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SPLIT DOMINATION IN DIRECTED GRAPHS

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ABSTRACT

Dominating sets in digraphs is analysed in the paper entitled "Total and connected domination in digraphs" by S.Arumugam, K.Jacob and Lutzvolkmann. Let D=(V, A) be a digraph. A subset S of V is called a dominating set of D if for every vertex $v \in$ V-S there exists a vertex $u \in S$ such that $(u, v) \in A$. Split dominating number introduced by Kulli and Janakiram, "Split domination number of a graph". A dominating set S of V is called Split dominating set of D if the induced subdigraph <V-S> is disconnected. In this paper we analyse the domination parameters corresponding to split domination in digraphs and obtain several results on these parameters.

Key words: *Dominating sets in digraphs, Split dominating sets in digraphs*

AMS Subject Classification: *O5C69.*

1.1 INTRODUCTION

Throughout this paper D=(V, A) is a finite directed graph with neither loops nor multiple arcs(but pairs of arcs are allowed) and G=(V,E) is a undirected graph with neither loops nor multiple edges. For basic terminology on graphs and digraphs, we refer to Chartrand and Lesniak [1].

Let G=(V, E) be a graph. A subset S of V is called dominating set of G if every vertex in V-S is adjacent to atleast one vertex in S. The minimum cardinality of dominating set of G is called domination number of G and is denoted by $\gamma(G)$.

Kulli V.R. has introduced the concept of split dominating set. A dominating set D $\subseteq$ V of a graph G is split dominating set if the induced subgraph<V-D> is disconnected. The split domination number $\gamma_s(G)$ is the minimum cardinality of a split dominating set.

Let D=(V,A) be a digraph. A subset S of V is called a dominating set of D if for every vertex $v \in$ V-S there exists a vertex $u \in S$ such that $(u, v) \in A$. The domination number $\gamma(D)$ is the minimum cardinality of dominating set D.

Let D=(V, A) be a digraph. For any vertex $u \in V$, the sets $O(u)=\{v/(u, v) \in A\}$ and $I(u)=\{v/(v, u) \in A\}$ are called outset and inset of u. The indegree and outdegree of u are defined by id(u)=$|I(u)|$ and od(u)=$|O(u)|$. The minimum indegree, the minimum outdegree, the maximum indegree and maximum outdegree of D is denoted by $\delta^-, \delta^+, \Delta^-,$ *and* Δ^+ respectively.

An out- domination set of digraph D is a set S^+ of vertices of D such that every vertex of V-S^+ is adjacent from some vertex of S. The minimum cardinality of out-domination set for D is the out-domination number $\gamma^+(D)$.

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The in-domintion number $\gamma^{-}(D)$ is defined as expected.

Although domination and other related concepts have extensively studied for undirected graphs, the respective analogue on digraphs have not received much attention.

The purpose of this paper is to introduce the concept of split domination in directed graphs

γ - set is the set of all vertices in dominating set with # $\gamma(D)$

γ^{+} - set is the set of all vertices in out dominating set with # $\gamma^{+}(D)$

γ^{-} - set is the set of all vertices in in-dominating set with # $\gamma^{-}(D)$

γ_s - set is set of all vertices in split dominating set with # $\gamma_s(D)$

γ_s^{+} - set is set of all vertices in split out dominating set with # $\gamma_s^{+}(D)$

γ_s^{-} - set is set of all vertices in split in dominating set with # $\gamma_s^{-}(D)$

1.1.1 Definition: A dominating set S of V is called Split dominating set of D if the induced sub digraph <V-S> is disconnected. The split domination number $\gamma_s(D)$ is the minimum cardinality of a split dominating set of directed graph.

1.2 RESULTS AND BOUNDS

Here we observed the exact values of $\gamma_s^{+}(D)$ *and* $\gamma^{+}(D)$ for some standard digraphs and proved some standard results.

1.2.1 Observation and Results:

Here we observed the exact values of $\gamma^{+}(D)$ and $\gamma_s^{+}(D)$

(i) For path p_n for n$\geq$ 5

$$\gamma_s^{+}(D) \leq n - \Delta^{+}$$

(ii) For any cycle C_n

$$\gamma_s^{+}(D) = \frac{n}{2} \quad for\, n = 4$$

$$\gamma_s^{+}(D) \leq n - \Delta^{+} \quad for\, n \geq 5$$

(iii) For complete digraph

$$\gamma_s^{+}(D) = 0$$

Theorem 1.3.1: For any digraph D $\gamma^{+}(D) \leq \gamma_s^{+}(D)$

Proof: Since every split dominating set of D is an dominating set of D, we have

$$\gamma^{+}(D) \leq \gamma_s^{+}(D)$$

Definition: A split dominating set S of digraph D is said to be minimal split dominating set if no proper subset S of S is a split dominating set of D with $|S'| < |S|$.

Theorem 1.3.2: A split dominating set S of D is minimal for each vertex v$\in$S, one of the following two conditions holds:

(i) v is an isolated vertex in<S>

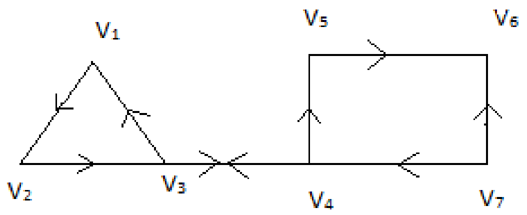
(ii) $<(V-S)\cup\{v\}>$ is connected.

Proof: Suppose S is minimal and there exists a vertex v$\in$S such that v does not hold any of above conditions. Then by condition (i) S_1=S-{v} is a dominating set of D. Also by (ii) <V-S> is disconnected. This implies that S_1 is split dominating set of D, a contradiction.

Theorem 1.3.3: Let D be a strongly connected block graph and each block is strong then

$$\gamma_s^+(D) \le \frac{n}{2}$$

Proof:



Let

$$S_s^+(D) = \{v_1, v_4, v_6\} \qquad V - S_s^+ = \{v_2, v_3, v_5, v_7\}$$

$$\textit{Therefore} \ <V - S_s^+> \textit{is di-subgraph of } D \textit{ induced by} <V - S_s^+> \textit{is disconnected}.$$

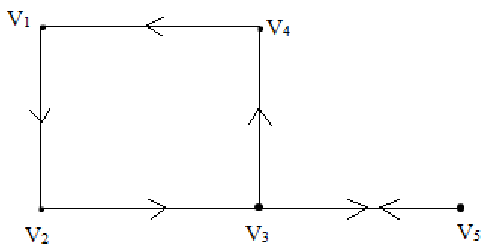
Hence S is a split dominating set of D with split domination number $\gamma_s^+(D) = 3$

Therefore $\gamma_s^+(D) \le \frac{n}{2}$

Theorem 1.3.4: Let D be a strongly connected digraph of cycle with an end vertex of deg2 (ie) od(v)=id(v)=1 then

$$\gamma^+(D) = \gamma_s^+(D) \le \frac{n+1}{2}$$

Proof:



$$S_s^+(D) = \{v_1, v_3\} \qquad V - S_s^+ = \{v_2, v_4, v_5\}$$

$$\textit{Therefore} \ <V - S_s^+> \textit{is di-subgraph of } D \textit{ induced by} <V - S_s^+> \textit{is disconnected}.$$

Hence S is a split dominating set of D with split domination number $\gamma_s^+(D) = 2$

Therefore $\gamma_s^+(D) \le \frac{n+1}{2}$

Theorem 1.3.5: Let T be a directed tree such that any two adjacent cut vertices u and v with atleast one of u and v is adjacent to an end vertex then

$$\gamma^+(T) = \gamma_s^+(T) \le n - \Delta^+$$

Proof: Let S be a γ^+ - set of T, then we consider the following two cases.

Case-(i): Suppose that atleast one of u, v $\in$ S, then <V-S> is disconnected with atleast one vertex.

Hence S is a γ_s^+ - set of T. Thus the theorem is true.

Case-(ii): Suppose u, v $\in$ V-S. Since there exists an end vertex w adjacent to either u or v say u, it implies that w $\in$ S. Thus it follows that S_1=S-{w} $\cup$ {u} is a γ^+-set of T.

Hence by case (i) the theorem is true.

Theorem 1.3.6: Let D be digraph which is not a cycle with atleast 5 vertices. Let H be a connected spanning subgraph of D then

$$\gamma_s^{+}(D) \le \gamma_s^{+}(H)$$

Proof: Since D is connected then any spanning tree T of D is minimally connected subdigraph of D such that

$$\gamma_s^{+}(D) \le \gamma_s^{+}(T) \le \gamma_s^{+}(H)$$

Hence the proof.

Theorem 1.3.7: If D is a digraph has one cut vertex v and atleast two blocks H_1 and H_2 with v adjacent to all vertices in H_1 and H_2 then v is in everysplit dominating set of D .

Proof: Let S be a split dominating set of D.

Suppose $v \in V-S$ then each of H_1 and H_2 contributes atleast one vertex to S say u and w respectively. This implies that $S' = S - \{u, w\} \cup \{v\}$ is a split dominating set of D.
Which is contradiction .Hence v is in every split dominating set of D.

Theorem 1.3.8: Let v be a cut vertex of D .If there is a block H in D such that v is the only cut vertex of H and v is adjacent to all vertices of H, then there is a split dominating set of D containing v.

Proof: If there exists two blocks in D satisfy the given condition then by **theorem 1.3.7** v is in every split dominating set of D and hence the result.

Suppose there exists only one block H in D satisfying given condition. Let S be a split dominating set of D. Suppose $v \in V-S$ then for some vertex $u \in H$, $\{u\} \subset D$.

This proves $S' = S - \{u\} \bigcup \{v\}$ is a split dominating set of D.

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