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FIXED POINT, COINCIDENCE POINT AND COMMON FIXED POINT THEOREMS UNDER VARIOUS EXPANSIVE CONDITIONS IN TRIANGULAR INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In this article, we establish some fixed point, common fixed point and coincidence point theorems for expansive type mappings in the triangular intuitionistic fuzzy metric spaces. The presented theorems extend, generalize and improve many existing results in the literature.

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Keywords: *intuitionistic fuzzy metric space; fixed point, common fixed point, coincidence point; expansive mappings.*

1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy set was introduced by Zadeh [1] in 1965. In 1975, Kramosil and Michalek [2] introduced the notion of fuzzy metric space, which can be regarded as a generalization of the statistical (probabilistic) metric space. This work has provided an important basis for the construction of fixed point theory in fuzzy metric spaces.

In 2004, Park introduced the notion of intuitionistic fuzzy metric space [12]. He showed that, for each intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, the topology generated by the intuitionistic fuzzy metric (M, N)coincides with the topology generated by the fuzzy metric M. For more details on intuitionistic fuzzy metric space and related results we refer the reader to [12–20]. The study of expansive mappings is a very interesting research area in fixed point theory. In 1984, Wang et.al [31] introduced the concept of expanding mappings and proved some fixed point theorems in complete metric spaces. In 1992, Daffer and Kaneko [30] defined an expanding condition for a pair of mappings and proved some common fixed point theorems for two mappings in complete metric spaces. Chintaman and Jagannath [32] introduced several meaningful fixed point theorems for one expanding mapping.

In this paper, we present some new fixed point, coincidence point and common fixed point theorems under various expansive conditions in triangular intuitionistic fuzzy metric space. These results improve and generalize some important known results in [23-33]. Some related results to highlight the realized improvements is also furnished.

Throughout this paper $\mathbb{R}$ and $\mathbb{R}_+$ will represents the set of real numbers and nonnegative real numbers, respectively. The following two definitions are required in the sequel which can be found in [12].

Definition 1.1: A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfying the following conditions:

(1). $*$ is commutative and associative;
(2). $*$ is continuous;
(3). $a * 1 = a, \forall\, a \in [0,1]$;
(4). $a * b \le c * d$ whenever $a \le c$ and $b \le d, \forall\, a \in [0,1]$.

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Definition 1.2: A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\diamond$ satisfying the following conditions:

(1). $\diamond$ is commutative and associative;
(2). $\diamond$ is continuous;
(3). $a \diamond 0 = a, \forall\, a \in [0,1]$;
(4). $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d, \forall\, a \in [0,1]$.

In 2004, Park [12] introduced the concept of intuitionistic fuzzy metric space as follows.

Definition 1.3: A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, and M, N are two fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$:

(IFMS1). $M(x,y,t) + N(x,y,t) \leq 1$;
(IFMS2). $M(x,y,t) > 0$;
(IFMS3). $M(x,y,t) = 1$ for all $t > 0 \Leftrightarrow x = y$;
(IFMS4). $M(x,y,t) = M(y,x,t)$;
(IFMS5). $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;
(IFMS6). $M(x,y,.): (0,\infty) \to [0,1]$ is left continuous;
(IFMS7). $\lim_{t\to\infty} M(x,y,t) = 1$;
(IFMS8). $N(x,y,t) > 0$;
(IFMS9). $N(x,y,t) = 0$ for all $t > 0 \Leftrightarrow x = y$;
(IFMS10). $N(x,y,t) = N(y,x,t)$;
(IFMS11). $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$;
(IFMS12). $N(x,y,.): (0,\infty) \to [0,1]$ is right continuous;
(IFMS13). $\lim_{t\to\infty} N(x,y,t) = 0$;

Then(M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree on nonnearness between x and y with respect to t, respectively.

Definition 1.4: (see [12]) Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

1) a sequence $\{x_n\}$ is said to be Cauchy sequence whenever
$$\lim_{m,n\to\infty} M(x_n, x_m, t) = 1$$
and
$$\lim_{m,n\to\infty} N(x_n, x_m, t) = 0$$
for all $t > 0$. That is, for each $\varepsilon > 0$ and $t > 0$, there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ and $N(x_n, x_m, t) < \varepsilon$ for all $n, m \geq n_0$.

2) $(X, M, N, *, \diamond)$ is called complete whenever every Cauchy sequence is convergent with respect to the topology $\tau_{(M,N)}$.

Remark 1.5: Note that, if (M, N) is called an intuitionistic fuzzy metric space on X and $\{x_n\}$ is a sequence in X such that $\lim_{m,n\to\infty} M(x_n, x_m, t) = 1$ and $\lim_{m,n\to\infty} N(x_n, x_m, t) = 0$ for all $t > 0$ as from (IFMS1) of Definition 1.3, we know that $M(x,y,t) + N(x,y,t) \leq 1$ for all $x, y \in X$ and $t > 0$.

Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. According to [8, 10], the fuzzy metric (M, N) is called triangular whenever $\frac{1}{M(x,y,t)} - 1 \leq \frac{1}{M(x,z,t)} - 1 + \frac{1}{M(z,y,t)} - 1$ and $N(x,y,t) \leq N(x,z,t) + N(z,y,t)$ for all $x, y, z \in X$ and $t > 0$.

Example 1.6: Let $X = \{(0,0), (0,4), (4,0), (4,5), (5,4)\}$ endowed with the metric $d: X \times X \to [0, +\infty)$ given by
$$d\big((x_1,x_2),(y_1,y_2)\big) = |x_1 - y_1| + |x_2 - y_2|$$
for all $(x_1,x_2), (y_1,y_2) \in X$. Define intuitionistic fuzzy metric by
$$M\big((x_1,x_2),(y_1,y_2),t\big) = \frac{t}{t + d\big((x_1,x_2),(y_1,y_2)\big)} \quad \text{and} \quad N\big((x_1,x_2),(y_1,y_2),t\big) = \frac{d\big((x_1,x_2),(y_1,y_2)\big)}{t + d\big((x_1,x_2),(y_1,y_2)\big)}$$
for all $(x_1,x_2), (y_1,y_2) \in X$ and $t > 0$, where
$$a * b = \min\{a,b\} \quad \text{and} \quad a \diamond b = \max\{a,b\}.$$

Then X is a complete triangular intuitionistic fuzzy metric space.

Example 1.4: Let $X = [0, 2-\sqrt{3})$ endowed with the usual distance $d(x,y) = |x-y|$. Consider

$$M(x,y,t) = \frac{t}{t+d(x,y)} \text{ and } N(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

for all $x, y \in X$ and $t > 0$. Then X is a complete triangular intuitionistic fuzzy metric space.

2. MAIN RESULT

Now, we are ready to state and prove our main results.

Theorem 2.1: Let $(X, M, N, \star, \lozenge)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a continuous self-mapping satisfying the condition

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,Tx,t)} - 1\right) + c\left(\frac{1}{M(y,Ty,t)} - 1\right) \tag{2.1}$$

for all $x, y \in X$, all $t > 0$ and $a > 1, b \in \mathbb{R}$ and $c \leq 1$ with $a + b + c > 1$. Then T has a unique fixed point.

Proof: Let x_0 be arbitrary in X, we define a sequence $\{x_n\}$ in X by the rule

$$x_0 = Tx_1, x_1 = Tx_2, \ldots\ldots, x_n = Tx_{n+1} \tag{2.2}$$

If $x_n = x_{n+1}$ for some n then we have nothing to prove. Assume that $x_n \neq x_{n+1}$ for all n. Consider

$$\frac{1}{M(x_n, x_{n-1}, t)} - 1 = \frac{1}{M(Tx_{n-1}, Tx_n, t)} - 1 \tag{2.3}$$

Now by (2.1) and definition of sequence

$$\frac{1}{M(x_n, x_{n-1}, t)} - 1 = \frac{1}{M(Tx_{n-1}, Tx_n, t)} - 1 \leq a\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) + b\left(\frac{1}{M(x_{n+1}, Tx_{n+1}, t)} - 1\right) + c\left(\frac{1}{M(x_n, Tx_n, t)} - 1\right)$$

$$\frac{1}{M(x_n, x_{n-1}, t)} - 1 \geq a\left(\frac{1}{M(x_{n+1}, x_n, t)} - 1\right) + b\left(\frac{1}{M(x_{n+1}, x_n, t)} - 1\right) + c\left(\frac{1}{M(x_n, x_{n-1}, t)} - 1\right) \tag{2.4}$$

By use of symmetric property, we have

$$\frac{1}{M(x_{n-1}, x_n, t)} - 1 \geq a\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) + b\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) + c\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right)$$

Thus $(1-c)\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) \geq (a+b)\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right)$

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leq \frac{1-c}{a+b}\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) \tag{2.5}$$

Let $k = \frac{1-c}{a+b} < 1$. So the above inequality, become

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leq k\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) \tag{2.6}$$

Also, we can show that

$$\frac{1}{M(x_{n-1}, x_n, t)} - 1 \leq k\left(\frac{1}{M(x_{n-2}, x_{n-1}, t)} - 1\right) \tag{2.7}$$

So that

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leq k^2\left(\frac{1}{M(x_{n-2}, x_{n-1}, t)} - 1\right) \tag{2.8}$$

Proceeding in similar way we can get,

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leq k^n\left(\frac{1}{M(x_0, x_1, t)} - 1\right) \tag{2.9}$$

for all n.

By using the triangular inequality, for each $m \geq n$, we obtain

$$\begin{aligned}\frac{1}{M(x_n, x_m, t)} - 1 &\leq \frac{1}{M(x_n, x_{n+1}, t)} - 1 + \frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 + \cdots\ldots\ldots + \frac{1}{M(x_{m-1}, x_m, t)} - 1 \\ &\leq (k^n + k^{n+1} + \cdots\ldots\ldots + k^{m-1})\left(\frac{1}{M(x_0, x_1, t)} - 1\right) \\ &\leq \frac{k^n}{1-k}\left(\frac{1}{M(x_0, x_1, t)} - 1\right)\end{aligned} \tag{2.10}$$

Assume that $M(x_0, x_1, t) < 1$ that is, $\frac{1}{M(x_0, x_1, t)} - 1 > 0$. Letting $n \to \infty$, $\{x_n\}$ is a Cauchy sequence. Also, if $M(x_0, x_1, t) = 1$, then $M(x_n, x_m, t) = 1$ for all $m > n$ and hence $\{x_n\}$ is a Cauchy sequence. Since X is complete. So there must exists $x^\star \in X$. such that

$$\lim_{n\to\infty} x_n = x^\star \tag{2.11}$$

Now to show that $x^\star$ is a fixed point of T. Since T is continuous so

$$\lim_{n\to\infty} Tx_n = Tx^\star \Rightarrow \lim_{n\to\infty} x_{n-1} = Tx^\star \Rightarrow Tx^\star = x^\star \tag{2.12}$$

Hence $x^\star$ is the fixed point of T. Now suppose that T has another fixed point $y^\star \neq x^\star$. then we have

$$\begin{aligned}\frac{1}{M(x^\star, y^\star, t)} - 1 &= \frac{1}{M(Tx^\star, Ty^\star, t)} - 1\\ &\geq a\left(\frac{1}{M(x^\star, y^\star, t)} - 1\right) + b\left(\frac{1}{M(x^\star, Tx^\star, t)} - 1\right) + c\left(\frac{1}{M(y^\star, Ty^\star, t)} - 1\right)\\ &= a\left(\frac{1}{M(x^\star, y^\star, t)} - 1\right)\end{aligned} \tag{2.13}$$

Since $a > 1$, so the above inequality is possible if

$$\frac{1}{M(x^\star, y^\star, t)} - 1 = 0 \tag{2.14}$$

which implies that $x^\star = y^\star$. Hence fixed point of T is unique.

Corollary 2.2: Let $(X, M, N, \star, \Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a continuous self-mapping satisfying the condition

$$\frac{1}{M(Tx, Ty, t)} - 1 \geq a\left(\frac{1}{M(x, y, t)} - 1\right) \tag{2.15}$$

for all $x, y \in X$ and $a > 1$. Then T has a unique fixed point.

Proof: Putting $b = c = 0$ in Theorem 2.1 one can get the required result without any difficulty.

Corollary 2.3: Let $(X, M, N, \star, \Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a continuous self-mapping satisfying the condition

$$\frac{1}{M(Tx, Ty, t)} - 1 \geq a\left(\frac{1}{M(x, y, t)} - 1\right) + b\left(\frac{1}{M(x, Tx, t)} - 1\right) \tag{2.16}$$

for all $x, y \in X$ and $a > 1, b \in \mathbb{R}$ with $a + b > 1$. Then T has a unique fixed point.

Proof: Putting $c = 0$ in Theorem 2.1 one can get the required result without any difficulty.

Theorem 2.4: Let $(X, M, N, \star, \Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a surjective self-mapping satisfying the condition (2.1). Then T has a unique fixed point.

Proof: Let x_0 be arbitrary in X, we define a sequence $\{x_n\}$ in X by the rule $x_0 = Tx_1, x_1 = Tx_2, \ldots\ldots, x_n = Tx_{n+1}$. Proceeding like Theorem 2.1, we obtain that $\{x_n\}$ is a Cauchy sequence in complete triangular intuitionistic fuzzy metric space. So there must exists $x^\star \in X$. such that $\lim_{n\to\infty} x_n = x^\star$. Now to show that $x^\star$ is a fixed point of T. Since T is surjective continuous mapping, so for any $p \in X, Tp = x^\star$. Consider

$$\begin{aligned}\frac{1}{M(x_n, x^\star, t)} - 1 &= \frac{1}{M(Tx_{n+1}, Tp, t)} - 1\\ &\geq a\left(\frac{1}{M(x_{n+1}, p, t)} - 1\right) + b\left(\frac{1}{M(x_{n+1}, Tx_{n+1}, t)} - 1\right) + c\left(\frac{1}{M(p, Tp, t)} - 1\right)\end{aligned}$$

$$\frac{1}{M(x_n, x^\star, t)} - 1 \geq a\left(\frac{1}{M(x_{n+1}, p, t)} - 1\right) + b\left(\frac{1}{M(x_{n+1}, x_n, t)} - 1\right) + c\left(\frac{1}{M(p, u, t)} - 1\right) \tag{2.17}$$

Taking limit $n \to \infty$, we get

$$0 \geq (a + c)\left(\frac{1}{M(p, x^\star, t)} - 1\right) \Rightarrow M(p, x^\star, t) = 1 \Rightarrow p = x^\star. \tag{2.18}$$

So $Tp = x^\star$ becomes $Tx^\star = x^\star$. Thus $x^\star$ is a fixed point of T.

Uniqueness Follows from Theorem 2.1.

Theorem 2.5: Let $T, S: X \to X$ be two surjectiuve self mappings of a complete triangular intuitionistic fuzzy metric space. Suppose that T and S satisfying the following inequalities

$$\left(\frac{1}{M(T(Sx),Sx,t)} - 1\right) + k\left(\frac{1}{M(T(Sx),x,t)} - 1\right) \geq a\left(\frac{1}{M(Sx,x,t)} - 1\right) \tag{2.19}$$

and $$\left(\frac{1}{M(S(Tx),Tx,t)} - 1\right) + k\left(\frac{1}{M(S(Tx),x,t)} - 1\right) \geq b\left(\frac{1}{M(Tx,x,t)} - 1\right) \tag{2.20}$$

for all $x, y \in X$ and some nonnegative real numbers a, b and k with $a > 1 + 2k$ and $b > 1 + 2k$. If T or S is continuous, then T and S have a common fixed point.

Proof: Let x_0 be arbitrary in X. since T is surjective, there exists $x_1 \in X$ such that $x_0 = Tx_1$. Also, since S is surjective, there exists $x_2 \in X$ such that $x_2 = Sx_1$. Continuing this process, we construct a sequence $\{x_n\}$ in X such that

$$x_{2n} = Tx_{2n+1} \text{ and } x_{2n+1} = Sx_{2n+2} \tag{2.21}$$

for all $n \in \mathbb{N} \cup \{0\}$. Now for $n \in \mathbb{N} \cup \{0\}$, we have

$$\left(\frac{1}{M(T(Sx_{2n+2}), Sx_{2n+2}, t)} - 1\right) + k\left(\frac{1}{M(T(Sx_{2n+2}), x_{2n+2}, t)} - 1\right) \geq a\left(\frac{1}{M(Sx_{2n+2}, x_{2n+2}, t)} - 1\right)$$

Thus, we have

$$\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) + k\left(\frac{1}{M(x_{2n},x_{2n+2},t)} - 1\right) \geq a\left(\frac{1}{M(x_{2n+1},x_{2n+2},t)} - 1\right)$$

which implies that

$$\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) + k\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) + k\left(\frac{1}{M(x_{2n+1},x_{2n+2},t)} - 1\right) \quad \geq a\left(\frac{1}{M(x_{2n+1},x_{2n+2},t)} - 1\right)$$

Hence $$\left(\frac{1}{M(x_{2n+1},x_{2n+2},t)} - 1\right) \leq \frac{1+k}{a-k}\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \tag{2.22}$$

On other hand, we have

$$\left(\frac{1}{M(S(Tx_{2n+1}), Tx_{2n+1}, t)} - 1\right) + k\left(\frac{1}{M(S(Tx_{2n+1}), x_{2n+1}, t)} - 1\right) \geq b\left(\frac{1}{M(Tx_{2n+1}, x_{2n+1}, t)} - 1\right)$$

Thus we have,

$$\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right) + k\left(\frac{1}{M(x_{2n-1},x_{2n+1},t)} - 1\right) \geq b\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \tag{2.23}$$

Since

$$\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right) + \left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \geq \left(\frac{1}{M(x_{2n-1},x_{2n+1},t)} - 1\right)$$

We have

$$\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right) + k\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right) + k\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \geq b\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \tag{2.24}$$

Hence, $$\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \leq \frac{1+k}{b-k}\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right)$$

Let $k = \max\left\{\frac{1+k}{a-k}, \frac{1+k}{b-k}\right\}$

Then by combining (2.22) and (2.24), we have

$$\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right) \leq \lambda\left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right) \tag{2.25}$$

Repeating (2.25) n-times, we get

$$\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right) \leq \lambda^n\left(\frac{1}{M(x_0,x_1,t)} - 1\right) \tag{2.26}$$

Thus, for $m > n$, we have

$$\frac{1}{M(x_n,x_m,t)} - 1 \leq \frac{1}{M(x_n,x_{n+1},t)} - 1 + \frac{1}{M(x_{n+1},x_{n+2},t)} - 1 + \cdots + \frac{1}{M(x_{m-1},x_m,t)} - 1 \tag{2.27}$$

$$\leq \left(\lambda^n + \lambda^{n+1} + \cdots \ldots \ldots + \lambda^{m-1}\right)\left(\frac{1}{M(x_0,x_1,t)} - 1\right)$$

$$\leq \frac{\lambda^n}{1-\lambda}\left(\frac{1}{M(x_0,x_1,t)} - 1\right) \tag{2.28}$$

Assume that $M(x_0, x_1, t) < 1$ that is, $\frac{1}{M(x_0, x_1,t)} - 1 > 0$. Letting $n \to \infty$, $\{x_n\}$ is a Cauchy sequence. Also, if $M(x_0, x_1, t) = 1$, then $M(x_n, x_m, t) = 1$ for all $m > n$ and hence $\{x_n\}$ is a Cauchy sequence in X. So there must exists $x^\star \in X$. such that $x_n \to x^\star$ as $n \to +\infty$. therefore $x_{2n+1} \to x^\star$ and $x_{2n+2} \to x^\star$ as $n \to +\infty$. Without loss of generality, we may assume that T is continuous, then $Tx_{2n+1} \to Tx^\star$ as $n \to +\infty$. But $Tx_{n+1} = x_{2n} \to x^\star$ as $n \to +\infty$. Thus we have $Tx^\star = x^\star$. Since S is surjective, there exists $y^\star \in X$ such that $Sy^\star = x^\star$.

Now,

$$\left(\frac{1}{M(T(Sy^\star), Sy^\star, t)} - 1\right) + k\left(\frac{1}{M(T(Sy^\star), y^\star, t)} - 1\right) \geq a\left(\frac{1}{M(Sy^\star, y^\star, t)} - 1\right)$$

implies that

$$k\left(\frac{1}{M(x^\star, y^\star, t)} - 1\right) \geq a\left(\frac{1}{M(x^\star, y^\star, t)} - 1\right)$$

Thus, $$\left(\frac{1}{M(x^\star,y^\star,t)} - 1\right) \leq \frac{k}{a}\left(\frac{1}{M(x^\star,y^\star,t)} - 1\right) \tag{2.29}$$

Since $a > k$, we conclude that $\frac{1}{M(x^\star,y^\star,t)} - 1 = 0$ and so $x^\star = y^\star$. Hence $Tx^\star = Sx^\star = x^\star$. Therefore $x^\star$ is a common fixed point of T and S.

By taking $b = a$ in Theorem 2.5, we have the following result.

Corollary 2.6: Let $T, S: X \to X$ be two surjectiuve self mappings of a complete triangular intuitionistic fuzzy metric space. Suppose that T and S satisfying the following inequalities

$$\left(\frac{1}{M(T(Sx),Sx,t)} - 1\right) + k\left(\frac{1}{M(T(Sx),x,t)} - 1\right) \geq a\left(\frac{1}{M(Sx,x,t)} - 1\right) \tag{2.30}$$

and $$\left(\frac{1}{M(S(Tx),Tx,t)} - 1\right) + k\left(\frac{1}{M(S(Tx),x,t)} - 1\right) \geq a\left(\frac{1}{M(Tx,x,t)} - 1\right) \tag{2.31}$$

for all $x, y \in X$ and some nonnegative real numbers a, b and k with $a > 1 + 2k$. If T or S is continuous, then T and S have a common fixed point.

By taking $S = T$ in Corollary 2.6, we have the following corollary.

Corollary 2.7: Let $S: X \to X$ be a continuous surjective self mappings of a complete triangular intuitionistic fuzzy metric space. Suppose that S satisfying the following inequalities

$$\left(\frac{1}{M(S(Sx),Sx,t)} - 1\right) + k\left(\frac{1}{M(S(Sx),x,t)} - 1\right) \geq a\left(\frac{1}{M(Sx,x,t)} - 1\right) \tag{2.32}$$

for all $x, y \in X$ and some nonnegative real numbers a, b and k with $a > 1 + 2k$. Then S has a common fixed point.

Theorem 2.8: Let $(X, M, N, \star, \lozenge)$ be a complete triangular intuitionistic fuzzy metric space. Let $T, f: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(fx,fy,t)} - 1\right) + b\left(\frac{1}{M(fx,Tx,t)} - 1\right) + c\left(\frac{1}{M(fy,Ty,t)} - 1\right) \tag{2.33}$$

for all $x, y \in X$, where $a, b, c \geq 0$, with $a + b + c > 1$. Suppose the following hypotheses:

(1) $b < 1$ or $c < 1$.
(2) $fX \subseteq TX$.
(3) TX is a complete subspace of X.

Then T and f have a coincidence point.

Proof: Let $x_0 \in X$. Since $fX \subseteq TX$, we choose $x_1 \in X$ such that $Tx_1 = fx_0$. Again, we can choose that $x_2 \in X$ such that $Tx_2 = fx_1$. continuing in this same way, we construct a sequence $\{x_n\}$ in X such that for all $n \in \mathbb{N} \cup \{0\}$. If $fx_{m-1} = fx_m$ for some $m \in \mathbb{N}$, then $Tx_m = fx_m$. Thus x_m is a coincidence point of T and f. Now, assume that $x_{n-1} \neq x_n$ for all $m \in \mathbb{N}$.

Case-1: Suppose $b < 1$, by (2.33), we have

$$\begin{aligned}\frac{1}{M(fx_{n-1}, fx_n, t)} - 1 &= \frac{1}{M(Tx_n, Tx_{n+1}, t)} - 1\\ &\geq a\left(\frac{1}{M(fx_n,fx_{n-1}`,t)} - 1\right) + b\left(\frac{1}{M(fx_n,Tx_n`,t)} - 1\right) + c\left(\frac{1}{M(fx_{n+1},Tx_{n+1}`,t)} - 1\right)\\ &= a\left(\frac{1}{M(fx_n,fx_{n-1}`,t)} - 1\right) + b\left(\frac{1}{M(fx_n,fx_{n-1}`,t)} - 1\right) + c\left(\frac{1}{M(fx_{n+1},fx_n`,t)} - 1\right) \qquad (2.34)\end{aligned}$$

Thus, we have

$$(1-b)\left(\frac{1}{M(fx_{n-1},fx_{n`},t)}-1\right) \geq (a+c)\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right) \tag{2.35}$$

Hence,

$$\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right) \leq \left(\frac{1-b}{a+c}\right)\left(\frac{1}{M(fx_{n-1},fx_{n`},t)}-1\right) \tag{2.36}$$

Case-2: Suppose $c < 1$, by (2.33), we have

$$\begin{aligned}\frac{1}{M(fx_n,fx_{n-1},t)}-1 &= \frac{1}{M(Tx_{n+1},Tx_n,t)}-1\\ &\geq a\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right)+b\left(\frac{1}{M(fx_{n+1},Tx_{n+1`},t)}-1\right)+c\left(\frac{1}{M(fx_n,Tx_{n`},t)}-1\right)\\ &= a\left(\frac{1}{M(fx_n,fx_{n+1`},t)}-1\right)+b\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right)+c\left(\frac{1}{M(fx_n,fx_{n-1`},t)}-1\right)\end{aligned} \tag{2.37}$$

Thus we have,

$$(1-c)\left(\frac{1}{M(fx_{n-1},fx_{n`},t)}-1\right) \geq (a+b)\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right) \tag{2.38}$$

Hence,

$$\left(\frac{1}{M(fx_{n+1},fx_{n`},t)}-1\right) \leq \left(\frac{1-c}{a+b}\right)\left(\frac{1}{M(fx_{n-1},fx_{n`},t)}-1\right)$$

In case (1), we get $\lambda = \frac{1-b}{a+c}$ and in case (2), we get $\lambda = \frac{1-c}{a+b}$. Thus in both cases, we have $\lambda < 1$ and

$$\frac{1}{M(fx_{n+1},fx_{n`},t)}-1 \leq \lambda\left(\frac{1}{M(fx_{n-1},fx_{n`},t)}-1\right) \tag{2.39}$$

Repeating (2.39) n-times we get,

$$\frac{1}{M(fx_{n+1},fx_{n`},t)}-1 \leq \lambda^n\left(\frac{1}{M(fx_0,fx_{1`},t)}-1\right) \tag{2.40}$$

So for $m > n$, we have

$$\begin{aligned}\frac{1}{M(fx_n,fx_{m`},t)}-1 &\leq \frac{1}{M(fx_n,fx_{n+1`},t)}-1+\frac{1}{M(fx_{n+1},fx_{n+2`},t)}-1+\cdots+\frac{1}{M(fx_{m-1},fx_{m`},t)}-1\\ &\leq \left(\lambda^n+\lambda^{n+1}+\cdots\ldots..+\lambda^{m-1}\right)\left(\frac{1}{M(fx_0,fx_{1`},t)}-1\right)\\ &\leq \frac{\lambda^n}{1-\lambda}\left(\frac{1}{M(fx_0,fx_{1`},t)}-1\right)\end{aligned} \tag{2.41}$$

Assume that $M(fx_0,fx_1,t) < 1$ that is, $\frac{1}{M(fx_0,fx_1,t)}-1 > 0$. Letting $n \to \infty$, $\{Tx_n\}$ is a Cauchy sequence. Also, if $M(fx_0,fx_1,t) = 1$, then $M(Tx_n,Tx_m,t) = 1$ for all $m > n$ and hence $\{Tx_n\}$ is a Cauchy sequence in X. Therefore $\{Tx_n\}$ is a Cauchy sequence in TX. Since TX is complete. There is $x^\star \in X$ such that $\{Tx_n\}$ converges to $Tx^\star$ as $n \to +\infty$. Hence fx_n converges to $Tx^\star$ as $n \to +\infty$. Since $a+b+c > 1$, we have a, b and c are not all 0. So we have the following cases.

Case-1: If $a \neq 0$, then

$$\begin{aligned}\frac{1}{M(Tx_n,Tx^\star,t)}-1 &\geq a\left(\frac{1}{M(fx_n,fx^\star,t)}-1\right)+b\left(\frac{1}{M(fx_n,Tx_n,t)}-1\right)+c\left(\frac{1}{M(fu,Tu,t)}-1\right)\\ &\geq a\left(\frac{1}{M(fx_n,fu,t)}-1\right)\end{aligned}$$

Hence, $$\left(\frac{1}{M(fx_n,fx^\star,t)}-1\right) \leq \frac{1}{a}\left(\frac{1}{M(Tx_n,Tx^\star,t)}-1\right) \tag{2.42}$$

Since $\frac{1}{a}\left(\frac{1}{M(Tx_n,Tx^\star,t)}-1\right) \to 0$ as $n \to +\infty$. Thus $fx_n \to fx^\star$ as $n \to +\infty$. By uniqueness of limit, we have $Tx^\star = fx^\star$. Therefore T and f have a coincidence point.

Case-2: If $b \neq 0$, then

$$\begin{aligned}\frac{1}{M(Tx^\star,Tx_n,t)}-1 &\geq a\left(\frac{1}{M(fx_n,fx^\star,t)}-1\right)+b\left(\frac{1}{M(fx^\star,Tx^\star,t)}-1\right)+c\left(\frac{1}{M(fx_n,Tx_n,t)}-1\right)\\ &\geq b\left(\frac{1}{M(fx^\star,Tx^\star,t)}-1\right)\end{aligned}$$

Hence, $$\frac{1}{b}\frac{1}{M(Tx^\star,Tx_n,t)}-1 \geq \frac{1}{M(fx^\star,Tx^\star,t)}-1 \tag{2.43}$$

As similar proof of case (1), we can show that $fx^\star = Tx^\star$. Thus f and T have a coincidence point.

Case-3: If $c \neq 0$, then

$$\frac{1}{M(Tx_n,Tu,t)} - 1 \geq a\left(\frac{1}{M(fx_n,fx^\star,t)} - 1\right) + b\left(\frac{1}{M(fx_n,Tx_n,t)} - 1\right) + c\left(\frac{1}{M(Tu,fu,t)} - 1\right)$$
$$\geq c\left(\frac{1}{M(fx^\star,Tx^\star,t)} - 1\right)$$

Hence, $$\frac{1}{M(fx^\star,Tx^\star,t)} - 1 \leq \frac{1}{c}\left(\frac{1}{M(Tx_n,Tx^\star,t)} - 1\right) \tag{2.44}$$

As similar proof of case (1), we can show that $fx^\star = Tx^\star$. Thus f and T have a coincidence point.

By taking $c = 0$ in Theorem 2.8, we have the following.

Corollary 2.9: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T, f: X \to X$ be a mapping satisfying;

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(fx,fy,t)} - 1\right) + b\left(\frac{1}{M(fx,Tx,t)} - 1\right) \tag{2.45}$$

for all $x, y \in X$, where $a, b \geq 0$, with $a + b > 1$. Suppose the following hypotheses:

(1) $b < 1$.
(2) $fX \subseteq TX$.
(3) TX is a complete subspace of X.

Then T and f have a coincidence point.

By taking $b = c = 0$ in Theorem 2.8, we have the following.

Corollary 2.10: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T, f: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(fx,fy,t)} - 1\right) \tag{2.46}$$

for all $x, y \in X$, where $a > 1$. Suppose the following hypotheses:

(1) $fX \subseteq TX$.
(2) TX is a complete subspace of X.

Then T and f have a coincidence point.

By taking $T = I$ (Identity on X) in Theorem 2.8, we have the following.

Corollary 2.11: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,Tx,t)} - 1\right) + c\left(\frac{1}{M(y,Ty,t)} - 1\right) \tag{2.47}$$

for all $x, y \in X$, where $a, b, c \geq 0$, with $a + b + c > 1$. Suppose that $b < 1$ or $c < 1$. Then T has a fixed point.

Corollary 2.12: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,Tx,t)} - 1\right) \tag{2.48}$$

for all $x, y \in X$, where $a, b \geq 0$, with $a + b > 1$. Suppose that $b < 1$. Then T has a fixed point.

Corollary 2.13: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(y,Ty,t)} - 1\right) \tag{2.49}$$

for all $x, y \in X$, where $a, b \geq 0$, with $a + b > 1$. Suppose that $b < 1$. Then T has a fixed point.

Corollary 2.14: Let $(X, M, N,\star,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space. Let $T: X \to X$ be a mapping satisfying

$$\frac{1}{M(Tx,Ty,t)} - 1 \geq a\left(\frac{1}{M(x,y,t)} - 1\right) \tag{2.50}$$

for all $x, y \in X$, where $a > 1$. Then T has a fixed point.

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