International Journal of Mathematical Archive-6(11), 2015, 135-141
Available online through www.ijma.info ISSN 2229 – 5046

ON SOME PROPERTIES OF αc-OPEN SETS IN TOPOLOGICAL SPACES

STELLA IRENE MARY.J*[1], SINDHU.M[2]

[1]Associate Professor, [2]M. Phil Scholar,
Department of Mathematics, PSG College of Arts & Science, Coimbatore, India.

(Received On: 29-10-15; Revised & Accepted On: 20-11-15)

ABSTRACT

In this article a new class of open sets called αc-open sets in topological spaces is introduced. This class contains the class of all θ-open sets and is contained in the class of all α-open sets. The inclusion relation of this new class with other known classes of open sets are investigated. Also their properties are analyzed.

Keywords: *α-open sets, θ-open sets, semi-θ-open sets, αc-open sets, Extremally Disconnected.*

AMS Subject Classification: *54A05, 54G05, 54H05.*

1. INTRODUCTION

In 1965 Njastad [13] introduced the notion of alpha open sets (briefly α-open sets). Followed by the class of α-open sets, several other related classes such as alpha generalized open sets and generalized alpha open sets (briefly αg-open sets and $g\alpha$-open sets) were defined by Maki *et.al* [10]. As an extension of α-closed sets, $\alpha\hat{g}$closed sets were defined by Abd El-Monsef, *et.al* [1]. As further study on the application of α-closed sets and $\alpha\hat{g}$-closed sets Mary and Nagajothi [11] and [12] introduced $b\alpha\hat{g}$-closed sets and $\alpha b\hat{g}$-closed sets and analyzed their properties. The following inclusion relation holds.

$\{\alpha$-closed sets$\}\subset\{\alpha b\hat{g}$-closed sets$\}\subset\{$ b$\alpha\hat{g}$-closed sets$\}$.

Velicko[16] defined θ-open sets in 1968. As an extension of this class, Di Maio and Noiri[5] introduced semi-θ-open sets in 1987. Following these classes, in this paper a new class of open sets namely class of αc-open sets is introduced which is contained in the class of α-open sets and the class of θ-open sets forms a subclass. The new class of αc-open sets satisfy the inclusion relation given below

$\{\theta$-open sets$\}\subset\{\alpha c$-open sets$\}\subset\{\alpha$-open sets$\}$.

2. PRELIMINARIES

Throughout this paper, (X,τ) denote a topological space with topology τ. For a subset A of X the interior of A and closure of A are denoted by $Int(A)$ and $Cl(A)$ respectively.

Definition 2.1: [8] A $topology$ on a set X is a collection τ of subsets of X having the following properties:

1) $\emptyset$ and X are in τ.
2) The union of the elements of any subcollection of τ is in τ.
3) The intersection of the elements of any finite subcollection of τ is in τ.

A set X for which a topology τ has been specified is called a $topological\ space$ and is denoted by (X,τ).

Definition 2.2: A subset A of a space X is said to be:

1) α-$open\ set$[13] if $A\subset Int(cl(int(A))$ and α-$closed\ set$ if $Int(cl(int(A))\subset A$.
2) $Regular$-$open\ set$[14] if $A = Int\big(cl(A)\big)$ and $Regular$-$closed\ set$ if $A = Cl\big(int(A)\big)$.
3) $Semi$-$open\ set$[9] if $A\subset Cl(int(A))$ and $Semi$-$closed\ set$ if $Cl(int(A))\subset A$.
4) Pre-$open$ set if $A\subset Int\big(cl(A)\big)$ and Pre- closed set if $Int\big(cl(A)\big)\subset A$.
5) b-$open\ set$[3] if $A\subset (Int\ Cl(A))\cup(Cl\ int(A))$ and b-$closed\ set$ if $\big(Int\ Cl(A)\big)\cup\big(Cl\ int(A)\big)\subset A$.

Corresponding Author: Stella Irene Mary.J*[1]
[1]Associate Professor, Department of Mathematics,
PSG College of Arts & Science, Coimbatore, India.

6) $Semi\text{-}\theta\text{-}open\ set$[9] if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset sCl(G) \subset A$.
7) $\theta\text{-}open\ set$[16] if for each $x \in A$, there exists an open set G such that $x \in G \subset Cl(G) \subset$ A.
8) $\delta\text{-}open$ set[16] if for each $x \in A$, there exists an open set G such that $x \in G \subset Int(cl(G)) \subset A$.
9) $\theta\text{-}semi\text{-}open$ set if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset Cl(G) \subset$ A.

Definition 2.3:
1) The intersection of all semi-closed sets containing A is called the $semi\text{-}closure$[4] $of\ A$ denoted by $sCl(A)$.
2) The intersection of all α -closed sets containing A is called $\alpha\text{-}closure$[13] $of\ A$ denoted by $\alpha Cl(A)$.
3) The intersection of all b-closed sets containing A is called the $b\text{-}closure$[3] $of\ A$ denoted by $bCl(A)$.

Definition 2.4: The family of all open sets, semi-open sets, α-open sets, pre-open sets, semi-θ-open sets, θ-open sets, δ-open sets, regular-open sets, semi-closed sets and regular closed sets are denoted by $O(X), SO(X), \alpha O(X), PO(X), S\theta O(X), \theta O(X), \delta O(X), RO(X), SC(X)$ and $RC(X)$ respectively.

Definition 2.5: A topological space (X, τ) is said to be:
(i) $T_1\ space$ if to each pair of distinct points x, y of X there exist a pair of open sets, one containing x but not y and other containing y but not x, as well as is T_1 if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed [17].
(ii) $T_2\ space$ if to each pair of distinct points x, y of X there exist a pair of disjoint open sets, one containing x other containing y, as well as is T_2 if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed.
(iii) $Locally\ indiscrete$[6], if every open subset of X is closed.
(iv) S**- $normal$[2] if and only if for every semi-closed set F and semi-open set G containing F, there exists an open set H such that $F \subset H \subset Cl(H) \subset G$.
(v) $Regular$ if for each $x \in X$ and for each open set A containing x, there exists an open set G containing x such that $x \in G \subset cl(G) \subset A$.

Definition 2.6: A space X is called $Extremally\ disconnected$[15], if closure of every open set is open.

3. αc-OPEN SETS:

In this section we introduce a new class of open sets called $\alpha c\text{-}open\ sets$ which lie between the class of θ-open sets and the class of α-open sets.

Definition 3.1: A subset A of a topological space X is called $\boldsymbol{\alpha c\text{-}open\ set}$ if for each $x \in A \in \alpha O(X)$, there exists a closed set F, such that $x \in F \subset A$.

The following Theorem gives a characterization for αc-open sets.

Theorem 3.1.1: A subset A of a space X is αc-open set if and only if A is α-open set and it is the union of closed sets. That is $A = \cup F_x$ where A is α-open set and F_x is closed sets for each x.

Proof: Let A be αc-open set. Then by definition (3.1), A is a α-open set.

Since for each $x \in A \in \alpha O(X)$, there exist a closed set F_x such that $x \in F_x \subset A$, we have $A = \cup F_x$.

Conversely, let A be a α-open set and $A = \cup F_x$. For each $x \in A$, there exists x such that $x \in F_x \subset A$.

Hence A is a αc-open set.

Corollary 3.1.1: Every θ-open set of a space X is αc-open.

Proof: Let A be a θ-open set in X. Then for each $x \in A$, there exists an open set G such that $x \in G \subset Cl(G) \subset A$.

So $\cup \{x\} \subset \cup G \subset \cup Cl(G) \subset A$, implies $A \subset \cup G \subset \cup Cl(G) \subset A$. Therefore $A = \cup G$ and $A = \cup Cl(G)$.

Since G is open and arbitrary union of open set is open, A is open. Since open implies α-open, by Theorem (3.1.1), A is αc-open.

Remark 3.1.1:
1) For any α-open set A to be αc-open set, it is necessary that A must be the union of closed sets.

For example, let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, the Closed sets are $\{\emptyset, X, \{c\}, \{b,c\}, \{a,c\}\}$.

Then $\alpha O(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\alpha cO(X) = \{\emptyset, X\}$.

Here $\{a\}$ is α-open, but it is not a αc-open set. Since $\{a\}$ is not a union of closed sets in X.

2) $S\theta O(X)$ need not be an $\alpha c0(X)$.

Let $X=\{a,b,c\}$ $\tau = \{X, \emptyset, \{a,b\}, \{a\}, \{b\}\}$, the Closed sets are $\{X, \emptyset, \{c\}, \{b,c\}, \{a,c\}\}$. Then $S\theta O(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,c\}\}$, and $\alpha cO(X) = \{X, \emptyset\}$. Here $\{a\}$ is $S\theta O(X)$ but not $\alpha cO(X)$.

Theorem 3.1.2: Let (X,τ) be a topological space and $\{A_j : j \in \Delta\}$ be a collection of αc-open sets in X. Then $\cup \{A_j : j \in \Delta\}$ is αc-open.

Proof: By definition (3.1), for each A_j is α-open set, $j \in \Delta$. Since union of α-open sets is α-open, $\cup \{A_j : j \in \Delta\}$ is an α-open. Let $x \in \cup \{A_j : j \in \Delta\}$, then there exists $j \in \Delta$ such that $x \in A_j$. Since A_j is an αc-open set, there exists a closed set F such that $x \in F \subset A_j \subset \cup \{A_j : j \in \Delta\}$. Hence $\cup \{A_j : j \in \Delta\}$ is an αc-open set in X.

The following Corollary gives another characterization of αc-open sets.

Corollary 3.1.2: The set A is αc-open in the space (X,τ) if and only if for each $x \in A$, there exists a αc-open set B such that $x \in B \subset A$.

Proof: Let A is αc-open. Then for each $x \in A$, choose $B = A$ so that $x \in B \subset A$, and B is αc-open.

Conversely, Assume that for each $x \in A$, there exists a αc-open set B_x such that $x \in B_x \subset A$.

Thus $A = \cup B_x$ where $B_x \in \alpha cO(X)$. By Theorem (3.1.2), A is αc-open set.

Theorem 3.1.3: If the family of all α-open sets of a space X is a topology on X, then the family of all αc-open sets is also a topology on X.

Proof:

(i) Clearly $\emptyset, X \in \alpha cO(X)$.

(ii) By Theorem(3.1.2), the union of all αc-open sets is αc-open.

(iii) We have to show that finite intersection of αc-open set is αc-open set.

Let $\{A_j : j = 1,2, \dots n\}$ be a collection of αc-open sets. Then by definition (3.1), $A_1, A_2, \dots\dots A_n$ are α-open sets.
Let $x \in \cap \{A_j : j = 1,2 \dots n\}$, then $x \in A_j$ for each j, and there exists a closed set F_j such that $x \in F_j \subset A_j$.
Then $x \in \cap F_j \subset \cap \{A_j : j=1,2\dots n\}$. Thus $\cap \{A_j : j = 1,2 \dots n\}$ is αc-open.
Hence the family of all αc-open sets is also a topology on X.

Lemma 3.1.4: For any spaces X and Y. If $A \subseteq X$ and $B \subseteq Y$ then,

(i) $\alpha Int_{X\times Y}(A \times B) = \alpha Int_X(A) \times \alpha Int_Y(B)$ [9].

(ii) $Cl_{X\times Y}(A \times B) = Cl_X(A) \times Cl_Y(B)$.

The following Theorem shows that the property of being αc-open is preserved by the product of two topological spaces.

Theorem 3.1.5: Let X and Y be two topological spaces and $X \times Y$ be the product topology. If $A \in \alpha cO(X)$ and $B \in \alpha cO(Y)$. Then $A \times B \in \alpha cO(X \times Y)$.

Proof: Let $(x, y) \in A \times B$, then $x \in A$ and $y \in B$. Since $A \in \alpha cO(X)$ and $B \in \alpha cO(Y)$, then $A \in \alpha O(X)$ and $B \in \alpha O(Y)$. Also there exists closed sets F and E in X and Y respectively, such that $x \in F \subseteq A$ and $y \in E \subseteq B$.

Therefore, $(x, y) \in F \times E \subseteq A \times B$. Since $A \in \alpha O(X)$ and $B \in \alpha O(Y)$. Then by Lemma 3.1.4(i),
$A \times B = \alpha Int_X(A) \times \alpha Int_Y(B) = \alpha Int_{X\times Y}(A \times B)$. Hence $A \times B \in \alpha O(X \times Y)$. Since F is closed in X and E is closed in Y.

Then by Lemma3.1.4(ii), $F \times E = Cl_X(F) \times Cl_Y(E) = Cl_{X\times Y}(F \times E)$. Hence $F \times E$ is closed in $X \times Y$.

Therefore, $A \times B \in \alpha cO(X \times Y)$.

Theorem 3.1.6: If the space X is a T_1-space (or) a T_2-space, then the family $\alpha c\ O(X)$= $\alpha O(X)$.

Proof: Let A be α- open set of X.

Since X is a T_1 space (or) a T_2 space, for each $x \in A \subset X$, $\{x\}$ is closed.

Therefore $x \in \{x\} \subset A$, $A \in \alpha cO(X)$. Hence $\alpha O(X) \subset \alpha cO(X)$.

By definition of αc-open sets, $\alpha cO(X) \subset \alpha O(X)$. Hence $\alpha O(X) = \alpha cO(X)$.

Theorem 3.1.7: [18] If X is S^{**}-normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$.

Theorem 3.1.8: If (X,τ) is an S**-normal space and if $A \in S\theta O(X)$, then $A \in \alpha cO(X)$.

Proof: Let A be an semi-θ-open set of X. If $A = \emptyset$, then $A \in \alpha cO(X)$.

Suppose $A \neq \emptyset$. Since the space A is S**-normal, by Theorem (3.1.7) $S\theta O(X) = \theta O(X)$.

Then A is θ-open of X. By Corollary (3.1.1), $A \in \alpha cO(X)$.

Remark 3.1.9: Every open set need not be αc-open. It is evident from the following example.

For example, let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,b,c\},\{b,c,d\}\}$ and

The closed sets are $\{\emptyset, X, \{a\},\{d\},\{a,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\},\{b,d\}\}$.

Then $\alpha cO(X) = \{\{a\},\{a,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\},\emptyset, X\}$. Here $\{b\}$ is open set but not αc-open.

The following Theorem gives conditions under which an open set is also αc-open.

Theorem 3.1.9: Every open set is αc-open set in X, if one of the following holds.
- (i) (X,τ) is Locally indiscrete.
- (ii) X is Regular.

Proof:
$(\boldsymbol{i})$ Let A be an open set of X. If A=$\emptyset$, then $A \in \alpha cO(X)$.

Suppose $A \neq \emptyset$, we know that $\tau \subset \alpha O(X)$. Therefore $A \in \alpha O(X)$.

If X is Locally indiscerte, then every open subset is closed.

Since A is open, we have A is closed. Therefore, $x \in A \subset A$ implies $A \in \alpha cO(X)$.

Hence every open is αc-open of X.

$(\boldsymbol{ii})$ Let A be an open set of X. If A=$\emptyset$, then $A \in \alpha cO(X)$. Suppose $A \neq \emptyset$, we know that $\tau \subset \alpha O(X)$. Therefore $A \in \alpha O(X)$. If X is Regular and since $A \in \tau$, we have for each $x \in A$, there exists an open set G containing x such that $x \in G \subset Cl(G) \subset A$ implies $x \in Cl(G) \subset A$. Thus $A \in \alpha cO(X)$.

Theorem 3.1.10: [18] A space X is called Extremally disconnected if and only if $\delta O(X) = \theta SO(X)$.

Theorem 3.1.11: Let (X,τ) be an Extremally disconnected and S**-normal space. Then
- (i) $\delta O(X) \subset \alpha cO(X)$.
- (ii) $RO(X) \subset \alpha cO(X)$.

Proof:
$(\boldsymbol{i})$ Let A be an δ-open set of X. If A=$\emptyset$, then $\in \alpha cO(X)$. Suppose $A \neq \emptyset$, since X is Extremally disconnected , by

Theorem (3.1.10), we have $\delta O(X)$=$\theta SO(X)$. Then $A \in \theta SO(X)$.

If X is S**-normal space, then by Theorem(3.1.7), $\theta SO(X)$=$\theta O(X)$. Hence $A \in \theta O(X)$.

By Corollary (3.1.1), A is αc-open of X. Hence $\delta O(X) \subset \alpha c O(X)$.

$(\boldsymbol{ii})$ Let A be an Regular open set of X. If A=$\emptyset$, then $\in \alpha c O(X)$. Suppose $A \neq \emptyset$, Since A be Regular open implies $A = Int(Cl(A))$, then for each $x \in A$, there exist a open set A such that $x \in A \subset Int(Cl(A)) \subset A$.
Then $A \in \delta O(X)$. By(i), A be an $\alpha c O(X)$.

Remark 3.1.11:
1) Every δ-open set need not be αc-open. It is evident from the following example.

Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$, the closed set are $\{X, \emptyset, \{b, c\}, \{a\}\}$.

Then $\delta O(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$ and $\alpha c O(X) = \{X, \emptyset, \{a\}, \{b, c\}\}$.

Here $\{b\}$ is $\delta O(X)$ but not $\alpha c O(X)$.

2) Every Regular-open set need not be αc-open. It is evident from the following example.
In Remark (3.1.9), we have Regular-open sets= $\{\{a\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \emptyset, X\}$ and $\alpha c O(X) = \{\{a\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \emptyset, X\}$. Here $\{d\}$ is $RO(X)$ but not $\alpha c O(X)$.

3.2 $\boldsymbol{\alpha c}$-Closed set:

Definition 3.2: A subset B of a space X is called αc-closed set if $X \backslash B$ is αc-open set. The family of all αc-closed subsets of a topological space (X,τ) is denoted by $\alpha c C(X)$.

The following Theorem gives a characterization of αc-closed sets.

Theorem 3.2.1: A subset B of a space X is αc-closed if and only if B is α-closed set and it is an intersection of open sets.

Proof: Let B be an αc-closed set in X.

Then $X \backslash B$ is αc-open set. Thus, $X \backslash B$ is α-open set and for all $y \in X \backslash B$, there exists a closed set F_y such that $y \in F_y \subset X \backslash B$. Then B is α-closed and $\cup \{y\} \subset \cup F_y \subset X \backslash B$, $X \backslash B \subset \cup F_y \subset X \backslash B$, $X \backslash B = \cup F_y$.

Then B= $X \backslash (\cup F_y)$ implies B=$\cap \left(X \backslash F_y\right)$, $X \backslash F_y$-is open set. B is an intersection of open sets. Hence B is α-closed and it is an intersection of open sets.

Conversely, Let B be α-closed set and intersection of open sets. B is α-closed implies $X \backslash B$ is α-open and B=$\cap F_i$ where F_i's are open set. $X \backslash B$= $X \backslash (\cap F_i)$=$\cup(X \backslash F_i)$, where $X \backslash F_i$ -is closed set.

Thus for all $y \in X \backslash B$, there exists some i such that $y \in X \backslash F_i$, where $X \backslash F_i$- is closed set.

i.e., $y \in X \backslash F_i \subset X \backslash$B implies $X \backslash B$ *is* αc-open. Hence B is αc-closed.

Corollary 3.2.1: For any subset B of a space, if $B \in \theta C(X)$, then $B \in \alpha c C(X)$.

Proof: Let B be an θ-closed set of X. Then $X \backslash B$-is an θ-open set.

By Corollary (3.1.1), we have $X \backslash B$ is an αc-open set. Thus B is αc-closed set. Hence $\theta C(X) \subset \alpha c C(X)$.

Theorem 3.2.2: Let $\{B_j : j \in \Delta\}$ be a collection of αc-closed sets in a topological space X. Then
$\cap \{B_j : j \in \Delta\}$ is αc-closed set.

Proof: Let B_j's be αc-closed set. Then X\\$B_j$ is αc-open set. By Theorem (3.1.2), $\cup \{X \backslash B_j : j \in \Delta\}$ is an αc-open set. Then $\{X \backslash (\cap B_j) : j \in \Delta\}$ is an αc-open set. Hence $\{\cap B_j : j \in \Delta\}$ is αc-closed set.

Theorem 3.2.3: If the space X is a T_1-space (or) T_2-space, then the family $\alpha c C(X) = \alpha C(X)$.

Proof: Let B be an αc-closed subset of X.

Then $X \backslash B$ *is* αc open. Since $\alpha c\ O(X)$= $\alpha O(X)$, we have $X \backslash B$ is α open. Hence B is α closed.

Remark 3.2.4: Every closed set need not be αc-closed. It is evident from the following example.

Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{b, c, d\}\}$.

The closed sets are $\{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{b, d\}\}$.

Then $\alpha cC(X) = \big\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, \emptyset, X\big\}$. Here $\{d\}$ is closed set but not αc-closed.

The following Theorem gives conditions under which an closed set is also αc-open.

Theorem 3.2.4: Every closed set is αc-closed in X, if one of the following condition holds:
  (i) X is Locally indiscrete.
  (ii) X is Regular.

Proof:
$(\boldsymbol{i})$ Let B be a closed subset of X. If A=$\emptyset$, then $A \in \alpha cC(X)$.

Suppose $A \neq \emptyset$, then $X\backslash B$ is open set. Since every open set is α-open, $X\backslash B$ is α-open of X.

Since X is Locally indiscrete, $X\backslash B$ is closed. Then for each $x \in X\backslash B \subset X\backslash B$, $X\backslash B \in \alpha cO(X)$. Hence $B \in \alpha cC(X)$.

$(\boldsymbol{ii})$ Let B be closed subset of X. Then $X\backslash B$- is open.

If B=$\emptyset$, then $B \in \alpha cC(X)$. Suppose $B \neq \emptyset$, then $X\backslash B \in \alpha O(X)$.

If X is Regular, then for each open set $X\backslash B$ containing x, there exists an open set G such that, $x \in G \subset Cl(G) \subset X\backslash B$, $x \in Cl(G) \subset X\backslash B$.

Therefore $X\backslash B \in \alpha cO(X)$ implies $B \in \alpha cC(X)$. Hence $C(X) \subset \alpha cC(X)$.

Remark 3.2.5: Every δ-closed set need not be αc-closed. It is evident from the following example.

Let $X = \{a, b, c\}$, $\tau = \big\{X, \emptyset, \{a\}, \{b, c\}\big\}$, the closed sets are $\big\{X, \emptyset, \{b, c\}, \{a\}\big\}$. Then δ-closed sets= $\big\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\big\}$ and $\alpha cC(X) = \big\{X, \emptyset, \{a\}, \{b, c\}\big\}$. Here $\{b\}$ is $\delta C(X)$ but not $\alpha cC(X)$.

The following Theorem gives conditions under which an δ-closed set is also αc-closed.

Theorem 3.2.5: Let (X, τ) be an Extremally disconnected and S^{**}-normal space. If $B \in \delta C(X)$, then $B \in \alpha cC(X)$.

Proof: Let B be an δ-closed subset of X. Then $X\backslash B$ is δ-open set. If B=$\emptyset$, then $A \in \alpha cC(X)$. $Suppose\ A \neq \emptyset$, let $X\backslash B \in \delta O(X)$. we have by 3.1.11$(i)$ $X\backslash B \in \alpha cO(X)$. Hence $B \in \alpha cC(X)$.

Remark 3.2.6: Every $S\theta C(X)$ need not be an αcC(X).

Let X=$\{a, b, c\}$ τ=$\{X$,$\emptyset$, $\{a, b\}$, $\{a\}$, $\{b\}\}$, the Closed sets are $\{X$, $\emptyset$,$\{c\}$,$\{b$,$c\}$,$\{a$,$c\}\}$
Semi-θ-closed=$\{\emptyset$, X,$\{a\}$,$\{b\}$,$\{c\}$,$\{b$,$c\}$,$\{a, c\}\}$, and $\alpha cC(X)$=$\{$ X, $\emptyset$ $\}$. Here $\{a\}$ is $S\theta C(X)$ but not $\alpha cC(X)$.

Theorem 3.2.6: Let $(X$,$\tau)$ be an S^{**}- normal space. If $B \in S\theta C(X)\ then\ B \in \alpha cC(X)$.

Proof: Let B be an semi-θ-closed subset of X, then $X\backslash B$-is semi-θ-open of X.

If B=$\emptyset$, then $B \in \alpha cC(X)$. Suppose $B \neq \emptyset$, as the space X is S^{**}- normal, By (3.1.7) $S\theta O(X)$= $\theta O(X)$, $X\backslash B \in \theta O(X)$. By Corollary(3.1.1), $X\backslash B$-is αc-open set. Hence B is αc-closed set of X.

The following diagram shows that the relations among $\alpha cO(X)$, $\alpha O(X)$, $RO(X)$, $\delta O(X)$, τ, and $\theta O(X)$.

$$\begin{tikzcd}
\text{Regular-open set} \arrow[r] & \delta\text{ -open set} \arrow[r] & \text{Open-set} \arrow[r] & \alpha\text{-open set} \\
& \theta\text{-open set} \arrow[u] \arrow[rr] & & \boldsymbol{\alpha c}\text{-open set} \arrow[u]
\end{tikzcd}$$

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Source of support: Nil, Conflict of interest: None Declared