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# THE DOMINATING GRAPH $DG^{abc}(G)$ OF A GRAPH G

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***ABSTRACT***

*The dominating graph $DG^{abc}(G)$ of a graph G is obtained from G with vertex set $V' = V(G) \cup S$, where $V = V(G)$ and S is the set of all minimal dominating sets of G. Then two elements in $V'$ are said to satisfy property 'a' if $u, v \in V$ and are adjacent in G. Two elements in $V'$ are said to satisfy property 'b' if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in $V'$ are said to satisfy property 'c' if $u \in V(G)$, $v = D \in S$ such that $u \in D$. Two elements in $V'$ are said to satisfy property 'd' if $u, v \in V(G)$ and there exists $D \in S$ such that $u, v \in D$. A graph having vertex set $V'$ and any two elements in $V'$ are adjacent if they satisfy any one of the property a, b, c is denoted by $DG^{abc}(G)$. In this paper, we obtain some basic properties of $DG^{abc}(G)$. Also, we establish the characterization of $DG^{abc}(G)$.*

**Keywords:** *dominating set, minimal dominating set, minimal dominating graph.*

***2010Mathematical Classification Code:*** *05C12, 05C69.*

## INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple. For graph theoretic terminology refer to Harary[3], Buckley and Harary[1]. For a graph, let V(G) and E(G) be its vertex and edge set respectively. A graph with p vertices and q edges is called a (p, q) graph. The *degree of a vertex* v in a graph G is the number of edges of G incident with v and it is denoted by deg(v).

The length of any shortest path between any two vertices u and v of a connected graph G is called the *distance between u and v* and it is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be $\infty$. For a connected graph G, The *eccentricity* e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$.The *radius* rad(G) is the minimum eccentricity of the vertices, whereas the *diameter* diam(G) is the maximum eccentricity. If these two are equal in a graph, that graph is called *self-centered* graph with radius r and is called an *r self-centered graph*. For any connected graph G, $rad(G) \le diam(G) \le 2rad(G)$. v is a central vertex if e(v) = r(G). The *center* C(G) is the set of all central vertices. For a vertex v, each vertex at a distance e(v) from v is an *eccentric vertex of v***.**

The *girth g(G)* of the graph G, is the length of the shortest cycle (if any) in G.

A graph G is *connected* if every two of its vertices are connected, otherwise G is *disconnected*. The *vertex connectivity* or simply *connectivity $\kappa(G)$* of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. The *edge connectivity $\lambda(G)$* of a graph G is the minimum number of edges whose removal from G results in a disconnected or trivial graph. A set S of vertices of G is *independent* if no two vertices in S are adjacent. The *independence number $\beta_o(G)$* of G is the maximum cardinality of an independent set.

The concept of distance in graph plays a dominant role in the study of structural properties of graphs in various angles using related concept of eccentricity of vertices in graphs.

The concept of domination in graphs was introduced by Ore [8]. The concept of domination in graphs originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. For details on $\gamma(G)$, refer to [2, 9].

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A set $D \subseteq V$ is said to be a *dominating set* in G, if every vertex in V–D is adjacent to some vertex in D. The cardinality of minimum dominating set is called the *domination number* and is denoted by $\gamma(G)$. A dominating set D is called a *minimal dominating* set if no proper subset of D is a dominating set. The *upper domination number* $\Gamma(G)$ of a graph G is the maximum cardinality of a minimal dominating set of G. *Domatic number d(G)* of a graph is the largest order of a partition of V(G) into dominating sets of G.

In [5, 6, 7], Kulli, Janakiram and Niranjan introduced the following concepts in the field of domination theory.

The *minimal dominating graph MD(G)*[5] of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices of G. The *vertex minimal dominating graph* $M_vD(G)$[6] of a graph G with $V(M_vD(G)) = V' = V \cup S$, where S is the collection of all minimal dominating sets of G with two vertices $u, v \in V'$ are adjacent if either they are adjacent in G or v = D is a minimal dominating set of G containing u.

The *dominating graph D(G)*[7] of a graph G= (V, E) is a graph with $V(D(G)) = V \cup S$, where S is the set of all minimal dominating sets of G and with two vertices $u, v \in V(D(G))$ are adjacent if $u \in V$ and v = D is a minimal dominating set of G containing u.

In this paper, we define a new dominating graph $DG^{abc}(G)$ with property a, b and c. we establish some basic properties of $DG^{abc}(G)$. We characterize the graph G for which $DG^{abc}(G)$ is completely disconnected, complete, self- centered of radius 2, etc.

We need the following results to study the dominating graph $DG^{abc}(G)$ of a graph G.

**Theorem 1.1[3]:** A graph G is Eulerian if and only if every vertex of G is of even degree.

**Theorem: 1.2[3]** If for all vertices v of G, $\deg(v) \geq p/2$ where $p \geq 3$, then G is Hamiltonian.

## 2. THE DOMINATING GRAPH $DG^{abc}(G)$ OF A GRAPH G

We define a new class of intersection graphs in the field of domination theory as follows.

**Definition 2.1:** A graph having vertex set $V' = V(G) \cup S$, where S is the set of all minimal dominating sets of G. Then two elements in V′ are said to satisfy property 'a' if $u, v \in V$ and are adjacent in G. Two elements in V′ are said to satisfy property 'b' if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in V′ are said to satisfy property 'c' if $u \in V(G)$, $v = D \in S$ such that $u \in D$. A graph having vertex set $V' = V(G) \cup S$, where S is the set of all minimal dominating sets of G and any two elements in V′ are adjacent if and only if they satisfy any one of the property a, b, c is denoted by $DG^{abc}(G)$.

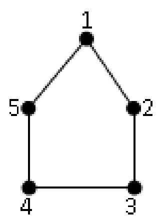
**Remark 2.1:**

(i) G is an induced sub graph of $DG^{abc}(G)$.
(ii) $M_vD(G)$ is an induced sub graph of $DG^{abc}(G)$.
(iii) MD(G) is an induced sub graph of $DG^{abc}(G)$.
(iv) Number of vertices in $DG^{abc}(G)$ = p + Number of minimal dominating sets of G.
(v) Number of edges $\geq$ q.
(vi) $\text{Deg}_{DG^{abc}(G)} v_j = \deg_G v_j + S_j$, $1 \leq j \leq p$, where $S_j$ is the number of minimal dominating set containing $v_j$.
(vii) $\text{Deg } D_i \leq |D_i| + \dfrac{|S|(|S|-1)}{2}$, $1 \leq i \leq n$, where S is the set of all minimal dominating sets of G.

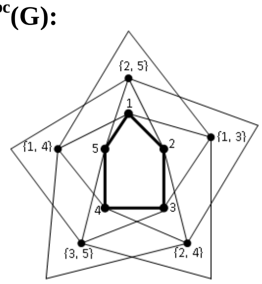
Here, the elements of V(G) are called as point vertices and the elements of S are known as set vertices.

**Example:**

**G:**




**$DG^{abc}(G)$:**




{1, 3}, {1, 4}, {2, 4}, {2, 5} and {3, 5} are minimal dominating sets of G.

**Theorem 2.1:** If $G = K_p$, Then $DG^{abc}(G)$ is $K_p \circ K_1$.

**Proof:** When $G = K_p$. Each vertex is a minimal dominating set of G. By the definition of $DG^{abc}(G)$, $DG^{abc}(G)$ is $K_p \circ K_1$.

**Theorem 2.2:** If $G = \overline{K_p}$, then $DG^{abc}(G)$ is $K_{1,\,p}$.

**Proof:** When $G = \overline{K_p}$. The whole vertex set is a minimal dominating set of G. By the definition of $DG^{abc}(G)$, $DG^{abc}(G)$ is $K_{1,\,p}$.

**Theorem 2.3:** For any graph G, $DG^{abc}(G)$ is connected.

**Proof:** Since for each vertex $v\in V(G)$ there exists a minimal dominating set containing v, every vertex in $DG^{abc}(G)$ is not an isolated vertex. Suppose $DG^{abc}(G)$ is disconnected. Let $G_1$ and $G_2$ be two components of $DG^{abc}(G)$. Then there exists two non-adjacent vertices $u,v\in V(G)$ such that $u\in V(G_1)$ and $v\in V(G_2)$. This implies that there is no minimal dominating set in G containing u and v, which is a contradiction, since there exists a maximal independent set containing u and v and every maximal independent set is a minimal dominating set. Hence $DG^{abc}(G)$ is connected.

We establish a necessary and sufficient condition on G for which $DG^{abc}(G)$ is complete.

**Theorem 2.4:** $DG^{abc}(G)$ is complete if and only if $G = K_1$.

**Proof:** Suppose $DG^{abc}(G)$ is complete. Then G is complete and has exactly one minimal dominating set. This implies that $G = K_1$.

Conversely, $G = K_1$. By the definition, we get $DG^{abc}(G) = K_2$, which is complete.

**Theorem 2.5:** For any graph G, $p + d(G) \le p' \le \frac{p(p+1)}{2}$**,** where d(G) is the domatic number of G and $p'$ denotes the number of vertices of $DG^{abc}(G)$. Further, the lower bound is attained if and only if $G = K_p$ or $\overline{K_p}$ or $K_{1,\,p-1}$ and the upper bound is attained if and only if G is (p–2) regular graph.

**Proof:** The lower bound follows from every graph has at least d(G) number of minimal dominating sets of G and the upper bound follows from every vertex is in at most (p–1) minimal dominating set of G.

Assume $p' = p+d(G)$. Suppose $uv\in E(G)$, then $u\in D_1$ and $v\in D_2$. So if $D_1$ and $D_2$ are disjoint minimal dominating sets and $|D_1|, |D_2| > 1$, elements of $D_1$ are dominated by $D_2$ and vice versa in G. Hence we can form a minimal dominating set with vertices from $D_1$ and $D_2$. But $p' = p+d(G)$ implies minimal dominating sets are all disjoint. This implies either $|D_1| = 1 .... |D_p| = 1$or $|D_1| = 1$ and $|D_2| = p-1$ or $|D_1| = p$. Thus $G = K_p$ or $\overline{K_p}$ or $K_{1,\,p-1}$.

Conversely, Suppose $G = K_p$ or $\overline{K_p}$ or $K_{1,\,p-1.}$ Then $d(K_p) = p$ or $d(\overline{K_p}) = 1$ or $d(K_{1,\,p-1}) = 2$. Thus, it follows that the order of $DG^{abc}(G)$ is $p+d(G)$.

Suppose the upper bound is attained. Then $p' = p(p+1)/2$.

Thus, number of minimal dominating sets = $\frac{p(p+1)}{2} - p = \frac{p(p-1)}{2} = pC_2$. Therefore any two vertices of G form a minimal dominating set. This implies that each vertex is in exactly (p–1) minimal dominating sets and hence G is (p–2) regular graph.

Conversely, G is a (p–2) regular graph. $G = K_p$–1 factor, p is even. Thus we get $1+2+3+ ...+ (p-1) = \frac{p(p-1)}{2}$ minimal dominating sets. Thus, it follows that $p' = p + \frac{p(p-1)}{2} = \frac{p(2+p-1)}{2} = \frac{p(p+1)}{2}$.

**Theorem 2.6:** For any graph G, $p + q \leq q' \leq p(p-1) + \frac{m(m-2)}{2} + q$, where $|S| = m$ and $q'$ is the number of edges in $DG^{abc}(G)$ .

**Proof:** The lower bound follows from every vertex in G is in at least one minimal dominating set. The upper bound follows from (i) Every vertex is in at most (p–1) minimal dominating set of G. (ii) Corresponding to each minimal dominating set there exists at least one non-trivial disjoint minimal dominating set when G has no isolated vertices, since, if D is a minimal dominating set V−D is also a dominating set. The lower bound is sharp for $G = K_p$. Upper bound is attained when $G = K_p$–1factor.

**Corollary 2.1:** If G is a connected (p, q) graph, then $2p-1 \leq q'$.

**Proposition 2.1:** For any graph G, $\beta_o(DG^{abc}(G)) \geq \max\{\beta_o(G), d(G)\}$, where $\beta_o(G)$ is the independence number of G and d(G) is the domatic partition of G.

**Observation 2.1:** Let G be a connected graph and let v be a full degree vertex. Since the minimal dominating set w = D = {v} contains v, this implies that, in $DG^{abc}(G)$, D is adjacent to v only. Hence, deg(w) = 1. Therefore, $DG^{abc}(G)$ has a pendent vertex.

**Observation 2.2:** For any graph G, $\chi(G) \leq \chi(DG^{abc}(G)) \leq \chi(G) + 2$.

Next we study the connectivity and edge connectivity of $DG^{abc}(G)$.

**Theorem 2.7:** For any graph G,
$\kappa(DG^{abc}(G)) \leq \min\{\min\{\deg_{DG^{abc}(G)}(D_i), 1 \leq i \leq n\}, \min\{\deg_{DG^{abc}(G)}(v_j), 1 \leq j \leq p\}\}$

**Proof:** We consider the following cases.

**Case-(i):** Let u = D be the minimal dominating set of G and is of minimum degree among all the vertices of $DG^{abc}(G)$. Then by deleting the vertices adjacent to u, the resulting graph is disconnected.

**Case-(ii):** Let $v \in V(G)$ and is of minimum degree among all the vertices of $DG^{abc}(G)$. Then by deleting the vertices adjacent to v, the resulting graph is disconnected.

Hence, the result is proved.

**Theorem 2.8:** For any graph G,
$\lambda(DG^{abc}(G)) \leq \min\{\min\{\deg_{DG^{abc}(G)}(D_i), 1 \leq i \leq n\}, \min\{\deg_{DG^{abc}(G)}(v_j), 1 \leq j \leq p\}\}$

**Proof:** Proof is similar to theorem 2.7.

Next we shall find out radius and diameter of $DG^{abc}(G)$, classify the graph G such that $DG^{abc}(G)$ is self-centered with diameter two, $DG^{abc}(G)$ is bi-eccentric etc.

**Theorem 2.9:** For any graph G, distance between any two vertices in $DG^{abc}(G)$ is at most three.

**Proof:** Suppose G has at least two vertices. Then $DG^{abc}(G)$ has at least three vertices. Let $u, v \in V'$. We consider the following cases.

**Case-(i): $u, v \in V(G)$.**

If u and v are adjacent in G, then in $DG^{abc}(G)$, d(u, v) = 1. Suppose u and v are not adjacent in G. Then there exists a minimal dominating set containing u and v. In $DG^{abc}(G)$, d(u, v) = 2. Hence $d_{DG^{abc}(G)}(u, v) \leq 2$.

**Case-(ii): $u \in V(G)$ and $v \notin V(G)$.**

In this case $v \notin V(G)$, thus v = D is a minimal dominating set of G. If $u \in D$, then in $DG^{abc}(G)$, d(u, v) = 1. If $u \notin D$, then there exists a vertex $w \in D$ adjacent to u and hence in $DG^{abc}(G)$, d(u, v) = d(u, w) + d(w, v) = 2.

**Case-(iii): Suppose u, v $\notin$ V(G).** Then u = $D_1$ and v = $D_2$ are two minimal dominating sets of G. If $D_1$ and $D_2$ are disjoint, then every vertex $w \in D_1$ is adjacent to some vertex in $x \in D_2$ and vice versa. This implies that in $DG^{abc}(G)$, d(u, v) = d(u, w) + d(w, x) + d(x, v) = 3.

If $D_1$ and $D_2$ are disjoint, then there exists a minimal dominating set $D_3$ such that $D_3$ is adjacent to both $D_1$ and $D_2$. Thus, in $DG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_3) + d(D_3, D_2) = 2$.

If $D_1$ and $D_2$ have a vertex in common, then in $DG^{abc}(G)$, $d(D_1, D_2) = 1$.

Thus, from all the three cases, distance between any two vertices in $DG^{abc}(G)$ is at most three.

**Theorem 2.10:** $rad(DG^{abc}(G)) = 1$ if and only if $G = \overline{K_p}$.

**Proof:** Suppose $rad(DG^{abc}(G)) = 1$, then there exists $x \in V'$ such that e(x) = 1, x is adjacent to all other vertices. We have to prove $G = \overline{K_p}$. On the contrary, assume $G \neq \overline{K_p}$. Then there exists at least two minimal dominating sets and $\Delta(DG^{abc}(G)) < p'-1$, a contradiction. Hence G must be $\overline{K_p}$.

Conversely, $G = \overline{K_p}$, then there exists exactly one minimal dominating set containing all the vertices of G. Thus the result follows from the definition of $DG^{abc}(G)$.

**Theorem 2.11:** If G is a disconnected graph with at least one edge then $DG^{abc}(G)$ is 2-self-centered.

**Proof:** Suppose G is a disconnected graph. Consider the following cases.

**Case-(i): G has an isolated vertex.**

Each minimal dominating set contains this isolated vertex. Let u be the isolated vertex. In $DG^{abc}(G)$, d (u, D) = 1, where D is any minimal dominating set of G. Let $v \in V(G)$. Then d(u, v) = d(u, D)+d(D, v) = 2. That is e(u) = 2.

Suppose $v \in V(G)$ and v is not isolated and $v' \in V'$, $v' \notin V(G)$. Then v′ = D is a minimal dominating set of G. If $v \in D$, then in $DG^{abc}(G)$, d(v, v′) = 1. If $v \notin D$, then there exists a vertex $w \in D$ adjacent to v and hence in $DG^{abc}(G)$, d(v, v′) = d(v, w) + d(w, v′) = 2. Let $v \in V$, $v \neq u$ and $x \in V$. In $DG^{abc}(G)$, d(v, x) = 1 if $vx \in E(G)$. If $vx \notin E(G)$, then there exists $D \in S$ such that $v, x \in D$. Therefore, d(v, x) = d(v, D)+d(D, x) = 2 in $DG^{abc}(G)$. Thus, e(v) = 2, where $v \in V$ is not an isolated vertex. d(x, y)=1 for $x, y \in S$, since $x = D_{1,}$ $y = D_2$ contains the isolated vertex u. Therefore, eccentricity of set vertices is also 2. Hence, $DG^{abc}(G)$ is 2-seld-centered.

**Case-(ii): G has no isolated vertex.**

Since G has at least two components. Let $v \in V(G_1)$ and $u \in V(G_2)$. Then there exists a minimal dominating set D such that D contains u and v. Then, in $DG^{abc}(G)$, d(u, v) = d(u, D) + d(D, v) = 2. Suppose $D_i$, $D_j$ for $1 \leq i \leq n$, $1 \leq j \leq m$, $i \neq j$ are not disjoint, then in $DG^{abc}(G)$, $d(D_i, D_j) = 1$.

Suppose $D_i$, $D_j$ are disjoint. Then there exists a minimal dominating set $D_3$ such that $D_3$ is adjacent to both $D_i$ and $D_j$. Then, in $DG^{abc}(G)$, $d(D_i, D_j) = d(D_i, D_3) + d(D_3, D_j)$. Hence, in $DG^{abc}(G)$, $d(D_i, D_j) \leq 2$.

Suppose $x \in V(G)$ and $x \notin D_i$ for $1 \leq i \leq n$. Then there exists a vertex $y \in V(G)$ such that y is adjacent to x and $y \in D_i$. Then, in $DG^{abc}(G)$, $d(x, D_i) = d(x, y) + d(y, D_i) = 2$. Therefore, eccentricity of set vertices is 2 and eccentricity of point vertices is also 2. Hence, $DG^{abc}(G)$ is 2-self-centered.

**Theorem 2.12:** $rad(DG^{abc}(G)) = 2$ and $diam(DG^{abc}(G)) = 3$ if G is any one of the following:

(i) $G = K_p$.
(ii) rad(G) = 1 and diam(G) = 2.

**Proof: Case-(i): $G = K_p$.**

For each vertex v, $\{v\} \subseteq V(G)$ form a minimal dominating set of G. By the definition of $DG^{abc}(G)$, each vertex is adjacent to exactly one minimal dominating set and $DG^{abc}(G) = K_p \circ K_1$. The eccentricityof pendent vertices is 3 and other vertices have eccentricity two. Hence, $rad(DG^{abc}(G)) = 2$ and $diam(DG^{abc}(G)) = 3$.

**Case-(ii): rad(G) = 1 and diam(G) = 2.**

In this case two sub cases arise.

**Sub case-(i): G has only one central vertex.**

Let v be the central vertex. Then {v} = D form a minimal dominating set of G. In $DG^{abc}(G)$, D is adjacent to v only. Let $D_1$ be the minimal dominating set of G that contains $u \in V(G)$ which is not a central vertex. Thus, it follows, in $DG^{abc}(G)$, $d(D, D_1) = d(D, v) + d(v, u) + d(u, D_1) = 3$. Hence, the eccentricity of set vertices is three.

$d(u, D) = d(u, v) + d(v, D) = 2$. Hence, the eccentricity of point vertices is two.

Therefore, $rad(DG^{abc}(G) = 2$ and $diam(DG^{abc}(G)) = 3$.

**Sub case-(ii): G has more than one central vertex.**

Let t be the number of central vertices and $D_i$, $1 \leq i \leq t$ be the corresponding minimal dominating sets of G. Let u and v be the central vertices of G and let $D_1$ and $D_2$ be the corresponding minimal dominating sets of G. In $DG^{abc}(G)$, $d(D_1, D_2) = d(D_1, u) + d(u, v) + d(v, D_2) = 3$.

Let $D' \neq D_i$ be the minimal dominating set that contains $w \in V(G)$ which is not a central vertex. Then in $DG^{abc}(G)$, $d(D_1, D') = d(D_1, w) + d(w, u) + d(u, D') = 3$. Hence the eccentricity of set vertices is three. $d(w, D_1) = d(w, u) + d(u, D_1) = 2$ and $d(u, D_2) = d(u, v) + d(v, D_2) = 2$. Hence, the centricity of point vertices is two. Therefore, $rad(DG^{abc}(G)) = 2$ and $diam(DG^{abc}(G)) = 3$.

**Theorem 2.13:** If G is a connected graph with $rad(G) \geq 2$, then $DG^{abc}(G)$ is self-centred with diameter two.

**Proof:** Suppose $rad(G) \geq 2$, then consider the following cases.

**Case-(i):** Suppose $u,v \in V(G)$ and $d_G(u, v) \geq 3$. Then {u, v} is a maximal independent set. Hence it is a minimal dominating set. Thus, in $DG^{abc}(G)$, $d(u, v) = d(u, D) + d(D, v) = 2$.

**Case-(ii):** Suppose $x \in V(G)$ and $x \notin D_1$, $D_1 \in S$. Then there exists a vertex $y \in V(G)$ such that y is adjacent to x and $y \in D_1$. Thus, it follows that, in $DG^{abc}(G)$, $d(x, D_1) = d(x, y) + d(y, D_1) = 2$.

**Case-(iii):** Suppose $u', v' \notin V(G)$. Then $u' = D_2$ and $v' = D_3$ are two minimal dominating sets of G. If $D_2$ and $D_3$ are adjacent, then, in $DG^{abc}(G)$, $d(u', v') = 1$.

Suppose $D_2$ and $D_3$ are disjoint. Then there exists a minimal dominating set $D_4$ such that $D_4$ is adjacent to both $D_2$ and $D_3$. Thus, in $DG^{abc}(G)$, $d(D_2, D_3) = d(D_2, D_4) + d(D_4, D_3) = 2$.

So in all cases, eccentricity of point vertices and eccentricity of set vertices is two.

Hence, $DG^{abc}(G)$ is self-centred with diameter two.

Next, we establish the necessary and sufficient condition on G for which $DG^{abc}(G)$ is a tree.

**Theorem 2.14:** For any graph G, $DG^{abc}(G)$ is a tree if and only if G = $\overline{K_p}$ or $K_2$.

**Proof:** Suppose $DG^{abc}(G)$ is tree, then we have to prove that G = $\overline{K_p}$ or $K_2$. On the contrary, if G ≠ $\overline{K_p}$ or $K_2$, then we consider the following cases. Since $DG^{abc}(G)$ is a tree, G is also a tree.

**Case-(i):** If $\Delta(G) = p-1$, $p \geq 3$, then G is a star. Then there exists exactly two minimal dominating sets D and D′, D contains the central vertex and D′ contains all pendent vertices of G. Clearly, by the definition of $DG^{abc}(G)$, $DG^{abc}(G)$ contains a cycle, a contradiction.

**Case-(ii):** If $\Delta(G) \leq p-2$, then there exists three vertices u, v and $w \in V(G)$ such that u and v are adjacent and w is not adjacent to both u and v. This implies that in $DG^{abc}(G)$, u and v are connected by at least two paths, which is a contradiction. Thus from the above cases G = $\overline{K_p}$ or $K_2$.

Conversely, suppose G = $\overline{K_p}$ or $K_2$. Then by definition, $DG^{abc}(G)$ is a tree.

**Theorem 2.15:** $\gamma(DG^{abc}(G)) = 1$ if and only if $G = \overline{K_p}$.

**Proof:** The proof follows from theorem 2.11.

**Theorem 2.16:** $\gamma(DG^{abc}(G)) = p$ if and only if $G = K_p$.

**Proof:** Suppose $\gamma(DG^{abc}(G)) = p$. We have to prove $G = K_p$. On the contrary, if $G \neq K_p$, then there exists at least two non-adjacent vertices u and v in G. Hence there exists D containing {u, v} and this D dominates u and v in $DG^{abc}(G)$ and other vertices dominates remaining vertices. This implies that $\gamma(DG^{abc}(G)) \leq p-1$, which is a contradiction. Hence $G = K_p$.

Conversely, suppose $G = K_p$, then each vertex form a minimal dominating set of G. By the definition each vertex is adjacent to exactly one minimal dominating set. That is $DG^{abc}(G) = K_p \circ K_1$. Hence, $\gamma(DG^{abc}(G)) = p$.

**Theorem 2.17:** For any graph G, $1 \leq \gamma(DG^{abc}(G)) \leq p$. The lower bound is attained if and only if $G = \overline{K_p}$, and the upper bound is attained if and only if $G = K_p$.

**Proof:** Proof follows from theorem 2.12 and 2.13.

**Theorem 2.18:** $\gamma(DG^{abc}(G)) \leq \gamma(G)$ if G has an isolated vertex.

**Proof:** Suppose G has an isolated vertex. Then every minimal dominating set contains this isolated vertex. Thus all minimal dominating sets are adjacent to each other. This isolated vertex dominates all set vertices. Remaining vertices of G is dominated by at most $\gamma(G)-1$ vertices. Hence at most $\gamma(G)-1+1= \gamma(G)$ vertices are needed to dominate $DG^{abc}(G)$. Thus, $\gamma(DG^{abc}(G)) \leq \gamma(G)$.

**Theorem 2.19:** For any graph G, $\gamma(G) + \gamma(DG^{abc}(G)) \leq 1+p$.

**Proof:** Suppose $\gamma(G) = 1$. All the point vertices form a dominating set in $DG^{abc}(G)$. Hence $\gamma(DG^{abc}(G)) \leq p$. Thus, $\gamma(G) + \gamma(DG^{abc}(G)) \leq 1+p$. Suppose $\gamma(G) = p$. Then $G = \overline{K_p}$, $\gamma(DG^{abc}(G))=1$. Hence $\gamma(G)+\gamma(DG^{abc}(G)) = 1+p$. Therefore, $\gamma(G) + \gamma(DG^{abc}(G)) \leq 1+p$. Suppose $1 < \gamma(G) < p$. Let $\gamma(G) = k$. Let D with $|D| = k$ be a minimum dominating set. In $DG^{abc}(G)$, x = D dominates k point vertices. The remaining $p-k$ vertices with D dominates $DG^{abc}(G)$. Therefore, $\gamma(DG^{abc}(G)) \leq 1+(p-k) = p-k+1$.

Therefore, $\gamma(G) + \gamma(DG^{abc}(G)) \leq k+1+p-k = 1+p$.

Next, traversability properties of the graph $DG^{abc}(G)$ is discussed.

**Theorem 2.20:** Let G be a (p−3) regular graph and $\beta_0(G) = 2$. If each minimal dominating set is independent, then $DG^{abc}(G)$ is Hamiltonian.

**Proof:** Let G be a (p−3) regular graph and $\beta_0(G) = 2$. Since every minimal dominating set has exactly two vertices and for each $v \in V(G)$, there exists two minimal dominating sets containing v. Thus, $DG^{abc}(G)$ contains a spanning cycle. Hence G is Hamiltonian.

**Theorem 2.21:** Let G be a (p−3) regular graph with $\beta_o(G) = 2$ and each minimal dominating set is independent. If p is odd, then $DG^{abc}(G)$ is Eulerian.

**Proof:** Let G be a (p−3) regular graph with $\beta_o(G) = 2$ and every minimal dominating set is independent. For every vertex $v \in V(G)$, there exists exactly two minimal dominating sets containing v. Also, every minimal dominating set contains two vertices. Thus, in $DG^{abc}(G)$, $\deg_{DG^{abc}(G)} v_i = p-3+2 = p-1$, $1 \leq i \leq p$ and $\deg_{DG^{abc}(G)} D_j = 4$, $1 \leq j \leq n$.

If p is odd, then every vertex of $DG^{abc}(G)$ has an even degree, then $DG^{abc}(G)$ is Eulerian.

Next, we find the girth of $DG^{abc}(G)$.

**Theorem 2.22:** Girth of $DG^{abc}(G)$ is three if and only if G has a triangle or there exists at least two minimal dominating sets which are not disjoint or there exists a minimal dominating set containing two adjacent vertices or there exists three dominating sets which are pair wise not disjoint.

**Proof: Case-(i): G has a triangle.**

Since G is an induced sub graph of $DG^{abc}(G)$. Then girth of G is the girth of $DG^{abc}$ (G). Hence, girth of $DG^{abc}(G)$ is three.

**Case-(ii):** Suppose there exists at least two minimal dominating sets $D_1$, $D_2$ which are not disjoint. Then they have a common vertex u. Thus, it follows that, in $DG^{abc}(G)$, $D_1$, $D_2$ and u form a triangle. Hence, girth of $DG^{abc}(G)$ is three.

**Case-(iii):** Suppose there exists a minimal dominating set $D_3$ containing two adjacent vertices u and v. In, $DG^{abc}(G)$, $D_3$, u and v form a triangle. Hence, girth of $DG^{abc}(G)$ is three.

**Case-(iv):** Suppose there exists three dominating sets $D_4$, $D_5$, $D_6$ which are pair wise not disjoint. In, $DG^{abc}(G)$, $D_4$, $D_5$ and $D_6$ form a triangle. Hence, girth of $DG^{abc}(G)$ is three.

Conversely, assume that girth of $DG^{abc}(G)$ is three. Consider the following cases.

**Case-(i): Three vertices are point vertices**

Since G is an induced sub graph of $DG^{abc}(G)$, these three form a triangle in G.

**Case-(ii): Two vertices are point vertices and one set vertex**

Let u, v be point vertices and D = w is a set vertex. Then {u, v, w} forms a triangle implies that u and v are adjacent and $u, v \in D$.

**Case-(iii): Two vertices are set vertices and one point vertex**

Suppose there exists at least two minimal dominating sets $u = D_1$, $v = D_2$ have a common vertex $w \in V(G)$. u, v are adjacent implies $D_1$, $D_2$ have a common vertex and uw, vw are adjacent implies $w \in D_1 \cap D_2$.

**Case-(iv): All three vertices are set vertices**

Let $u = D_1$, $v = D_2$ and $w = D_3$ are three minimal dominating sets of G. $D_i \cap D_j \neq \phi$ for $i \neq j$.

## CONCLUSION

In this paper, we have defined and studied the new dominating graph $DG^{abc}(G)$. Eccentricity properties of $DG^{abc}(G)$ are also studied.

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