Theorem 3.40: If A is a $g^{\wedge\wedge} p$ -closed set of (X, τ) such that $A \subseteq B \subseteq pcl(A)$, then B is also a $g^{\wedge\wedge} p$ –closed set of (X, τ).

Proof: Let U be a $g^{\wedge\wedge}$ -open set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$. Since $A \subseteq U$ and A is $g^{\wedge\wedge} p$ -closed set, $pcl(A) \subseteq U$. Then $pcl(B) \subseteq pcl(pcl(A)) = pcl(A)$, since $B \subseteq pcl(A)$. Thus $pcl(B) \subseteq pcl(A) \subseteq U$. Hence B is also $g^{\wedge\wedge} p$ -closed set.

Remark 3.41: From the above discussions we have the following implications where $A \rightarrow B$ (resp. $A = B$) represents A implies B but not conversely (resp. A and B are independent of each other).



4. g^^ p -LOCALLY CLOSED SETS

Definition 4.1: A subset A of (X, τ) is called g^^ p -locally closed (briefly g^^ plc) if A = U ∩ F, where U is g^^ p -open and F is g^^ p -closed in (X, τ). The class of all g^^p -locally closed sets in X is denoted by G^^PLC(X).

Example 4.2: Let X = {a, b, c} and τ = {φ, {a}, X}. Then g^^ p -lc set = P(X).

Theorem 4.3: Every locally closed set is ^^gp -lc set.

Proof: Let A be lc set in (X, τ). Then there exist an open set U and closed set F such that A = U ∩ F. Since every closed set is g^^ p -closed and, its complement is g^^p –open, A is g^^p - lc set.

Remark 4.4: The converse need not be true as it can be seen from the following example.

Example 4.5: In example 4.2 g^^p -lc = P(X). Here the set {a, b} is g^^p - locally closed set but not locally closed set in (X, τ).

Theorem 4.6: Every ^g -lc set is g^^p -lc set in (X, τ).

Proof: Let A be g^-lc set. Then there exist an g^ -open set U and g^ -closed set F such that A = U ∩ F. Since every g^-closed set is g^^p -closed set, its complement is g^^p –open, A is g^^p - lc set.

Remark 4.7: The converse need not be true as seen from the following example.

Example 4.8: Let (X, τ) be in example 3.13, g^^p -lc = P(X) . Here the set {b} is g^^p -locally closed but not g^ -locally closed set.

Theorem 4.9: Every g^*-lc set is g^^p -lc set.

Proof: Let A be g^*-lc set. Then there exist an g^* -open set U and g^* -closed set F such that A = U ∩ F. Since every g^* -closed set is g^^p –closed, and its complement is g^* -open, A is g^^p - lc set.

Remark 4.10: The converse need not be true as seen from the following example.

Example 4.11: In example 4.2, g^^p -lc = P(X). Here the set {c} is g^^p - locally closed set but not g^*-lc set in (X, τ).

Theorem 4.12: Every $g^\#$ -lc set is g^^p -lc set.

Proof: Let A be $g^\#$ - lc set. Then there exist an g# -open set U and g# -closed set F such that A = U ∩ F. Since every $g^\#$ -closed set is g^^p -closed and its complement is g^^p -open. Then A is g^^p - lc set.

Remark 4.13: The converse need not be true as seen from the following example.

Example 4.14: Let X = {a, b, c} with τ = {φ, {a}, {b, c}, X}. Then the set {a, b} is g^^plc but not $g^\#$ - lc set.

Definition 4.15: A subset A of a space (X, τ) is called
(i) g^^p –lc^* set if A = S ∩ G, where S is g^^p -open in (X, τ) and G is closed in(X, τ).
(ii) g^^p –lc^{**} set if A = S ∩ G, where S is open in (X, τ) and G is g^^p –closed in (X, τ).

The class of all g^^p -lc _ (resp. g^^p –lc^{**}) sets in a topological space (X, τ) is denoted by G^^ PLC^* (X) (resp.^^ $GPLC^{**}$ (X)).

Theorem 4.16: Every locally closed set is g^^p -lc^* set in (X, τ).

Proof: Let A be lc set. Then there exist U and closed set F such that A = U ∩ F. Since every open set is g^^p -open, A is g^^p - lc_ set.

Remark 4.17: The converse need not be true as seen from the following example.

Example 4.18: Let X = {a, b, c} with the topology τ = {φ, {a}, X}. Then the set {a, c} is g^^ plc^* set is not locally closed set.

Theorem 4.19: Every locally closed set is g^^ p –lc** in (X, τ).

Proof: Let A be locally closed set. Then there exist an open set U and closed set F such that A = U ∩ F. Since every closed set is g^^ p -closed, A is g^^ p –lc**.

Remark 4.20: The converse of the above theorem need not be true as seen from the following example.

Example 4.21: In example 4.18, the set {a, c} is g^^ plc** set but not locally closed set.

Theorem 4.22: Let A and B be any two subsets of (X, τ). If A in G^^ PLC(X) and B is g^^ p -open, then A ∩ B ∈ G^^ PLC(X).

Proof: Let A in G^^ PLC(X). Then there exist an g^^ p -open set U and g^^ p –closed set F such that A = U ∩ F. So, A ∩ B = (U ∩ F) ∩ B = (U ∩ B) ∩ F in G^^ PLC(X).

Theorem 4.23: Let A and B be any two subsets of (X, τ). If A in G^^ PLC** (X) and B in G^^ PLC* (X), then A ∩ B in G^^ PLC(X).

Proof: Let A = S ∩ G, where S is open and G is g^^ p -closed and B = P ∩ Q, where P is g^^ p -open and Q is closed. Then A ∩ B = (S ∩ G) ∩ (P ∩ Q) = (S ∩ P) ∩ (G∩Q) where S ∩ P is g^^ p -open and G∩Q is g^^ p -closed. Therefore, A ∩ B in G^^ PLC(X).

Theorem 4.24: Let A and B be any two subsets of (X, τ). If A∈ G^^ PLC** (X) and B is open or closed, then A ∩ B ∈ G^^ PLC** (X, τ).

Proof: If A in G^^ PLC** (X, τ). Then there exist an open set U and g^^ p –closed set F such that A = U ∩F. If B is open, then A∩B = (U ∩F)∩B = (U ∩B)∩F in G^^ PLC** (X, τ) . If B is closed, then A ∩B = (U ∩ F) ∩B = U ∩ (B ∩ F) in G^^ PLC **(X, τ).

Theorem 4.25: Let A and B be any two subsets of (X, τ). If A ∈ G^^ PLC(X) and B is g^^ p -open or g^^ p -closed, then A ∩ B ∈ G^^ PLC(X, τ).

Proof: Let A in G^^ PLC(X, τ). Then there exist an g^^ p -open set U and g^^ p -closed set F such that A = U ∩B. If B is g^^ p -open, then A∩B = (U ∩F)∩B = (U ∩ B) ∩ F in G^^ PLC(X, τ). If B is g^^ p -closed, then A ∩ B = (U ∩ F) ∩ B = U ∩ (B ∩ F) in G^^ PLC(X, τ).

Theorem 4.26: For a subset A of (X, τ) the following are equivalent:

(1) A ∈ G^^ PLC* (X, τ)
(2) A = P ∩ cl(A) for some g^^ p -open set P
(3) cl(A) − A is g^^ p -closed
(4) A ∪ (X − cl(A)) is g^^ p -open

Proof:
(1) ⇒ (2): Let A∈G^^ PLC* (X, τ). Then there exist an g^^ p -open set P and a closed set F in (X, τ) such that A = P ∩F. Since A ⊆ P and A⊆cl(A), we have A ⊆ P ∩cl(A). Conversely, since cl(A) ⊆ F, P ⊆ cl(A) ⊆ P ∩F = A, we have that A = P ∩ cl(A) .

(2) ⇒ (1): Since P is g^^p -open and cl(A) is closed, we have P ∩ cl(A) ∈ G^^PLC* (X, τ).

(3) ⇒ (4): Let F = cl(A) − A. By assumption F is g^^ p -closed. X − F = X ∩ F^{c} = X ∩ (cl(A) − A)^{c} = A ∩ (X − cl(A)). Since X − F is g^^ p -open, we have that A ∪ (X − cl(A)) is g^^ p -open.

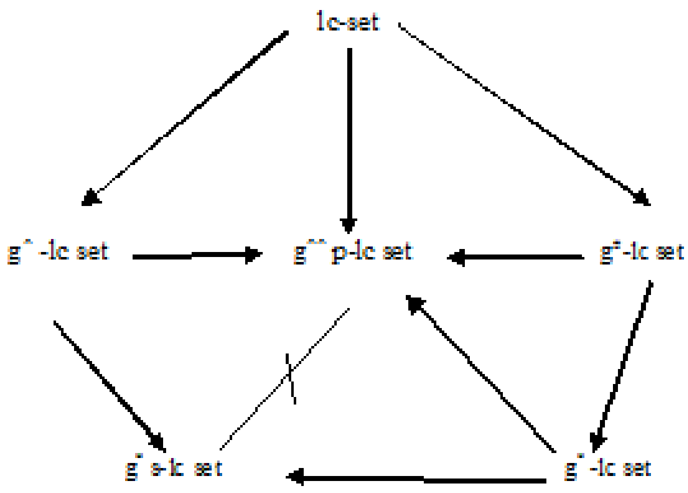
(4) ⇒ (3): Let U = A ∪ (X − cl(A)). By assumption U is g^^ p -open. Then X − U is g^^ p -closed. X − U = X − (A ∪ (X − cl(A)) = cl(A) ∩ (X − A) = cl(A) − A, cl(A) − A is g^^ p -closed.

(4) ⇒ (2): Let U = A ∪ (X − cl(A)) . By assumption, U is g^^ p -open. Now U ∩ cl(A) = A ∪ (X − cl(A)) ∩ cl(A) = (cl(A) ∩ A) ∪ (cl(A) ∩ (X − cl(A))) = A∪ φ = A. Therefore A = U ∩ cl(A) for the g^^ p -open set U.

(2) ⇒ (4): Let A = P ∩ cl(A) for some g^^ p -open set P.

Now A ∪ (X − cl(A)) =P ∩cl(A)∪(X−cl(A)) = P ∩(cl(A)∪(X−cl(A))) = P ∩X = P is g^^ p -open.

Remark 4.27: From the above discussions and known results, we have the following implications where A → B (resp. A = B) represents A implies B but not conversely (resp. A and B are independent of each other).



5. APPLICATIONS OF gˆˆ p -CLOSED SET

Now we introduce new type of spaces namely Tpˆ spaces, Tpˆˆ spaces, ˆ Tp spaces, αTpˆ spaces αTpˆˆ spaces, sTpˆ spaces .

Definition 5.1: A space (X, τ) is called
1. Tp ˆ space if every gˆˆ p -closed set is closed.
2. ˆTp space if every g α-closed set is gˆˆ p -closed.
3. Tpˆˆ space if gˆˆ p -closed set is gp-closed.
4. α Tpˆˆ space if every gˆˆ p -closed set is α-closed.
5. p Tpˆˆ space if every gˆˆ p -closed set is pre-closed.
6. s Tpˆˆ space if every gsp-closed set is gˆˆ p -closed.

Theorem 5.2:
1. Every Tp ˆ space is Tpˆˆ space.
2. Every Tpˆ space is α Tpˆˆ space.
3. Every p Tpˆˆ space is Tpˆˆ space.

Proof:
1. Follows from the fact that every closed set is gp -closed and pre-closed set.
2. Follows from the fact that every closed set is α-closed.
3. Since every pre-closed set is gp -closed.

Remark 5.3: The converses of the above theorem need not be true as seen from the following examples.

Example 5.4: Let X = {a, b, c} and τ = {φ, {c}, {a, c}, X}. Here (X, τ) is Tpˆˆ space but not Tpˆ space.

Example 5.5: In example 5.4, (X, τ) is αTpˆˆ space but not Tpˆ space.

Example 5.6: Let X = {a, b, c} and τ = {φ, {a}, {a, b}, X}. So (X, τ) is Tpˆˆ space but not p Tpˆˆ space.

Theorem 5.7:
1. Every s Tpˆˆ space is ˆTp space.
2. Every α Tpˆˆ space is p Tpˆˆ space.

Proof:
1. Since every gα-closed set is gsp -closed.
2. Since pcl(A) ⊆ cl(A) (2) follows.

Remark 5.8: The class of Tpˆ space is properly contained in the class of α Tpˆˆ space and class of p Tpˆˆ space. The class of s Tpˆˆ is properly contained in the class of ˆTp space. The class of p Tpˆˆ space is properly contains in the class of α Tpˆˆ space.

Theorem 5.9: Every semi-pre-T 1/2 space is ˆTp and Tpˆˆ space.

Proof. Every gα-closed set is gsp -closed and also gˆˆ p -closed set is gsp –closed in (X, τ).

Remark 5.10: s Tpˆˆ is independent from Tpˆ, Tpˆ ˆ, α Tpˆ ˆ, p Tpˆˆ and semipre- T 1/2. Also Tpˆ is independent from semi-pre-T 1/2 as it can be from the following examples.

Example 5.11: Let X = {a, b, c} with τ ={φ, {a},{a, b},X}. Then (X, τ) is a s Tpˆˆ space but it is not Tpˆ, Tpˆ ˆ, α Tpˆ ˆ, p Tpˆˆ, semi-pre-T 1/2.

Example 5.12: Let X = {a, b, c} with τ ={φ,{a},{b},{a, b}, X}. Then (X, τ) is a Tpˆ, Tpˆ ˆ, α Tpˆ ˆ, p Tpˆ ˆ, semi-pre-T 1/2 but it is not s Tpˆ ˆ.

Example 5.13: Let X = {a, b, c} and τ ={φ, {a},{b, c}, X}. Then (X, τ) is semi-pre-T 1/2 space but it is not Tpˆ space.

Example 5.14: Let X = {a, b, c} with τ ={φ,{a},{a, c}, X}. Then (X, τ) is Tpˆ space but it is not semi-pre-T 1/2 space.

Definition 5.15: A subset A of a space (X, τ) is called gˆˆ p -dense if gˆˆ p -cl (A) = X.

Example 5.16: In example 5.4, the set {a, b} is gˆˆ p -dense in (X, τ).

Theorem 5.17: Every gˆˆ p -dense set is dense.

Proof: Let A be an gˆˆ p -dense in (X, τ). Then gˆˆ p -cl(A)= X. Since gˆˆ p - cl(A) ⊆ cl(A) , we have X ⊆ cl(A). Also cl(A) ⊆ X. So cl(A) = X. Thus A is dense.

Remark 5.18: The converse need not be true as it can be from the following example.

Example 5.19: In example 5.4, D(X, τ) = {φ, {a}, {a, b}, {a, c}, X}.

Here the set {a} is not gˆˆ p -dense in (X, τ).

Definition 5.20: A topological space (X, τ) is called gˆˆp -submaximal if every dense subset in it is g ˆˆp -open in (X, τ).

Example 5.21: Let X and τ be in example 5.4, gˆˆ p -open= {φ, {a}, {b}, {a, b}, {a, c}, X}. We have every dense subset is g ˆˆp -open and hence (X, τ) is gˆˆ p -submaximal.

Theorem 5.22: Every Submaximal space is gˆˆ p -submaximal.

Proof: Let (X, τ) be a submaximal space and A be a dense subset. Then A is open. But every open set is gˆˆ p -open and so A is gˆˆ p -open. Therefore, (X, τ) is gˆˆ p -submaximal.

Remark 5.23: The converse need not be true from the following example.

Example 5.24: Let X = {a, b, c} and τ = {φ, {a}, {b}, {a, b}, X}. Then gˆˆ p -submaximal= {φ, {a}, {b}, {a, b}, X}. Here the set {a} is not submaximal.

Theorem 5.25: A space (X, τ) is gˆˆ p -submaximal if and only if P(X) = Gˆˆ PLC* (X, τ).

Proof: Necessity. Let A∈P(X) and let V = A ∪ (cl(A))^c. This implies that cl(V) = cl(A) ∪ (cl(A))^c = X. Hence cl(V) = X. Therefore V is a dense subset of X. Since (X, τ) is gˆˆ p -submaximal, V is gˆˆ p -open.

Thus A ∪ (cl(A))^c is gˆˆ p -open and by Theorem 4.26, we have A∈ Gˆˆ PLC* (X, τ).

Sufficiency. Let A be a dense subset of (X, τ). This implies A ∪ (cl(A))^c = A ∪ X^c = A∪ φ = A.

Now A ∈ Gˆˆ PLC* (X) implies that A = a ∪ (cl(A))^c is gˆˆ p -open by Theorem 4.26.

Hence (X, τ) is gˆˆ p -submaximal.

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