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ON A SEMI$-$SYMMETRIC NON$-$METRIC CONNECTION IN AN (ε) $-$KENMOTSU MANIFOLD

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ABSTRACT

The object of the present paper is to study semi$-$symmetric non$-$metric connection in an (ε) $-$Kenmotsu manifold. We have also studied a semi-symmetric non-metric connection in an (ε) $-$Kenmotsu manifold with projective curvature tensor satisfying certain curvature conditions and obtained many interesting results.

***AMS Classification*[2000]:** $53C15, 53C25$.

Keywords: *(ε) $-$Kenmotsu manifold, semi$-$symmetric non$-$metric connection, quasi$-$projectively flat (ε) $-$Kenmotsu manifold, φ $-$projectively flat (ε) $-$Kenmotsu manifold, space like, time like.*

1. INTRODUCTION

In 1924, Friedmann and Schouten [2] introduced the idea of semi$-$symmetric linear connection on a differentiable manifold. In 1930, H. A. Hayden [6] defined a semi$-$symmetric metric connection on a Riemannian manifold and this was further developed by K. Yano [7]. He proved that a Riemannian manifold with respect to the semi$-$symmetric metric connection has vanishing curvature tensor if and only if it is conformally flat. This result was generalized for vanishing Ricci tensor of the semi$-$symmetric metric connection by T. Imai ([16], [17]).Various properties of such connection have studied by many geometers. In[11], Agashe and Chafle introduced a semi$-$symmetric non$-$metric connection on a Riemannian manifold and this was further studied by U. C. De and D. Kamilya[19], S. C. Biswas and U. C. De [13], B. B. Chaturvedi and P. N. Pandey [5], De and Sengupta [20] defined new type of semi-symmetric non-metric connections on a Riemannian manifold and studied some geometrical properties with respect to such connections. In this connection, the properties of semi-symmetric non-metric connections have studied by Ahmad and Ozgur[8], Kumar and Chaubey[3], Dubey, Chaubey and Ojha [4] and many other geometers. In [10], the semi$-$symmetric non$-$metric connection in a Kenmotsu manifold was studied by M. M. Tripathi and N. Nakkar. Also in[9], M. M. Tripathi proved the existence of a new connection and he showed that in particular cases, this connection reduces to semi$-$symmetric connections; even some of them are not introduced so far. In [14,15], Chaubey defined semi$-$symmetric non$-$metric connections on an almost contact metric manifold and studied its different geometrical properties. Some properties of such connection have further studied by many others.

A. Bejancu and K. L. Duggal[1] introduced the concept of (ε) $-$Sasakian manifolds. Also Xufeng and Xiaoli [21] showed that every (ε) $-$Sasakian manifold must be a real hypersurface of some indefinite Kahler manifold. (ε) $-$Sasakian manifold have also been studied by R. Kumar, R.Rani and R.Nagaich [12]. Since Sasakian manifolds with indefinite metric play significant role in physics and relativity, our natural trend is to study various contact manifolds with indefinite metric. Manifolds with indefinite metrics have been studied by several authors. Recently De and Sarkar[18] introduced (ε) $-$Kenmotsu manifolds and studied conformally flat, Weyl semi$-$symmetric, φ $-$recurrent (ε) $-$Kenmotsu manifolds. The present paper is organized as follows:

In Section 2 we review some preliminary results. In section3, we establish the relation between Levi$-$Civita connection and semi$-$symmetric non$-$metric connection in an (ε) $-$Kenmotsu manifold and we find the relation between curvature tensors of Levi$-$Civita connection and semi$-$symmetric non$-$metric connection. In section4, Quasi$-$projectively flat (ε) $-$Kenmotsu manifold with respect to semi$-$symmetric non$-$metric connection is studied. In section5, it is shown that φ $-$projectively flat (ε) $-$Kenmotsu manifold with respect to a semi$-$symmetric non$-$metric connection is an Einstein manifold. Section 6 is devoted to study (ε) $-$Kenmotsu manifold with respect to a semi$-$symmetric non$-$metric connection satisfying $\tilde{P}.\tilde{S} = 0$.

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2. PRELIMINARIES

An n −dimensional smooth manifold (M^n, g) is said to be an (ε) −almost contact metric manifold, if it admits a (1,1) tensor field φ, a structure vector field ξ, a 1 −form η and an indefinite metric g such that

$$\varphi^2 X = -X + \eta(X)\xi \tag{2.1}$$
$$\eta(\xi) = 1 \tag{2.2}$$
$$g(\xi, \xi) = \varepsilon \tag{2.3}$$
$$\eta(X) = \varepsilon g(X, \xi) \tag{2.4}$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \varepsilon\eta(X)\eta(Y) \tag{2.5}$$

where ε is 1 or -1 according as ξ is space−like or time−like and rank φ is $n - 1$.

It is important to mention that in the above definition ξ is never a light-like vector field. If

$$d\eta(X, Y) = g(X, \varphi Y) \tag{2.6}$$

for every $X, Y \in TM^n$, then we say that M^n is an (ε) −contact metric manifold. It follows that

$$\varphi\xi = 0,\ \eta\varphi = 0 \tag{2.7}$$

If moreover, the manifold satisfies

$$(\nabla_X \varphi)Y = -g(X, \varphi Y)\xi - \varepsilon\eta(Y)\varphi X \tag{2.8}$$

where ∇ denotes the Riemannian connection of g, then we shall call the manifold an (ε) −Kenmotsu manifold [18].

An (ε) −almost contact metric manifold is an (ε) −Kenmotsu manifold [18] if and only if

$$\nabla_X \xi = \varepsilon(X - \eta(X)\xi) \tag{2.9}$$

In an (ε) −Kenmotsu manifold, the following relations holds [9]

$$(\nabla_X \eta)(Y) = g(X, Y) - \varepsilon\eta(X)\eta(Y) \tag{2.10}$$
$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X \tag{2.11}$$
$$R(\xi, X)Y = \eta(Y)X - \varepsilon g(X, Y)\xi \tag{2.12}$$
$$R(X, Y)\varphi Z = \varphi R(X, Y)Z + \varepsilon\{g(Y, Z)\varphi X - g(X, Z)\varphi Y + g(X, \varphi Z)Y - g(Y, \varphi Z)X\} \tag{2.13}$$
$$\eta(R(X, Y)Z) = \varepsilon[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)] \tag{2.14}$$
$$S(X, \xi) = -(n-1)\eta(X) \tag{2.15}$$
$$S(\varphi X, \varphi Y) = S(X, Y) + \varepsilon(n-1)\eta(X)\eta(Y) \tag{2.16}$$

EXAMPLE OF (ε) −KENMOTSU MANIFOLD

Example 1: We consider the three dimensional manifold $M^3 = \{(x, y, z) \in R^3, z \neq 0\}$, where (x, y, z) are the standard coordinates in R^3. The vector fields

$$e_1 = z\frac{\partial}{\partial x}, \quad e_2 = z\frac{\partial}{\partial y}, \quad e_3 = -z\frac{\partial}{\partial z}$$

are the linearly independent at each point of the manifold [18]. Let g be the Riemannian metric defined by

$$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0,$$
$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = \varepsilon,$$

where $\varepsilon = \pm 1$. Let η be the 1 −form defined by $\eta(Z) = \varepsilon g(Z, e_3)$ for any $Z \in TM^n$.

Let φ be the $(1,1)$ −tensor field defined by

$$\varphi(e_1) = -e_2,\ \ \varphi(e_2) = e_1,\ \ \ \varphi(e_3) = 0.$$

Then using the linearity of φ and g, we get

$$\eta(e_3) = 1,\ \ \varphi^2 Z = -Z + \eta(Z)e_3,$$
$$g(\varphi Z, \varphi W) = g(Z, W) - \eta(Z)\eta(W) \text{ for any } Z, W \in TM^n.$$

Let ∇ be the Levi−Civita connection with respect to metric g. Then, we obtain

$$[e_1, e_2] = 0, \qquad [e_1, e_3] = \varepsilon e_1, \qquad [e_2, e_3] = \varepsilon e_2.$$

The Riemannian connection ∇ of the metric g is given by

$$2g(\nabla_X Y, Z) \ = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [(Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]),$$

which is known as Koszul's formula. Then Koszul's formula yields

$$\nabla_{e_1} e_3 = \varepsilon e_1, \qquad \nabla_{e_2} e_3 = \varepsilon e_2, \qquad \nabla_{e_3} e_3 = 0,$$
$$\nabla_{e_1} e_2 = 0, \qquad \nabla_{e_2} e_2 = -\varepsilon e_3, \qquad \nabla_{e_3} e_2 = 0,$$
$$\nabla_{e_1} e_1 = -\varepsilon e_3, \qquad \nabla_{e_2} e_1 = 0, \qquad \nabla_{e_3} e_1 = 0.$$

These results shows that the manifold satisfies

$$\nabla_X \xi = \varepsilon(X - \eta(X)\xi)$$

for $\xi = e_3$. Hence the manifold under consideration is an (ε) −Kenmotsu manifold of dimension three.

3. SEMI-SYMMETRIC NON-METRIC CONNECTIONS

Let M^n be an n −dimensional (ε) −Kenmotsu manifold and ∇ be the Levi−Civita connectionon M^n. A linear connection $\tilde{\nabla}$ on M^n is given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X \tag{3.1}$$

Using equation (3.1), the torsion tensor $\tilde{T}$ of M^n with respect to the connection $\tilde{\nabla}$ is given by

$$\tilde{T}(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X \tag{3.2}$$

Satisfies

$$\tilde{T}(X,Y) = \eta(Y)X - \eta(X)Y \tag{3.3}$$

which shows that the linear connection defined in equation (3.1) is a semi−symmetric connection.

Moreover, using equation (3.1) we have, for all vector fields X, Y, Z.

$$\begin{aligned}(\tilde{\nabla}_X g)(Y,Z) &= \tilde{\nabla}_X g(Y,Z) - g(\tilde{\nabla}_X Y, Z) - g(Y, \tilde{\nabla}_X Z) \\ &= -\eta(Y)g(X,Z) - \eta(X)g(X,Y)\end{aligned} \tag{3.4}$$

A linear connection $\tilde{\nabla}$ defined in equation (3.1) satisfies equation (3.2), (3.3) and (3.4) and therefore we call $\tilde{\nabla}$ a semi−symmetric non−metric connection.

Let $\tilde{R}$ be the curvature tensor of semi−symmetric non−metric connection $\tilde{\nabla}$ is given by

$$\tilde{R}(X,Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z \tag{3.5}$$

On using equation (3.1) in the above equation we get

$$\tilde{R}(X,Y)Z = R(X,Y)Z + \eta(Z)\eta(Y)X - \eta(Z)\eta(X)Y + [(\nabla_X \eta)Z]Y - [(\nabla_Y \eta)Z]X \tag{3.6}$$

where

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{3.7}$$

is the Riemannian curvature tensor of connection ∇. Using equation (2.10) in equation (3.6) we obtain

$$\tilde{R}(X,Y)Z = R(X,Y)Z + g(X,Z)Y - g(Y,Z)X + (1+\varepsilon)[\eta(Y)X - \eta(X)Y]\eta(Z) \tag{3.8}$$

which is the relation between curvature tensors of connection $\tilde{\nabla}$ and ∇. From equation (3.8), we have

$$`\tilde{R}(X,Y,Z,U) = `R(X,Y,Z,U) + g(X,Z)g(Y,U) - g(Y,Z)g(X,U) + (1+\varepsilon)[\eta(Y)g(X,U - \eta(X)g(Y,U)]\eta(Z) \tag{3.9}$$

where $`\tilde{R}(X,Y,Z,U) = g(\tilde{R}(X,Y)Z,U)$ and $`R(X,Y,Z,U) = g(R(X,Y)Z,U)$.

Substituting $X = U = e_i$ in the above equation and taking summation over $i, 1 \le i \le n$, we obtain

$$\tilde{S}(Y,Z) = S(Y,Z) + (1-n)g(Y,Z) + (1+\varepsilon)(n-\varepsilon)\eta(Y)\eta(Z) \tag{3.10}$$

Where $\tilde{S}$ and S are the Ricci tensors of connection $\tilde{\nabla}$ and ∇, respectively in M^n. Contracting the above equation, we obtain

$$\tilde{r} = r + n(1-n) + (1+\varepsilon)(n-\varepsilon) \tag{3.11}$$

Where $\tilde{r}$ and r are the scalar curvatures of the connection $\tilde{\nabla}$ and ∇, respectively in M^n.

Writing two more equations by the cyclic permutations of X, Y and Z, we obtain

$$\tilde{R}(Y,Z)X = R(Y,Z)X + g(Y,X)Z - g(Z,X)Y + (1+\varepsilon)[\eta(Z)Y - \eta(Y)Z]\eta(X) \tag{3.12}$$

and

$$\tilde{R}(Z,X)Y = R(Z,X)Y + g(Z,Y)X - g(X,Y)Z + (1+\varepsilon)[\eta(X)Z - \eta(Z)X]\eta(Y) \tag{3.13}$$

Adding equations (3.8), (3.12) and (3.13) and using the Bianchi's first identity, we obtain

$$\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y = 0 \tag{3.14}$$

This leads to the following result.

Theorem 1: An (ε) −Kenmotsu manifold M^n with semi−symmetric non−metric connection satisfies the equation (3.14).

Now interchanging X and Y in the equation(3.9), we obtain

$$`\tilde{R}(Y,X,Z,U) = `R(Y,X,Z,U) + g(Y,Z)g(X,U) - g(X,Z)g(Y,U) + (1+\varepsilon)\,[\eta(X)g(Y,U) - \eta(Y)g(X,U)]\eta(Z) \tag{3.15}$$

Adding equation (3.9) and (3.15) with the fact that

$$`R(X,Y,Z,U) + `R(Y,X,Z,U) = 0$$

We obtain

$$`\tilde{R}(X,Y,Z,U) + `\tilde{R}(Y,X,Z,U) = 0$$

This leads to the following result.

Theorem 2: The curvature tensor of type (0,4) of a semi−symmetric non−metric connection in an (ε) −Kenmotsu manifold is Skew−symmetric in first two slots.

Now, let $\tilde{R}(X,Y)Z = 0$, which by virtue of the equation (3.8) yields

$$R(X,Y)Z = g(Y,Z)X - g(X,Z)Y + (1+\varepsilon)\,[\eta(X)Y - \eta(Y)X]\eta(Z) \tag{3.16}$$

Taking the inner product of the above equation with ξ, we obtain

$$\varepsilon\eta(R(X,Y)Z) = \varepsilon[g(Y,Z)(X) - g(X,Z)(Y)] \tag{3.17}$$

which by virtue of equation (2.4) gives

$$R(X,Y)Z = [g(Y,Z)X - g(X,Z)Y] \tag{3.18}$$

The above equation can be written as

$$`R(X,Y,Z,U) = g(Y,Z)g(X,U) - g(X,Z)g(Y,U) \tag{3.19}$$

This leads to the following result.

Theorem 3: If the curvature tensor of a semi−symmetric non−metric connection in an (ε) −Kenmotsu manifold M^n vanishes, then the manifold is of constant curvature.

Now suppose Ricci tensor of a semi−symmetric metric connection in M^n vanishes, i.e., $\tilde{S}(Y,Z) = 0$, then from the equation (3.10) we have

$$S(Y,Z) = (n-1)g(Y,Z) + (1+\varepsilon)(\varepsilon - n)\eta(Y)\eta(Z) \tag{3.20}$$

which is of the form

$$S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)$$

Where $a = (n-1)\ and\ b = (1+\varepsilon)(\varepsilon - n)$.

This leads to the following result.

Theorem 4: If the Ricci tensor of the semi−symmetric non−metric connection $\tilde{\nabla}$ in an(ε) −Kenmotsu manifold vanishes, then the manifold M^n is η −Einstein manifold.

4. QUASI−PROJECTIVELY FLAT (ε) −KENMOTSU MANIFOLD WITH RESPECT TO SEMI−SYMMETRIC NON−METRIC CONNECTION

The projective curvature tensor is an important tensor having one−one correspondence between each coordinate neighbourhood of an n −dimensional Riemannian manifold and a domain of Euclidean space such that there is one−one correspondence between geodesics of Riemannian manifold with straight line in Euclidean space. A manifold of dimension n, $(n \geq 3)$ is projectively flat if the tensorial relation of projective curvature tensor vanishes. The projective curvature tensor P with respect to semi−symmetric non−metric connection is defined by

$$\tilde{P}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{n-1}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y\} \tag{4.1}$$

An (ε) −Kenmotsu manifold M^n is said to be quasi−projectively flat with respect to semi−symmetric non-metric connection, if

$$g(\tilde{P}(X,Y)Z,W) = 0, \tag{4.2}$$

where $\tilde{P}$ is the projective curvature tensor with respect to semi−symmetric metric connection.

In view of the equation (4.1), we have

$$g(\tilde{P}(X,Y)Z,W) = g(\tilde{R}(X,Y)Z,W) - \frac{1}{n-1}[\tilde{S}(Y,Z)g(X,W) - \tilde{S}(X,Z)g(Y,W)] \tag{4.3}$$

Substituting $X = \varphi X$ and $W = \varphi W$ in the above equation, we obtain

$$g(\tilde{P}(\varphi X,Y)Z,\varphi W) = g(\tilde{R}(\varphi X,Y)Z,\varphi W) - \frac{1}{n-1}[\tilde{S}(Y,Z)g(\varphi X,\varphi W) - \tilde{S}(\varphi X,Z)g(Y,\varphi W)] \tag{4.4}$$

Now suppose that M^n is quasi−projectively flat with respect to semi−symmetric non−metric connection.

Using equation (4.2) in equation (4.4), we have

$$g(\tilde{R}(\varphi X,Y)Z,\varphi W) = \frac{1}{n-1}[\tilde{S}(Y,Z)g(\varphi X,\varphi W) - \tilde{S}(\varphi X,Z)g(Y,\varphi W)] \tag{4.5}$$

Using equations (3.8) and (3.10) in the equation (4.5), we obtain

$$g(R(\varphi X,Y)Z,\varphi W) = \frac{1}{n-1}[S(Y,Z)g(\varphi X,\varphi W) - S(\varphi X,Z)g(Y,\varphi W)] \tag{4.6}$$

Let $\{e_1,e_2,\dots,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M^n. Then $\{\varphi e_1,\varphi e_2,\dots,\varphi e_{n-1},\xi\}$ is also a local orthonormal basis of M^n.

Putting $X=W=e_i$ in the equation (4.6) and taking summation over $i, 1\le i\le n-1$, we obtain

$$\sum_{i=1}^{n-1} g(R(\varphi e_i,Y)Z,\varphi e_i) = \frac{1}{n-1}\sum_{i=1}^{n-1}[S(Y,Z)g(\varphi e_i,\varphi e_i) - S(\varphi e_i,Z)g(Y,\varphi e_i)] \tag{4.7}$$

Also

$$\sum_{i=1}^{n-1} g(R(\varphi e_i,Y)Z,\varphi e_i) = S(Y,Z)+g(Y,Z) \tag{4.8}$$

$$\sum_{i=1}^{n-1}[S(\varphi e_i,Z)g(Y,\varphi e_i)] = S(Y,Z) \tag{4.9}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i,\varphi e_i) = n-1 \tag{4.10}$$

Hence, by virtue of equations (4.8), (4.9) and (4.10) the equation (4.7) reduces

$$S(Y,Z) = (1-n)g(Y,Z) \tag{4.11}$$

which is of the form

$$S(Y,Z) = ag(Y,Z)$$

where $a=(1-n)$. This shows that M^n is an Einstein manifold.

This leads to the following result.

Theorem 5: A quasi−projectively flat (ε) −Kenmotsu manifold with respect to a semi−symmetric non−metric connection is an Einstein manifold.

5. φ −PROJECTIVELY FLAT (ε) −KENMOTSU MANIFOLD WITH RESPECT TO A SEMI−SYMMETR - IC NON−METRIC CONNECTION

An (ε) −Kenmotsu manifold with respect to a semi−symmetric non−metric connection is said to be φ −projectively flat if

$$\varphi^2(\tilde{P}(\varphi X,\varphi Y)\varphi Z) = 0, \tag{5.1}$$

Where $\tilde{P}$ is the projective curvature tensor of the manifold M^n with respect to semi−symmetric non−metric connection.

Let M^n be a φ −projectively flat (ε) −Kenmotsu manifold with respect to semi−symmetric metric connection. It is easy to see that $\varphi^2(\tilde{P}(\varphi X,\varphi Y)\varphi Z)=0$ holds if and only if

$$g(\tilde{P}(\varphi X,\varphi Y)\varphi Z,\varphi W) = 0 \tag{5.2}$$

for any $X,Y,Z,W\in TM^n$

Substituting $Y=\varphi Y$ and $Z=\varphi Z$ in the equation (4.4), we obtain

$$g(\tilde{P}(\varphi X,\varphi Y)\varphi Z,\varphi W) = g(\tilde{R}(\varphi X,\varphi Y)\varphi Z,\varphi W) - \frac{1}{n-1}[\tilde{S}(\varphi Y,\varphi Z)g(\varphi X,\varphi W) - \tilde{S}(\varphi X,\varphi Z)g(\varphi Y,\varphi W)] \tag{5.3}$$

Using the equation (5.3) in the equation (5.2), we get

$$g\big(\tilde{R}(\varphi X,\varphi Y)\varphi Z,\varphi W\big) = \frac{1}{n-1}[\tilde{S}(\varphi Y,\varphi Z)g(\varphi X,\varphi W) - \tilde{S}(\varphi X,\varphi Z)g(\varphi Y,\varphi W)] \tag{5.4}$$

On using equations (3.8) and (3.10) in the equation (5.4), we have

$$g(R(\varphi X,\varphi Y)\varphi Z,\varphi W) = \frac{1}{n-1}[S(\varphi Y,\varphi Z)g(\varphi X,\varphi W) - S(\varphi X,\varphi Z)g(\varphi Y,\varphi W)] \tag{5.5}$$

Let $\{e_1,e_2,\dots,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M^n. Then $\{\varphi e_1,\varphi e_2,\dots,\varphi e_{n-1},\xi\}$ is also a local orthonormal basis of M^n.

Putting $X=W=e_i$ in the equation (4.6) and taking summation over $i, 1\le i\le n-1$, we obtain

$$\sum_{i=1}^{n-1} g(R(\varphi e_i,\varphi Y)\varphi Z,\varphi e_i) = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n-1}[S(\varphi Y,\varphi Z)g(\varphi e_i,\varphi e_i) - S(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)] \tag{5.6}$$

Using equations (4.8), (4.9) and (4.10) in the above equation, we get

$$S(\varphi Y,\varphi Z)=(1-n)g(\varphi Y,\varphi Z) \tag{5.7}$$

Using equations (2.5) and (2.16) in the above equation, we obtain

$$S(Y,Z)=(1-n)g(Y,Z) \tag{5.8}$$

which is of the form

$$S(Y,Z)=ag(Y,Z)$$

Where $a=(1-n)$. This shows that M^n is an Einstein manifold.

This leads to the following result.

Theorem 6: φ −Projectively flat (ε) −Kenmotsu manifold with respect to a semi−symmetric non−metric connection is an Einstein manifold.

6. (ε) −KENMOTSU MANIFOLD WITH RESPECT TO A SEMI−SYMMETRIC NON−METRIC CONNEC - TION SATISFYING $\tilde{P}.\tilde{S}=0$

Consider (ε) −Kenmotsu manifold with respect to a semi−symmetric non−metric connection satisfying

$$(\tilde{P}(X,Y).\tilde{S})(Z,U)=0, \tag{6.1}$$

Where $\tilde{S}$ is the Ricci tensor with respect to a semi−symmetric non−metric connection. Then, we have

$$\tilde{S}(\tilde{P}(X,Y)Z,U)+\tilde{S}(Z,\tilde{P}(X,Y)U)=0 \tag{6.2}$$

Putting $X=\xi$ in the equation (6.2), we get

$$\tilde{S}(\tilde{P}(\xi,Y)Z,U)+\tilde{S}(Z,\tilde{P}(\xi,Y)U)=0 \tag{6.3}$$

In view of the equation (4.1), we have

$$\tilde{P}(\xi,Y)Z=\tilde{R}(\xi,Y)Z-\frac{1}{n-1}\{\tilde{S}(Y,Z)\xi-\tilde{S}(\xi,Z)Y\} \tag{6.4}$$

By virtue of equations (3.8) and (3.10), we have

$$\tilde{R}(\xi,Y)Z \ = \ R(\xi,Y)Z-g(Y,Z)\xi-\eta(Z)Y+(1+\varepsilon)\eta(Y)\eta(Z)\xi \tag{6.5}$$

and

$$\tilde{S}(\xi,Z)=S(\xi,Z)+(n-1)\eta(Z) \tag{6.6}$$

Using equations (6.5) and (6.6) in the equation (6.4), we obtain

$$\tilde{P}(\xi,Y)Z \ = \ -(1+\varepsilon)g(Y,Z)\xi+(1+\varepsilon)\eta(Y)\eta(Z)\xi-\frac{1}{n-1}S(Y,Z)\xi \tag{6.7}$$

Now using the equation (6.7) in the equation(6.3), we obtain

$$(1+\varepsilon)(n-1)[g(Y,Z)\eta(U)+g(Y,U)\eta(Z)]-2(1+\varepsilon)(n-1)\eta(Y)\eta(Z)\eta(U)+S(Y,Z)\eta(U)+S(Y,U)\eta(Z)=0$$

Putting $U=\xi$ in the above equation and again using equations (2.2) and (2.15), we obtain

$$S(Y,Z) \ = \ (1+\varepsilon)(1-n)g(Y,Z)+(2+\varepsilon)(n-1)\eta(Y)\eta(Z) \tag{6.8}$$

which is of the form

$$S(Y,Z)=ag(Y,Z)+b(Y)(Z),$$

Where $a=(1+\varepsilon)(1-n)$ and $b=(2+\varepsilon)(n-1)$. This shows that M^n is an η −Einstein manifold.

This leads to the following result.

Theorem 7: An (ε) −Kenmotsu manifold M^n with a semi−symmetric non−metric connection satisfying $\tilde{P}.\tilde{S}=0$, is an η −Einstein manifold.

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