ON SOME PROPERTIES OF $\alpha\gamma$-OPEN SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this article a new class of open sets called $\alpha\gamma$-open sets in topological spaces is introduced. This class contains the class of all $\theta$-open sets and is contained in the class of all $\alpha$-open sets. The inclusion relation of this new class with other known classes of open sets are investigated. Also their properties are analyzed.

Keywords: $\alpha$-open sets, $\theta$-open sets, semi-$\theta$-open sets, $\alpha\gamma$-open sets, Extremally Disconnected.

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1. INTRODUCTION

In 1965 Njastad [13] introduced the notion of alpha open sets (briefly $\alpha$-open sets). Followed by the class of $\alpha$-open sets, several other related classes such as alpha generalized open sets and generalized alpha open sets (briefly $ag$-open sets and $ga$-open sets) were defined by Maki et.al [10]. As an extension of $\alpha$-closed sets, $ag\gamma$-closed sets were defined by Abd El-Monsef, et.al [1]. As further study on the application of $\alpha$-closed sets and $ag\gamma$-closed sets Mary and Nagajothi [11] and [12] introduced $ba\gamma$-closed sets and $ab\gamma$-closed sets and analyzed their properties. The following inclusion relation holds.

$$\{\alpha$-closed sets$\} \subset \{ab\gamma$-closed sets$\} \subset \{ba\gamma$-closed sets$\}.$$  

Velicki[16] defined $\theta$-open sets in 1968. As an extension of this class, Di Maio and Noiri[5] introduced semi-$\theta$-open sets in 1987. Following these classes, in this paper a new class of open sets namely class of $\alpha\gamma$-open sets is introduced which is contained in the class of $\alpha$-open sets and the class of $\theta$-open sets forms a subclass. The new class of $\alpha\gamma$-open sets satisfy the inclusion relation given below

$$\{\theta$-open sets$\} \subset \{\alpha\gamma$-open sets$\} \subset \{\alpha$-open sets$\}.$$  

2. PRELIMINARIES

Throughout this paper, $(X, \tau)$ denote a topological space with topology $\tau$. For a subset $A$ of $X$ the interior of $A$ and closure of $A$ are denoted by $Int(A)$ and $Cl(A)$ respectively.

Definition 2.1: [8] A topology on a set $X$ is a collection $\tau$ of subsets of $X$ having the following properties:

1) $\emptyset$ and $X$ are in $\tau$.
2) The union of the elements of any subcollection of $\tau$ is in $\tau$.
3) The intersection of the elements of any finite subcollection of $\tau$ is in $\tau$.

A set $X$ for which a topology $\tau$ has been specified is called a topological space and is denoted by $(X, \tau)$.

Definition 2.2: A subset $A$ of a space $X$ is said to be:

1) $\alpha$-open set[13] if $A \subset Int(cl(int(A)))$ and $\alpha$-closed set if $Int(cl(int(A)) \subset A$.
3) Semi-open set[9] if $A \subset Cl(int(A))$ and Semi-closed set if $Cl(int(A)) \subset A$.
4) Pre-open set if $A \subset Int(cl(A))$ and Pre-closed set if $Int(cl(A)) \subset A$.
5) b-open set[3] if $A \subset (Int(Cl(A)) \cup (Cl int(A)) and b-closed set if $(Int(Cl(A)) \cup (Cl int(A)) \subset A$.
6) **Semi-**$\theta$-**open set**[9] if for each $x \in A$, there exists a semi-open set $G$ such that $x \in G \subset \text{Cl}(G) \subset A$.
7) **$\theta$-**open set[16] if for each $x \in A$, there exists an open set $G$ such that $x \in G \subset \text{Cl}(G) \subset A$.
8) **$\delta$-**open set[16] if for each $x \in A$, there exists an open set $G$ such that $x \in G \subset \text{Int}(\text{cl}(G)) \subset A$.
9) **$\theta$-**semi-**open set** if for each $x \in A$, there exists a semi-open set $G$ such that $x \in G \subset \text{Cl}(G) \subset A$.

**Definition 2.3:**
1) The intersection of all semi-closed sets containing $A$ is called the **semi-closure**[4] of $A$ denoted by $s\text{Cl}(A)$.
2) The intersection of all $\alpha$-closed sets containing $A$ is called $\alpha$-**closure**[13] of $A$ denoted by $\alpha\text{Cl}(A)$.
3) The intersection of all $b$-closed sets containing $A$ is called the $b$-**closure**[3] of $A$ denoted by $b\text{Cl}(A)$.

**Definition 2.4:** The family of all open sets, semi-open sets, $\alpha$-open sets, pre-open sets, semi-$\theta$-open sets, $\theta$-open sets, $\delta$-open sets, regular-open sets, semi-closed sets and regular closed sets are denoted by $O(X), SO(X), aO(X), PO(X), SBD(X), BD(X), BD(X), RBD(X), SC(X)$ and $RC(X)$ respectively.

**Definition 2.5:** A topological space $(X, \tau)$ is said to be:
(i) $T_1$ space if to each pair of distinct points $x, y$ of $X$ there exist a pair of open sets, one containing $x$ but not $y$ and other containing $y$ but not $x$, as well as is $T_1$ if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed [17].
(ii) $T_2$ space if to each pair of distinct points $x, y$ of $X$ there exist a pair of disjoint open sets, one containing $x$ other containing $y$, as well as is $T_2$ if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed.
(iii) **Locally indiscrete**[6], if every open subset of $X$ is closed.
(iv) **$S^\ast$**-**normal**[2] if and only if for every semi-closed set $F$ and semi-open set $G$ containing $F$, there exists an open set $H$ such that $F \subset H \subset \text{Cl}(H) \subset G$.
(v) **Regular** if for each $x \in X$ and for each open set $A$ containing $x$, there exists an open set $G$ containing $x$ such that $x \in G \subset \text{Cl}(G) \subset A$.

**Definition 2.6:** A space $X$ is called **Extremally disconnected**[15], if closure of every open set is open.

3. **$ac$-OPEN SETS:**

In this section we introduce a new class of open sets called **$ac$-open sets** which lie between the class of $\theta$-open sets and the class of $\alpha$-open sets.

**Definition 3.1:** A subset $A$ of a topological space $X$ is called **$ac$-open set** if for each $x \in A \in aO(X)$, there exists a closed set $F$, such that $x \in F \subset A$.

The following Theorem gives a characterization for $ac$-open sets.

**Theorem 3.1.1:** A subset $A$ of a space $X$ is $ac$-open set if and only if $A$ is $\alpha$-open set and it is the union of closed sets. That is $A=\bigcup F_x$ where $A$ is $\alpha$-open set and $F_x$ is closed sets for each $x$.

**Proof:** Let $A$ be $ac$-open set. Then by definition (3.1), $A$ is a $\alpha$-open set.

Since for each $x \in A \in aO(X)$, there exist a closed set $F_x$ such that $x \in F_x \subset A$, we have $A=\bigcup F_x$.

Conversely, let $A$ be a $\alpha$-open set and $A=\bigcup F_x$. For each $x \in A$, there exists $x$ such that $x \in F_x \subset A$.

Hence $A$ is a $ac$-open set.

**Corollary 3.1.1:** Every $\theta$-open set of a space $X$ is $ac$-open.

**Proof:** Let $A$ be a $\theta$-open set in $X$. Then for each $x \in A$, there exists an open set $G$ such that $x \in G \subset \text{Cl}(G) \subset A$.

So $\bigcup \{x\} \subset \bigcup G \subset \bigcup \text{Cl}(G) \subset A$, implies $A \subset \bigcup G \subset \bigcup \text{Cl}(G) \subset A$. Therefore $A=\bigcup G$ and $A=\bigcup \text{Cl}(G)$.

Since $G$ is open and arbitrary union of open set is open, $A$ is open. Since open implies $\alpha$-open, by Theorem (3.1.1), $A$ is $ac$-open.

**Remark 3.1.1:**
1) For any $\alpha$-open set $A$ to be $ac$-open set, it is necessary that $A$ must be the union of closed sets.

For example, let $X=\{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, the Closed sets are $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$.
Then \( aO(X) = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\} \} \) and \( \alpha O(X) = \{ \emptyset, X \} \).

Here \( \{a\} \) is \( a \)-open, but it is not a \( \alpha \)-open set. Since \( \{a\} \) is not a union of closed sets in \( X \).

2) \( S0\alpha(X) \) need not be an \( \alpha c0(X) \).

Let \( X=\{a, b, c\} \) with \( \tau = \{X, \emptyset, \{a, b\}, \{a\}, \{b\}\} \), the closed sets are \( \{X, \emptyset, \{c\}, \{b, c\}, \{a, c\}\} \). Then \( S0\alpha(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \) and \( \alpha O(X) = \{X, \emptyset\} \). Here \( \{a\} \) is \( S0\alpha(X) \) but not \( \alpha O(X) \).

**Theorem 3.1.2:** Let \((X, \tau)\) be a topological space and \( \{A_j : j \in \Delta\} \) be a collection of \( \alpha c\)-open sets in \( X \). Then \( \bigcup \{A_j : j \in \Delta\} \) is \( \alpha c\)-open.

**Proof:** By definition (3.1), for each \( A_j \) is \( \alpha \)-open set, \( j \in \Delta \). Since union of \( \alpha \)-open sets is \( \alpha \)-open, \( \bigcup \{A_j : j \in \Delta\} \) is \( \alpha \)-open. Let \( x \in \bigcup \{A_j : j \in \Delta\} \), then there exists \( j \in \Delta \) such that \( x \in A_j \). Since \( A_j \) is \( \alpha c\)-open set, there exists a closed set \( F \) such that \( x \in F \subset A_j \cup \{A_j : j \in \Delta\} \). Hence \( \bigcup \{A_j : j \in \Delta\} \) is an \( \alpha c\)-open set in \( X \).

The following Corollary gives another characterization of \( \alpha c\)-open sets.

**Corollary 3.1.2:** The set \( A \) is \( \alpha c\)-open in the space \((X, \tau)\) if and only if for each \( x \in A \), there exists a \( \alpha c\)-open set \( B \) such that \( x \in B \subset A \).

**Proof:** Let \( A \) is \( \alpha c\)-open. Then for each \( x \in A \), choose \( B = A \) so that \( x \in B \subset A \), and \( B \) is \( \alpha c\)-open.

Conversely, Assume that for each \( x \in A \), there exists a \( \alpha c\)-open set \( B_x \) such that \( x \in B_x \subset A \).

Thus \( A = \bigcup B_x \) where \( B_x \in \alpha O(X) \). By Theorem (3.1.2), \( A \) is \( \alpha c\)-open set.

**Theorem 3.1.3:** If the family of all \( \alpha \)-open sets of a space \( X \) is a topology on \( X \), then the family of all \( \alpha c\)-open sets is also a topology on \( X \).

**Proof:**

(i) Clearly \( \emptyset, X \in \alpha O(X) \).

(ii) By Theorem(3.1.2), the union of all \( \alpha c\)-open sets is \( \alpha c\)-open.

(iii) We have to show that finite intersection of \( \alpha c\)-open set is \( \alpha c\)-open set.

Let \( \{A_j : j = 1, 2, \ldots, n\} \) be a collection of \( \alpha c\)-open sets. Then by definition (3.1), \( A_1, A_2, \ldots, A_n \) are \( \alpha \)-open sets.

Let \( x \in \bigcap \{A_j : j = 1, 2, \ldots, n\} \), then \( x \in A_j \) for each \( j \), and there exists a closed set \( F_j \) such that \( x \in F_j \subset A_j \).

Then \( x \in \bigcap F_j \subset \bigcap \{A_j : j = 1, 2, \ldots, n\} \). Thus \( \bigcap \{A_j : j = 1, 2, \ldots, n\} \) is \( \alpha c\)-open.

Hence the family of all \( \alpha c\)-open sets is also a topology on \( X \).

**Lemma 3.1.4:** For any spaces \( X \) and \( Y \). If \( A \subseteq X \) and \( B \subseteq Y \) then,

(i) \( aInt_{X\times Y}(A \times B) = aInt_{X}(A) \times aInt_{Y}(B) \) [9].

(ii) \( Cl_{X\times Y}(A \times B) = Cl_{X}(A) \times Cl_{Y}(B) \).

The following Theorem shows that the property of being \( \alpha c\)-open is preserved by the product of two topological spaces.

**Theorem 3.1.5:** Let \( X \) and \( Y \) be two topological spaces and \( X \times Y \) be the product topology. If \( A \in \alpha O(X) \) and \( B \in \alpha O(Y) \). Then \( A \times B \in \alpha O(X \times Y) \).

**Proof:** Let \((x, y) \in A \times B \), then \( x \in A \) and \( y \in B \). Since \( A \in \alpha O(X) \) and \( B \in \alpha O(Y) \), then \( A \in \alpha O(X) \) and \( B \in \alpha O(Y) \). Also there exists closed sets \( F \) and \( E \) in \( X \) and \( Y \) respectively, such that \( x \in F \subseteq A \) and \( y \in E \subseteq B \).

Therefore, \((x, y) \in F \times E \subseteq A \times B \). Since \( A \in \alpha O(X) \) and \( B \in \alpha O(Y) \). Then by Lemma 3.1.4(i), \( A \times B = aInt_{X}(A) \times aInt_{Y}(B) = aInt_{X\times Y}(A \times B) \). Hence \( A \times B \in \alpha O(X \times Y) \). Since \( F \) is closed in \( X \) and \( E \) is closed in \( Y \).

Then by Lemma 3.1.4(ii), \( F \times E = Cl_{X}(F) \times Cl_{Y}(E) = Cl_{X\times Y}(F \times E) \). Hence \( F \times E \) is closed in \( X \times Y \).

Therefore, \( A \times B \in \alpha O(X \times Y) \).
Theorem 3.1.6: If the space $X$ is a $T_1$-space (or) a $T_2$-space, then the family $ac\ O(X) = aO(X)$.

Proof: Let $A$ be $\alpha$- open set of $X$.

Since $X$ is a $T_1$ space (or) a $T_2$ space, for each $x \in A \subset X$, $\{x\}$ is closed.

Therefore $x \in \{x\} \subset A$, $A \in \alpha O(X)$. Hence $aO(X) \subset acO(X)$.

By definition of $ac$-open sets, $acO(X) \subset aO(X)$. Hence $aO(X) = acO(X)$.

Theorem 3.1.7: [18] If $X$ is $S^{**}$-normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$.

Theorem 3.1.8: If $(X, \tau)$ is an $S^{**}$-normal space and if $A \in S\theta O(X)$, then $A \in acO(X)$.

Proof: Let $A$ be an semi-$\theta$-open set of $X$. If $A = \emptyset$, then $A \in acO(X)$.

Suppose $A \neq \emptyset$. Since the space $A$ is $S^{**}$-normal, by Theorem (3.1.7) $S\theta O(X) = \theta O(X)$.

Then $A$ is $\theta$-open of $X$. By Corollary (3.1.1), $A \in acO(X)$.

Remark 3.1.9: Every open set need not be $ac$-open. It is evident from the following example.

For example, let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\},\{a, b, c\},\{b, c, d\}\}$ and

The closed sets are $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\},\{a, b, c\},\{b, c, d\}\}$.

Then $acO(X) = \{\{a\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{b, d\}\}$. Here $\{b\}$ is open set but not $ac$-open.

The following Theorem gives conditions under which an open set is also $ac$-open.

Theorem 3.1.9: Every open set is $ac$-open set in $X$, if one of the following holds.

(i) $(X, \tau)$ is Locally indiscrte.

(ii) $X$ is Regular.

Proof:

(i) Let $A$ be an open set of $X$. If $A = \emptyset$, then $A \in acO(X)$.

Suppose $A \neq \emptyset$, we know that $\tau \subset aO(X)$. Therefore $A \in aO(X)$.

If $X$ is Locally indiscrete, then every open subset is closed.

Since $A$ is open, we have $A$ is closed. Therefore, $x \in A \subset A$ implies $A \in acO(X)$.

Hence every open is $ac$-open of $X$.

(ii) Let $A$ be an open set of $X$. If $A = \emptyset$, then $A \in acO(X)$. Suppose $A \neq \emptyset$, we know that $\tau \subset aO(X)$. Therefore $A \in aO(X)$. If $X$ is Regular and since $A \in \tau$, we have for each $x \in A$, there exists an open set $G$ containing $x$ such that $x \in G \subset Cl(G) \subset A$ implies $x \in Cl(G) \subset A$. Thus $A \in acO(X)$.

Theorem 3.1.10: [18] A space $X$ is called Extremally disconnected if and only if $\delta O(X) = \theta SO(X)$.

Theorem 3.1.11: Let $(X, \tau)$ be an Extremally disconnected and $S^{**}$-normal space. Then

(i) $\delta O(X) \subset acO(X)$.

(ii) $RO(X) \subset acO(X)$.

Proof:

(i) Let $A$ be an $\delta$-open set of $X$. If $A = \emptyset$, then $A \in acO(X)$. Suppose $A \neq \emptyset$, since $X$ is Extremally disconnected, by Theorem (3.1.10), we have $\delta O(X) = \theta SO(X)$. Then $A \in \theta SO(X)$.

If $X$ is $S^{**}$-normal space, then by Theorem(3.1.7), $\theta SO(X) = \theta O(X)$. Hence $A \in \theta O(X)$. 

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By Corollary (3.1.1), A is $\alpha$-open of X. Hence $\delta O(X) \subseteq a\alpha O(X)$.

(ii) Let A be a Regular open set of X. If A = $\emptyset$, then $\subseteq a\alpha O(X)$. Suppose $A \neq \emptyset$, Since A be Regular open implies $A = Int(Cl(A)$, then for each $x \in A$, there exist an open set A such that $x \in A \subseteq Int(Cl(A)) \subseteq A$.

Then $A \in \delta O(X)$. By (i), A be an $a\alpha O(X)$.

**Remark 3.1.11:**
1) Every $\delta$-open set need not be $\alpha$-open. It is evident from the following example.

Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a, b, c\}, \{a\}\}$. the closed set are $\{X, \emptyset, \{b, c\}, \{a\}\}$.

Then $\delta O(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $a\alpha O(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}\}$.

Here $\{b\}$ is $\delta O(X)$ but not $a\alpha O(X)$.

2) Every Regular-open set need not be $\alpha$-open. It is evident from the following example.

In Remark (3.1.9), we have Regular-open sets $\{\{a\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \emptyset, X\}$ and $a\alpha O(X) = \{\{a\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \emptyset, X\}$. Here $\{d\}$ is $RO(X)$ but not $a\alpha O(X)$.

**3.2 $\alpha$-Closed set:**

**Definition 3.2:** A subset B of a space X is called $\alpha$-closed set if $X \setminus B$ is $\alpha$-open set. The family of all $\alpha$-closed subsets of a topological space $(X, \tau)$ is denoted by $a\alpha C(X)$.

The following Theorem gives a characterization of $\alpha$-closed sets.

**Theorem 3.2.1:** A subset $B$ of a space $X$ is $\alpha$-closed if and only if $B$ is $\alpha$-closed set and it is an intersection of open sets.

**Proof:** Let $B$ be an $\alpha$-closed set in $X$.

Then $X \setminus B$ is $\alpha$-open set. Thus, $X \setminus B$ is $\alpha$-open set and for all $y \in X \setminus B$, there exists a closed set $F_y$ such that $y \in F_y \subset X \setminus B$. Then $B$ is $\alpha$-closed and $\cup \{y\} \subset F_y \subset X \setminus B$, $X \setminus B \subseteq \cup F_y$.

Then $B = X \setminus \cup F_y$ implies $B \cap \cap \{X \setminus F_y\}$ is open set. $B$ is an intersection of open sets. Hence $B$ is $\alpha$-closed set and it is an intersection of open sets.

Conversely, Let $B$ be $\alpha$-closed set and intersection of open sets. $B$ is $\alpha$-closed implies $X \setminus B$ is $\alpha$-open and $B = \cap F_i$ where $F_i$’s are open set. $X \setminus B = X \setminus (\cap F_i) = \cup (X \setminus F_i)$, where $X \setminus F_i$ is closed set.

Thus for all $y \in X \setminus B$, there exists some $i$ such that $y \in X \setminus F_i$, where $X \setminus F_i$ is closed set.

i.e., $y \in X \setminus F_i \subset X \setminus B$ implies $X \setminus B$ is $\alpha$-open. Hence $B$ is $\alpha$-closed.

**Corollary 3.2.1:** For any subset $B$ of a space, if $B \in \theta C(X)$, then $B \subseteq a\alpha C(X)$.

**Proof:** Let $B$ be a $\theta$-closed set of $X$. Then $X \setminus B$ is an $\alpha$-open set.

By Corollary (3.1.1), we have $X \setminus B$ is an $\alpha$-open set. Thus $B$ is $\alpha$-closed set. Hence $\theta C(X) \subseteq a\alpha C(X)$.

**Theorem 3.2.2:** Let $\{B_j : j \in \Delta\}$ be a collection of $\alpha$-closed sets in a topological space $X$. Then $\cap \{B_j : j \in \Delta\}$ is $\alpha$-closed set.

**Proof:** Let $B_j$’s be $\alpha$-closed set. Then $X \setminus B_j$ is $\alpha$-open set. By Theorem (3.1.2), $\cup \{X \setminus B_j : j \in \Delta\}$ is an $\alpha$-open set. Then $\{X \setminus (\cap B_j) : j \in \Delta\}$ is an $\alpha$-open set. Hence $\cap B_j : j \in \Delta$ is $\alpha$-closed set.

**Theorem 3.2.3:** If the space $X$ is an $T_1$-space (or) $T_2$-space, then the family $a\alpha C(X) = \alpha C(X)$.

**Proof:** Let $B$ be an $\alpha$-closed subset of $X$.

Then $X \setminus B$ is $\alpha$ open. Since $\alpha O(X) = aO(X)$, we have $X \setminus B$ is $\alpha$ open. Hence $B$ is $\alpha$ closed.
Theorem 3.2.4: Every closed set need not be $ac$-closed. It is evident from the following example.

Let $X = \{a, b, c, d\}$, $\tau = \emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. The closed sets are $\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $acC(X) = \{\{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, c, d\}, \emptyset, X\}$. Here $\{d\}$ is closed set but not $ac$-closed.

The following Theorem gives conditions under which an closed set is also $ac$-open.

Theorem 3.2.4: Every closed set is $ac$-closed in $X$, if one of the following condition holds:

(i) $X$ is Locally indiscrete.

(ii) $X$ is Regular.

Proof: (i) Let $B$ be a closed subset of $X$. If $A = \emptyset$, then $A \in acC(X)$.

Suppose $A \neq \emptyset$, then $X\setminus B$ is open set. Since every open set is $a$-open, $X\setminus B$ is $a$-open of $X$.

Since $X$ is Locally indiscrete, $X\setminus B$ is closed. Then for each $x \in X\setminus B \subset X\setminus B$, $X\setminus B \in acO(X)$. Hence $B \in acC(X)$.

(ii) Let $B$ be closed subset of $X$. Then $X\setminus B$- is open.

If $B = \emptyset$, then $B \in acC(X)$. Suppose $B \neq \emptyset$, then $X\setminus B \in acO(X)$.

If $X$ is Regular, then for each open set $X\setminus B$ containing $x$, there exists an open set $G$ such that, $x \in G \subset Cl(G) \subset X\setminus B$, $x \in Cl(G) \subset X\setminus B$.

Therefore $X\setminus B \in acO(X)$ implies $B \in acC(X)$. Hence $C(X) \subset acC(X)$.

Remark 3.2.5: Every $\delta$-closed set need not be $ac$-closed. It is evident from the following example.

Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$, the closed sets are $\{X, \emptyset, \{b, c\}, \{a\}\}$. Then $\delta$-closed sets=$\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$ and $acC(X) = \{X, \emptyset, \{a\}, \{b, c\}\}$. Here $\{b\}$ is $\delta C(X)$ but not $acC(X)$.

The following Theorem gives conditions under which an $\delta$-closed set is also $ac$-closed.

Theorem 3.2.5: Let $(X, \tau)$ be an Extremally disconnected and $S^{**}$-normal space. If $B \in \delta C(X)$, then $B \in acC(X)$.

Proof: Let $B$ be an $\delta$-closed subset of $X$. Then $X\setminus B$ is $\delta$-open set. If $B = \emptyset$, then $A \in acC(X)$. Suppose $A \neq \emptyset$, let $X\setminus B \in \delta O(X)$. we have by 3.1.11(i) $X\setminus B \in acO(X)$. Hence $B \in acC(X)$.

Remark 3.2.6: Every $S\theta C(X)$ need not be an $acC(X)$.

Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}\}$, the closed sets are $\{X, \emptyset, \{c\}, \{b, c\}, \{a, c\}\}$
Semi-$\theta$-closed=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$, and $acC(X) = \{X, \emptyset\}$. Here $\{a\}$ is $S\theta C(X)$ but not $acC(X)$.

Theorem 3.2.6: Let $(X, \tau)$ be an $S^{**}$- normal space. If $B \in S\theta C(X)$ then $B \in acC(X)$.

Proof: Let $B$ be an semi-$\theta$-closed subset of $X$, then $X\setminus B$-is semi-$\theta$-open of $X$.

If $B = \emptyset$, then $B \in acC(X)$. Suppose $B \neq \emptyset$, as the space $X$ is $S^{**}$- normal, By (3.1.7) $S\theta O(X) = \theta O(X)$, $X\setminus B \in \theta O(X)$. By Corollary(3.1.1), $X\setminus B$-is $\alpha$-open set. Hence $B$ is $ac$-closed set of $X$.

The following diagram shows that the relations among $acO(X)$, $\alpha O(X)$, $RO(X)$, $\delta O(X)$, $\tau$, and $\theta O(X)$.
REFERENCES


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