ON STRONGER FORMS OF SEMI $\#\alpha$-GENERALIZED $\alpha$ – IRRESOLUTE FUNCTIONS

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ABSTRACT
In this paper we introduce some stronger forms of semi $\#\alpha$-irresolute functions namely strongly semi $\#\alpha$-irresolute functions and almost semi $\#\alpha$-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions.

Key words: semi $\#\alpha$-closed set, semi $\#\alpha$-open set, semi $\#\alpha$-irresolute functions, strongly semi $\#\alpha$-irresolute functions, strongly *semi $\#\alpha$-irresolute functions and almost semi $\#\alpha$-irresolute functions.

1. INTRODUCTION
The concept of strongly $\alpha$-irresolute functions was introduced by G. Lo Fara \textsuperscript{[10]} in 1987. Later in 2003, Y. Beceren \textsuperscript{[3]} introduced the notions of almost $\alpha$-irresolute functions and $\beta$-preirresolute functions. R.Devi \textit{et.al.} \textsuperscript{[5]} introduced and investigated the notions of new classes of functions namely strongly $g^\alpha$-irresolute functions, strongly semi $g^\alpha$-irresolute functions and almost $g^\alpha$-irresolute functions.

In this paper we introduce some stronger forms of semi $\#\alpha$-irresolute functions namely strongly semi $\#\alpha$-irresolute functions and almost semi $\#\alpha$-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions. Throughout this paper X and Y denote the topological spaces $(X,\tau)$ and $(Y,\sigma)$ on which no separation axioms are assumed unless otherwise mentioned. For a subset $A$ of $X$ its closure and interior are denoted by $\text{cl}(A)$ and $\text{int}(A)$ respectively.

2. PRELIMINARIES

\textbf{Definition 2.1:} A subset $A$ of $X$ is said to be

(i) semi-open \textsuperscript{[8]} (resp. $\alpha$-open \textsuperscript{[15]}, $\beta$-open \textsuperscript{[11]}) if $A \subseteq \text{cl}(\text{int}(A))$ (resp. $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, $A \subseteq \text{cl}(\text{cl}(\text{cl}(A)))$.

(ii) generalized closed (briefly g-closed) set \textsuperscript{[9]} if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $X$. The complement of the g-closed set is called a g-open set.

(iii) $g^\alpha$-closed \textsuperscript{[16]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is g-open in $X$. The complement of the $g^\alpha$-closed set is called a $g^\alpha$-open set.

(iv) $g^\alpha$-closed \textsuperscript{[4]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $g^\alpha$-open in $X$. The complement of the $g^\alpha$-closed set is called a $g^\alpha$-open set.

(v) $\text{semi}^g\alpha$-closed \textsuperscript{[7]} if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $g^\alpha$-open in $X$. The complement of the semi $g^\alpha$-closed set is called a semi $g^\alpha$-open set.

(vi) $\text{g}^\alpha$-closed \textsuperscript{[12]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha$-open in $X$. The complement of the $g^\alpha$-closed set is called a $g^\alpha$-open set.

(vii) $g^\alpha$-closed \textsuperscript{[20]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha$-open in $X$. The complement of the $g^\alpha$-closed set is called a $g^\alpha$-open set.

(viii) $\alpha\text{g}$-closed \textsuperscript{[11]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $X$. The complement of the $\alpha\text{g}$-closed set is called a $\alpha\text{g}$-open set.

(ix) $g^\alpha\text{g}$-closed \textsuperscript{[20]} if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha\text{g}$-open in $X$. The complement of the $g^\alpha\text{g}$-closed set is called a $g^\alpha\text{g}$-open set.

(x) $\text{gs}$-closed \textsuperscript{[2]} if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $X$. The complement of the $\text{gs}$-closed set is called a $\text{gs}$-open set.
(xi) strongly g*s-closed [17] if scl(A) ⊆ U, whenever A ⊆ U and U is gs-open in X. The complement of the strongly g*s-closed set is called a strongly g*s-open set.

(xii) gsp-closed [6] if spcl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the gsp-closed set is called a gsp-open set.

**Definition 2.2:** A function f: (X, τ) → (Y, σ) is said to be

(i) *α*-continuous [20] if f⁻¹(V) is *α*-open in X for every open set V of Y.
(ii) g*-continuous [19] if f⁻¹(V) is g*-open in X for every open set V of Y.
(iii) strongly g*-continuous [18] if f⁻¹(V) is strongly g*-open in X for every open set V of Y.
(iv) gs-continuous [2] if f⁻¹(V) is gs-open in X for every open set V of Y.
(v) gsp-continuous [6] if f⁻¹(V) is gsp-open in X for every open set V of Y.
(vi) semi *α*-continuous [7] if f⁻¹(V) is semi *α*-open in X for every open set V of Y.

**Definition 2.3:** A function f: (X, τ) → (Y, σ) is said to be semi *α*-irresolute [7] (resp. *α*-irresolute [13], *β*-irresolute [14]) if f⁻¹(V) is semi *α*-closed (resp. *α*-closed, *β*-closed) in (X, τ) for every semi *α*-closed set V (resp. *α*-closed, *β*-closed) of (Y, σ).

**Definition 2.4:** A function f: (X, τ) → (Y, σ) is said to be strongly *α*-irresolute [10] if f⁻¹(V) is open in (X, τ) for every *α*-open set V of (Y, σ).

**Definition 2.5:** A function f: (X, τ) → (Y, σ) is said to be almost *α*-irresolute [3] if f⁻¹(V) is *β*-open in (X, τ) for every *α*-open set V of (Y, σ).

3. STRONGLY SEMI *α*-IRRESOLUTE FUNCTIONS

**Definition 3.1:** A function f: (X, τ) → (Y, σ) is said to be strongly semi *α*-irresolute if f⁻¹(V) is open in X for every semi *α*-open set V of Y.

**Theorem 3.2:** If f: (X, τ) → (Y, σ) is a strongly semi *α*-irresolute function then it is semi *α*-irresolute.

**Proof:** Let V be a semi *α*-open set in Y. Since f is strongly semi *α*-irresolute function, f⁻¹(V) is open in X and hence it is semi *α*-open. Therefore f is semi *α*-irresolute.

The Converse of the above theorem need not be true by the following example.

**Example 3.3:** Let X = Y = {a, b, c} with τ = {ϕ, X, {a}} and σ = {ϕ, Y, {a}, {a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

Semi *α*-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}.
Semi *α*-open sets in (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}.

Here f⁻¹(V) is semi *α*-open in X for every semi *α*-open set V of Y. Hence f is semi *α*-irresolute. But f⁻¹({a, c}) = {a, c} is semi *α*-open in Y, not open in X. Thus f is not strongly semi *α*-irresolute.

**Theorem 3.4:** If f: (X, τ) → (Y, σ) is a strongly semi *α*-irresolute function then it is continuous (resp. *α*-continuous, semi continuous, *β*-continuous).

**Proof:** Let V be an open set in Y and hence it is a semi *α*-open set in Y. Since f is strongly semi *α*-irresolute function, f⁻¹(V) is open in X (and hence it is *α*-open, semi open and *β*-open respectively). Therefore f is continuous (resp. *α*-continuous, semi continuous, *β*-continuous).

The following example shows that the converse of the above theorem need not be true.

**Example 3.5:** Let X = Y = {a, b, c} with τ = {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

*a*-open sets in (X, τ) = Semi-open sets in (X, τ) = *β*-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}
Semi *α*-open sets in (Y, σ) = {ϕ, X, {a}, {b}, {a, b}}.
Here \( f^{-1}(V) \) is open (resp. \( \alpha \)-open, semi open and \( \beta \)-open) in \( X \) for every open set \( V \) of \( Y \).

Hence \( f \) is continuous (resp. \( \alpha \)-continuous, semi continuous, \( \beta \)-continuous). But \( f^{-1}(\{b\}) = \{b\} \) semi \( \mathcal{g}\alpha \)-open in \( Y \), not open in \( X \). Thus \( f \) is not strongly semi \( \mathcal{g}\alpha \)-irresolute.

**Theorem 3.6:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a strongly semi \( \mathcal{g}\alpha \)-irresolute function then it is \( \mathcal{g}\alpha \)-continuous (resp. \( \mathcal{g}\alpha \)-continuous, strongly \( \mathcal{g}s\)-continuous, \( \mathcal{g}s\)-continious, \( \mathcal{g}s\)-open).

**Proof:** Let \( V \) be an open set in \( Y \) and hence it is a semi \( \mathcal{g}\alpha \)-open set in \( Y \). Since \( f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute function, \( f^{-1}(V) \) is open in \( X \) and hence it is \( \mathcal{g}\alpha \)-open (resp. \( \mathcal{g}s\)-open, strongly \( \mathcal{g}s\)-open, \( \mathcal{g}s\)-open). Therefore \( f \) is \( \mathcal{g}\alpha \)-continuous (resp. \( \mathcal{g}s\)-continuous, strongly \( \mathcal{g}s\)-continuous, \( \mathcal{g}s\)-open).

The Converse of the above theorem need not be true by the following example.

**Example 3.7:** Let \( X = Y = \{a, b, c\} \) with \( \tau = \{\emptyset, X, \{a\} \} \) and \( \sigma = \{\emptyset, Y, \{a, b\}\} \)

Define \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b \) and \( f(c) = c \).

\( \mathcal{g}\alpha \)-open sets in \( (X, \tau) = \mathcal{g}\alpha \)-open sets in \( (Y, \sigma) = \{\emptyset, X, \{a\}, \{a, b\}\} \).

Strongly \( \mathcal{g}s \)-open sets in \( (X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\} \).

\( \mathcal{g}s \)-open sets in \( (X, \tau) = \mathcal{g}s \)-open sets in \( (Y, \sigma) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \).

Semi \( \mathcal{g}\alpha \)-open sets in \( (X, \sigma) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\} \)

Here \( f^{-1}(V) \) is \( \mathcal{g}\alpha \)-open (resp. \( \mathcal{g}s\)-open, strongly \( \mathcal{g}s \)-open, \( \mathcal{g}s \)-open, \( \mathcal{g}s \)-open) in \( X \) for every open set \( V \) of \( Y \). Hence \( f \) is \( \mathcal{g}\alpha \)-continuous (resp. \( \mathcal{g}s \)-continuous, strongly \( \mathcal{g}s \)-continuous, \( \mathcal{g}s \)-continuous, \( \mathcal{g}s \)-continuous). But \( f^{-1}(\{b\}) = \{b\} \) semi \( \mathcal{g}\alpha \)-open in \( Y \), not open in \( X \). Thus \( f \) is not strongly semi \( \mathcal{g}\alpha \)-irresolute.

**Theorem 3.8:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a strongly semi \( \mathcal{g}\alpha \)-irresolute function then it is strongly \( \alpha \)-irresolute.

**Proof:** Let \( V \) be \( \alpha \)-open in \( Y \) and hence it is semi \( \mathcal{g}\alpha \)-open. Since \( f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute, \( f^{-1}(V) \) is open in \( X \). Thus \( f \) is strongly \( \alpha \)-irresolute.

The reverse implication need not be true which can be seen from the following example.

**Example 3.9:** Let \( X = Y = \{a, b, c\} \) with \( \tau = \{\emptyset, X, \{a\} \} \) and \( \sigma = \{\emptyset, Y, \{a, b\}\} \)

Define \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b \) and \( f(c) = c \).

\( \alpha \)-open sets in \( (Y, \sigma) = \{\emptyset, X, \{a\}, \{b\}\} \).

Semi \( \mathcal{g}\alpha \)-open sets in \( (Y, \sigma) = \{\emptyset, Y, \{a\}, \{b\}\} \).

Here \( f^{-1}(V) \) is open in \( X \) for every \( \alpha \)-open set \( V \) of \( Y \). Hence \( f \) is strongly \( \alpha \)-irresolute.

But \( f^{-1}(\{b\}) = \{b\} \) semi \( \mathcal{g}\alpha \)-open in \( Y \), not open in \( X \). Thus \( f \) is not strongly semi \( \mathcal{g}\alpha \)-irresolute.

**Theorem 3.10:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a strongly semi \( \mathcal{g}\alpha \)-irresolute function then for each \( x \in X \) and each semi \( \mathcal{g}\alpha \)-open set \( V \) of \( Y \) containing \( f(x) \) there exists an open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subset V \).

**Proof:** Let \( x \in X \) and \( V \) be any semi \( \mathcal{g}\alpha \)-open and semi \( \mathcal{g}\alpha \)-open set of \( Y \) containing \( f(x) \). Since \( f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute, \( f^{-1}(V) \) is open in \( X \) and contains \( x \). Let \( U = f^{-1}(V) \) then \( U \) is an open subset of \( X \) containing \( x \) and \( f(U) \subset V \).

**Theorem 3.11:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions, then their composition \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is

(i) strongly semi \( \mathcal{g}\alpha \)-irresolute if \( f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute and \( g \) is semi \( \mathcal{g}\alpha \)-irresolute

(ii) semi \( \mathcal{g}\alpha \)-irresolute if \( f \) is semi \( \mathcal{g}\alpha \)-continuous and \( g \) is strongly semi \( \mathcal{g}\alpha \)-irresolute.

**Proof:**

(i) Let \( V \) be a semi \( \mathcal{g}\alpha \)-open subset of \( Z \). Since \( g \) is semi \( \mathcal{g}\alpha \)-irresolute, \( g^{-1}(V) \) is semi \( \mathcal{g}\alpha \)-open in \( Y \). Since \( f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute, \( (g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is open in \( X \) and hence \( g \circ f \) is strongly semi \( \mathcal{g}\alpha \)-irresolute.

(ii) Let \( V \) be a semi \( \mathcal{g}\alpha \)-open subset of \( Z \). Since \( g \) is strongly semi \( \mathcal{g}\alpha \)-irresolute, \( g^{-1}(V) \) is open in \( Y \). Since \( f \) is semi \( \mathcal{g}\alpha \)-continuous, \( (g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is semi \( \mathcal{g}\alpha \)-open in \( X \). Thus \( g \circ f \) is semi \( \mathcal{g}\alpha \)-irresolute.
4. STRONGLY *SEMI $^g \alpha$-IRRESOLUTE FUNCTIONS

**Definition 4.1:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly *semi $^g \alpha$-irresolute if $f^{-1}(V)$ is semi open in $X$ for every semi $^g \alpha$-open set $V$ of $Y$.

**Theorem 4.2:** If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a strongly semi $^g \alpha$-irresolute function then it is strongly *semi $^g \alpha$-irresolute.

**Proof:** Let $V$ be a semi $^g \alpha$-open set in $Y$. Since $f$ is strongly semi $^g \alpha$-irresolute function, $f^{-1}(V)$ is open in $X$ and hence it is semi open in $X$. Therefore $f$ is strongly *semi $^g \alpha$-irresolute.

The Converse of the above theorem need not be true by the following example.

**Example 4.3:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$

Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$.

Semi-open sets in $(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$.

Semi $^g \alpha$-open sets in $(Y, \sigma) = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$.

Here $f^{-1}(V)$ is semi-open in $X$ for every semi $^g \alpha$-open set $V$ of $Y$. Hence $f$ is strongly *semi $^g \alpha$-irresolute. But $f^{-1}(\{a, c\}) = \{a, c\}$ is semi $^g \alpha$-open in $Y$, not open in $X$. Thus $f$ is not strongly semi $^g \alpha$-irresolute.

**Theorem 4.4:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions, then their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is strongly *semi $^g \alpha$-irresolute if $f$ is strongly *semi $^g \alpha$-irresolute and $g$ is semi $^g \alpha$-irresolute.

**Proof:** It is similar to the proof of Theorem 3.11.

**Theorem 4.5:** For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent.

(i) $f$ is strongly *semi $^g \alpha$-irresolute.

(ii) For each $x \in X$ and each semi $^g \alpha$-open set $V$ of $Y$ containing $f(x)$ there exists a semi open set $U$ of $X$ containing $x$ such that $f(U) \subseteq V$.

(iii) $f^{-1}(V) \subseteq \text{cl}(\text{int}(f^{-1}(V)))$ for every semi $^g \alpha$-open set $V$ of $Y$.

(iv) $f^{-1}(M)$ is semi-closed in $X$ for every semi $^g \alpha$-closed set $M$ of $Y$.

**Proof:**

(i) $\Rightarrow$ (ii): Let $x \in X$ and $V$ be any semi $^g \alpha$-open set of $Y$ containing $f(x)$. Since $f$ is strongly *semi $^g \alpha$-irresolute, $f^{-1}(V)$ is semi open in $X$ and contains $x$. Let $U = f^{-1}(V)$. Thus there exists a semi open set $U$ of $X$ containing $x$ such that $f(U) \subseteq V$.

(ii) $\Rightarrow$ (iii): Let $V$ be any semi $^g \alpha$-open set of $Y$ containing $f(x)$. By (ii), there exists a semi open set $U$ of $X$ containing $x$ such that $f(U) \subseteq V$. Thus we have $x \in U \subseteq \text{cl}(\text{int}(U)) \subseteq \text{cl}(\text{int}(f^{-1}(V)))$ and hence $f^{-1}(V) \subseteq \text{cl}(\text{int}(f^{-1}(V)))$.

(iii) $\Rightarrow$ (iv): Let $M$ be any semi $^g \alpha$-closed subset of $Y$. Let $V = Y \setminus M$. Then $V$ is semi $^g \alpha$-open in $Y$. By (iii), we have $f^{-1}(V) \subseteq \text{cl}(\text{int}(f^{-1}(V)))$ and hence $f^{-1}(M) = X \setminus f^{-1}(V \setminus M) = X \setminus f^{-1}(V)$ is semi-closed in $X$.

(iv) $\Rightarrow$ (i): Let $M$ be any semi $^g \alpha$-open subset of $Y$. Let $V = Y \setminus M$. Then $V$ is semi $^g \alpha$-closed in $Y$. By (iv), we have $f^{-1}(V)$ is semi-closed. Then $f^{-1}(M) = X \setminus f^{-1}(V \setminus M) = X \setminus f^{-1}(V)$ is semi-open in $X$. Therefore $f$ is strongly *semi $^g \alpha$-irresolute.

5. ALMOST SEMI $^g \alpha$-IRRESOLUTE FUNCTIONS

**Definition 5.1:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost semi $^g \alpha$-irresolute if $f^{-1}(V)$ is $\beta$-open in $X$ for every semi $^g \alpha$-open set $V$ of $Y$.

**Theorem 5.2:** If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost semi $^g \alpha$-irresolute function then it is $\beta$-continuous.

**Proof:** Let $V$ be an open set in $Y$ and hence it is a semi $^g \alpha$-open set in $Y$. Since $f$ is almost semi $^g \alpha$-irresolute function, $f^{-1}(V)$ is $\beta$-open in $X$. Hence $f$ is $\beta$-continuous.

The following example shows that the converse of the above theorem need not be true.
Example 5.3: Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$

Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$.

$\beta$-open sets in $(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$.

Semi $^\alpha_g\alpha$-open sets in $(Y, \sigma) = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$.

Here $f^{-1}(V)$ is $\beta$-open in X for every open set V of Y. Hence f is $\beta$-continuous. But $f^{-1}(\{b\}) = \{b\}$ is semi $^\alpha_g\alpha$-open in Y, not $\beta$-open in X. Thus f is not almost semi $^\alpha_g\alpha$-irresolute.

Theorem 5.4: If $f: (X, \tau) \to (Y, \sigma)$ is almost semi $^\alpha_g\alpha$-irresolute function then it is almost $\alpha$-irresolute.

Proof: Let V be $\alpha$-open in Y and hence it is semi $^\alpha_g\alpha$-open. Since f is almost semi $^\alpha_g\alpha$-irresolute, $f^{-1}(V)$ is $\beta$-open in X. Thus f is almost $\alpha$-irresolute.

The reverse implication need not be true which can be seen from the following example.

Example 5.5: Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$

Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$.

$\beta$-open sets in $(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$.

$\alpha$-open sets in $(Y, \sigma) = \{\emptyset, Y, \{a, b\}\}$.

Semi $^\alpha_g\alpha$-open sets in $(Y, \sigma) = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$.

Here $f^{-1}(V)$ is $\beta$-open in X for every $\alpha$-open set V of Y. Hence f is almost $\alpha$-irresolute.

But $f^{-1}(\{b\}) = \{b\}$ is semi $^\alpha_g\alpha$-open in Y, not $\beta$-open in X. Thus f is not almost semi $^\alpha_g\alpha$-irresolute.

Theorem 5.6: Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions, then their composition $g \circ f: (X, \tau) \to (Z, \eta)$ is

(i) almost $\alpha$-irresolute if f is almost semi $^\alpha_g\alpha$-irresolute and g is $\alpha$-irresolute

(ii) almost semi $^\alpha_g\alpha$-irresolute if f is $\beta$-irresolute and g is almost semi $^\alpha_g\alpha$-irresolute.

Proof: The Proof is similar to that of Theorem 3.11.

Theorem 5.7: For a function $f: (X, \tau) \to (Y, \sigma)$, the following are equivalent.

(i) $f$ is almost semi $^\alpha_g\alpha$-irresolute.

(ii) For each $x \in X$ and each semi $^\alpha_g\alpha$-open set $V$ of $Y$ containing $f(x)$ there exists a $\beta$-open set $U$ of $X$ containing $x$ such that $f(U) \subset V$.

(iii) $f^{-1}(V) \subset cl(int(f^{-1}(V)))$ for every semi $^\alpha_g\alpha$-open set $V$ of $Y$.

(iv) $f^{-1}(M)$ is $\beta$-closed in $X$ for every semi $^\alpha_g\alpha$-closed set $M$ of $Y$.

Proof: It is similar to the proof of Theorem 4.5.

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