ON INFRA TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduced and investigate infra-topological spaces which deduced from topological spaces and studies the properties of subsets of infra-topological spaces such as infra-topological space, infra-derived set, infra-interior set, infra-closure set, infra-exterior set and infra-boundary set.

Keywords: infra-topological space, infra-derived set, infra-interior point, infra-closure, infra-exterior point and infra-boundary set.

1. INTRODUCTION

In 1983, A.S. Mashhour et al. [1] introduced the supra-topological space and studied $s$-continuous functions and $s^*$-continuous functions. In this paper we introduced Infra-Topological Space (ITS) and analogue concepts associated with infra-topological space. Such as, infra-derived set (resp. infra-closure, infra-interior, infra-exterior and infra-boundary) of subset $A$ of infra-space $X$. We will be denoted by $\text{ids}(A)$ (resp. $\text{icp}(A)$, $\text{iip}(A)$, $\text{iep}(A)$ and $\text{ibp}(A)$).

Many results of topological space remain valid in infra-topological space, Whereas some become invalid in infra-topological space.

2. INFRA-TOPOLOGICAL SPACES

Definition 2.1: Let $X$ be any arbitrary set. An Infra-topological space on $X$ is a collection $\tau_{IX}$ of subsets of $X$ such that the following axioms are satisfying:

Ax-1 $\emptyset, X \in \tau_{IX}$.

Ax-2 The intersection of the elements of any subcollection of $\tau_{IX}$ in $X$.

i.e., If $O_i \in \tau_{IX}, 1 \leq i \leq n \rightarrow \bigcap O_i \in \tau_{IX}$.

Terminology, the ordered pair $(X, \tau_{IX})$ is called Infra-Topological Space. We simply say $X$ is an Infra-space.

Definition 2.2: Let $(X, \tau_{IX})$ be an (ITS) and $A \subset X$. $A$ is called infra-open set (IOS) if $A \in \tau_{IX}$.

Definition 2.3: Let $X$ be any arbitrary set and $\tau = \{\emptyset, X\}$, then $(X, \tau_{IX})$ is called indiscrete infra-topology space or is called trivial infra-topological space.

Definition 2.4: Let $X$ be any countable arbitrary set and $\tau = P(X)$ the set of all subsets of $X$, then $(X, \tau)$ is called discrete infra-topology space or is called maximal infra-topological space.

Theorem 2.1: Let $(X, \tau)$ be a topological space (TS), then $(X, \tau)$ is an infra-topological space (IITS).

Proof: Suppose that $(X, \tau)$ is a topological space, then by axioms it is clear that $(X, \tau)$ is infra-topological space. The converse of above theorem is not true.

Example 2.1: If $X = \{a, b, c\}$ and $\tau_{IX} = \{\emptyset, X, \{a\}, \{b\}\}$, then $(X, \tau_{IX})$ is infra-topological space, but not topological space.
Theorem 2.2: Let \((X, \tau_{ix})\) be infra-topological space. Then:
1. \(\emptyset, X\) are infra-open set.
2. Any arbitrary intersections of infra-open sets are infra-open sets.
3. Finite union of infra-open sets may not be infra-open sets.

Proof:
1. It is clear that \(\emptyset, X\) are infra open set by Ax-1 and definition 2.1.
2. Let \(C_{ix} \in \tau_{ix}\), by Ax-2 and definition 2.1 \(\bigcap C_{ix} \in \tau_{ix}\) are infra open set.
3. By counter example 2.1 \(\{a, b\} \in \tau_{ix}\), but \(\{a, b\} \notin \tau_{ix}\).

Theorem 2.3: Let \((X, \tau_{ix})\) and \((X, \tau_{ix}^*)\) be two infra-topological Spaces on set \(X\). Then the intersection \(\tau_{ix}\) and \(\tau_{ix}^*\) is an infra-topological space.

Proof:
Let \((X, \tau_{ix})\) and \((X, \tau_{ix}^*)\) be two infra-topological Spaces on set \(X\).

By Ax-1, if \(X \in \tau_{ix}\) and \(X \in \tau_{ix}^*\), so \(X \in \tau_{ix} \cap \tau_{ix}^*\). Suppose that \(O_i \in \tau_{ix} \cap \tau_{ix}^*\), where \(1 \leq i \leq n\) implies that \(O_i \in \tau_{ix}\) and \(O_i \in \tau_{ix}^*\). Consequently, \(\bigcap_{i=1}^{n} O_i \in \tau_{ix}\) and \(\bigcap_{i=1}^{n} O_i \in \tau_{ix}^*\), and hence \(\bigcap_{i=1}^{n} O_i \in \tau_{ix} \cap \tau_{ix}^*\).

Theorem 2.4: Let \((X, \tau_{ix})\) and \((X, \tau_{ix}^*)\) be two infra-topological Spaces on set \(X\). Then the union \(\tau_{ix}\) and \(\tau_{ix}^*\) is an infra-topological space.

Proof:
Let \((X, \tau_{ix})\) and \((X, \tau_{ix}^*)\) be two infra-topological infra spaces on set \(X\). By Ax-1, if \(X \in \tau_{ix}\) and \(X \in \tau_{ix}^*\), so \(X \in \tau_{ix} \cup \tau_{ix}^*\).

Suppose that \(O_i \in \tau_{ix} \cup \tau_{ix}^*\), where \(1 \leq i \leq n\), implies that \(O_i \in \tau_{ix}\) or \(O_i \in \tau_{ix}^*\) for some \(i, 1 \leq i \leq n\). So \(\bigcap_{i=1}^{n} O_i \in \tau_{ix}\) or \(\bigcap_{i=1}^{n} O_i \in \tau_{ix}^*\) and hence \(\bigcap_{i=1}^{n} O_i \in \tau_{ix} \cup \tau_{ix}^*\).

Remark: The union of infra-topological spaces may not be infra-topological space, in general, by the following example.

Example 2.2: Let \(X\) be a set \(X = \{a, b, c, d\}\), \(\tau_{ix} = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}\) and \(\tau_{ix}^* = \{\emptyset, X, \{c\}, \{d\}, \{b, c\}, \{c, d\}\}\).

Now \(\tau_{ix} \cup \tau_{ix}^* = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}\}\) is not infra-topological space.

3. PROPERTIES OF SUBSETS ON INFRA TOPOLOGICAL SPACES

Definition 3.1: Let \((X, \tau_{ix})\) be an (ITS) and \(A \subset X\). A point \(x \in X\) is called Infra-Cluster Point (ICP) of \(A\), if for all Infra-open set \(O\) containing \(x\), then \(O \cap \{x\} \neq \emptyset\).

Definition 3.2: Let \((X, \tau_{ix})\) be an (ITS) and \(A \subset X\). The set of all Infra-Cluster Point (ICP) of \(A\) is called the Infra-Derived Set (IDS) of \(A\) and is denoted by \(ids(A)\).

Definition 3.3: Let \((X, \tau_{ix})\) be infra-topological space. A subset \(C \subset X\) is called infra-closed set in \(X\) if \(X \setminus C\) is infra-open set in \(X\). That is \(C\) is infra-closed set (ICS) if \(X \setminus C \in \tau_{ix}\).

Theorem 3.1: Let \((X, \tau_{ix})\) be infra-topological space. Then:
1. \(\emptyset, X \in \tau_{ix}\) are infra-closed sets.
2. Any arbitrary finite intersections of infra-closed sets is an infra-closed sets.

Proof:
1. Since \(X \setminus \emptyset = X \in \tau_{ix}\) and \(X \setminus X = \emptyset \in \tau_{ix}\) are infra-closed sets.
2. Let \(\{C_i : i \in I\}\) be an arbitrary family of infra-closed sets such that \(C_i \in \tau_{ix}\) for all \(i \in I\). Now, \(X \setminus C_i \in \tau_{ix}\) is infra-open set for all \(i \in I\).
    
    But \(X \setminus C_i = \bigcap C_i \in \tau_{ix}\), then \(\bigcap C_i = \bigcap (X \setminus C_i) = X \setminus \bigcap C_i \in \tau_{ix}\) \(\forall i \in I\). Hence \(\bigcap C_i \in \tau_{ix}\) \(\forall i \in I\) is infra-closed set.

Remark: Finite union of infra-closed sets may not be infra-closed sets, in general.

Definition 3.4: Let \((X, \tau_{ix})\) be an (ITS) and \(A \subset X\). The Infra Closure Point (ICP) of \(A\) is a set denoted by \(icp(A)\) and given by: \(icp(A) = \cap \{C_i : A \subset C_i, X \setminus C_i \in \tau_{ix}\}\). That is, \(icp(A)\) is the intersection of all infra-closed set contained the set \(A\).
Remark: Since \( icp(A) \) is the intersection of all infra closed sets containing in \( A \), then \( A \subset icp(A) \) and \( icp(A) \) is the smallest infra closed sets.

**Definition 3.5:** Let \( (X,\tau_{\mathcal{I}}) \) be an (ITS) and \( A \subset X \). The Infra-Interior Points (IIP) of \( A \) is a set denoted by \( iip(A) \) and given by: \( iip(A) = \cup \{ O; O \subset A, O \subset \tau_{\mathcal{I}} \} \). That is, \( iip(A) \) is the union of all infra-open set contained in the set \( A \).

Remark: Since \( iip(A) \) is the union of all infra-open sets contained in \( A \), then \( iip(A) \subset A \) and \( icp(A) \) is the smallest infra-open sets. Also if \( O \) is infra-open set contained in \( A \), then \( O \subset iip(A) \).

**Definition 3.6** Let \( (X,\tau_{\mathcal{I}}) \) be an (ITS) and \( A \subset X \). The Infra-Exterior Points (IEP) of \( A \) is a set denoted by \( iep(A) \) and given by: \( iep(A) = iip(A^c) \). That is, Set of all infra-interior point of complement of \( A \).

**Definition 3.7:** Let \( (X,\tau_{\mathcal{I}}) \) be an (ITS) and \( A \subset X \). The Infra-Boundary Points (IBP) of \( A \) is a set denoted by \( ibp(A) \) and given by: \( ibp(A) = \partial ibp(A) \).

**Theorem 3.2:** Let \( (X,\tau_{\mathcal{I}}) \) be an (ITS) and \( A,B \subset X \). The Infra-Derived Set Axioms (IDSA) satisfies the followings:

\[ (IDSA)_1 : \quad ids(\emptyset) = \emptyset. \]
\[ (IDSA)_2 : \quad \text{If } A \subset B \text{, then } ids(A) \subset ids(B). \]
\[ (IDSA)_3 : \quad \text{if } x \in ids(A), \text{ then } x \in ids(A\setminus\{x\}). \]
\[ (IDSA)_4 : \quad ids(A \cap B) \subset ids(A) \cap ids(B). \]
\[ (IDSA)_5 : \quad ids(A \cup B) = ids(A) \cup ids(B). \]

Proof:
\[ (IDSA)_1 : \quad \text{Suppose that } ids(\emptyset) \neq \emptyset \rightarrow \exists x \in ids(A) \ni \emptyset \cap (O\setminus\{x\}) \neq \emptyset \]
\[ \rightarrow x \in \emptyset \text{ and } x \notin \emptyset. \text{ That is contradiction.} \]
\[ \rightarrow ids(\emptyset) = \emptyset. \]

\[ (IDSA)_2 : \quad \text{Suppose that } A \subset B \text{. Let } x \in ids(A) \rightarrow \forall O \exists x, A \cap (O\setminus\{x\}) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, B \cap (O\setminus\{x\}) \neq \emptyset. \]
\[ \rightarrow x \in ids(B) \]
\[ \rightarrow ids(A) \subset ids(B). \]

\[ (IDSA)_3 : \quad \text{Assume that } x \in ids(A) \rightarrow \forall O \exists x, A \cap (O\setminus\{x\}) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, A \cap (O \cap \{x\}) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, A \cap (\{x\}^c \cap O \cap \{x\}) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, A \cap (\{x\}^c \cap O) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, (A \cap \{x\}) \cap (O \setminus \{x\}) \neq \emptyset. \]
\[ \rightarrow x \in ids(A\setminus\{x\}). \]

\[ (IDSA)_4 : \quad \text{Since } A \cap B \subset A \setminus A \cap B \subset B \]
\[ \rightarrow ids(A \cap B) \subset ids(A \cap ids(A \cap B) \subset ids(B)). \]
\[ \rightarrow ids(A \cap B) \subset ids(A \cap ids(B)). \]

It can be easily shown by example the equality is not hold like topological space.

\[ (IDSA)_5 : \quad \text{Since } A \subset A \cup B \text{ and } B \subset A \cup B \text{, then } ids(A) \subset ids(A \cup B) \text{ and } ids(B) \subset ids(A \cup B), \text{ hence } \]
\[ ids(A) \cup ids(B) \subset ids(A \cup B). \text{ Conversely,} \]
\[ \text{Suppose that } \exists x \in ids(A \cup B) \rightarrow \forall O \exists x, (A \cup B) \cap (O\setminus\{x\}) \neq \emptyset. \]
\[ \rightarrow \forall O \exists x, A \cap (O\setminus\{x\}) \neq \emptyset \cup B \cap (O\setminus\{x\}) \neq \emptyset. \]
\[ \rightarrow x \in ids(A \cup B) \text{. Hence,} \]
\[ ids(A \cup B) = ids(A) \cup ids(B). \]

**Theorem 3.3:** Let \( (X,\tau_{\mathcal{I}}) \) be an (ITS) and \( A,B \subset X \). The Infra Closure Point Axioms (ICPA) satisfying the following conditions:

\[ (ICPA)_1 : \quad A \text{ is infra-closed if } \emptyset = icp(A). \]
\[ (ICPA)_2 : \quad icp(\emptyset) = \emptyset \text{ and } icp(X) = X. \]
\[ (ICPA)_3 : \quad icp(icp(A)) = icp(A). \]
\[ (ICPA)_4 : \quad \text{If } A \subset B \text{, then } icp(A) \subset icp(B). \]
\[ (ICPA)_5 : \quad icp(A \cap B) \subset icp(A) \cap icp(B). \]
Proof:

(ICPA)_1: Suppose that \( A \) is infra-closed set. Since \( A \subset A \) and \( A \cap A = A \rightarrow icp(A) \subset A \), Also \( A \subset icp(A) \rightarrow A = icp(A) \). Conversely, Let \( A = icp(A) \), obviously, icp(\( A \)) is the smallest infra-closed set. Hence \( A \) is infra-closed set.

(ICPA)_2: Since \( X \) and \( \emptyset \) are infra-closed sets, so by (ICPA)_1icp(\( \emptyset \)) = \( \emptyset \) and icp(\( X \)) = \( X \). (ICPA)_3: Since icp(\( A \)) is the intersection of all infra-closed sets are closed sets, then icp(icp(\( A \))) = icp(\( A \)).

(ICPA)_4: Consider \( A \subset B \). Since \( A \subset icp(A) \) and \( B \subset icp(B) \), so icp(\( A \)) \( \subset icp(B) \).

(ICPA)_5: Since \((A \cap B) \subset (A \subset icp(A)) \cap (B \subset icp(B)) \), then icp(\( A \cap B \)) \( \subset icp(A \cap B) \) \( \subset icp(A) \cap icp(B) \).

Remark: In (ICPA)_5 the equality does not work like topological space.

Theorem 3.4: Let \((X, \tau_{iX})\) be an (ITS) and \( A, B \subset X \).The Infra-Interior Points Axioms (IIPA) given by:

\[ \text{(IIPA)}_1: \quad \text{A is infra-open set iff } \text{iip}(A) = A. \]
\[ \text{(IIPA)}_2: \quad \text{iip}(X) = X \text{ and } \text{iip}(\emptyset) = \emptyset. \]
\[ \text{(IIPA)}_3: \quad \text{iip}(\text{iip}(A)) = \text{iip}(A). \]
\[ \text{(IIPA)}_4: \quad \text{If } A \subset B, \text{ then } \text{iip}(A) \subset \text{iip}(B). \]
\[ \text{(IIPA)}_5: \quad \text{\text{iip}(A \cap B) = \text{iip}(A) \cap \text{iip}(B).} \]

Proof:

(IIPA)_1: Suppose that \( A \) is infra-open set. Since \( A \subset A \), then \( A \) is infra-open set containing itself, so \( A \subset \text{iip}(A) \) and \( \text{iip}(A) \subset A \), that implies \( A = \text{iip}(A) \). Conversely, Let \( A = \text{iip}(A) \), suppose that \( A = \text{iip}(A) \) is infra-open set, then \( A \subset \text{iip}(A) \).

(IIPA)_2: Since \( X, \emptyset \) are infra-open sets, by (IIPA)_1, we have \( \text{iip}(X) = X \) and \( \text{iip}(\emptyset) = \emptyset \).

(IIPA)_3: Since \( \text{iip}(A) \) is infra-open set. so by (IIPA)_1\( \text{iip}(\text{iip}(A)) = \text{iip}(A) \).

(IIPA)_4: Suppose that \( A \subset B \). Let \( O_i \subset \text{iip}(A) \rightarrow O_i \subset A \rightarrow O_i \subset B \rightarrow O_i \subset \text{iip}(B) \).

Therefore \( \text{iip}(A) \subset \text{iip}(B) \).

(IIPA)_5: Let \( O_i \subset \text{iip}(A) \cap \text{iip}(B) \rightarrow O_i \subset \text{iip}(A) \land O_i \subset \text{iip}(B) \).

\[ \rightarrow \cup O_i \subset A, \forall i \land \cup O_i \subset B, \forall i. \]
\[ \rightarrow O_i \subset \text{iip}(A \cap B), \forall i. \]

Theorem 3.5: Let \((X, \tau_{iX})\) be an (ITS) and \( A, B \subset X \). The Infra-Exterior points Axioms (IEPA) given by:

\[ \text{(IEPA)}_1: \quad \text{iep}(X) = \emptyset \text{ and } \text{iep}(\emptyset) = X. \]
\[ \text{(IEPA)}_2: \quad \text{iep}(A) \subset A^c. \]
\[ \text{(IEPA)}_3: \quad \text{iep}(A \cup B) = \text{iep}(A) \cap \text{iep}(B). \]
\[ \text{(IEPA)}_4: \quad \text{If } A \subset B, \text{ then } \text{iep}(B) \subset \text{iep}(A). \]
\[ \text{(IEPA)}_5: \quad \text{\text{iep}(A \cap B) \subset \text{iep}(A) \cup \text{iep}(B).} \]

Proof:

(IEPA)_1: \( \text{iep}(X) = \text{iep}(X^c) = \text{iep}(\emptyset) = \emptyset \text{ and } \text{iep}(\emptyset^c) = \text{iep}(X) = X. \)

(IEPA)_2: \( \text{iep}(A) = \text{iep}(A^c) \subset A^c. \)

(IEPA)_3: \( \text{iep}(A \cup B) = \text{iep}(A \cup B)^c = \text{iep}(A^c \cap B^c) = \text{iep}(A^c) \cap \text{iep}(B^c) = \text{iep}(A) \cap \text{iep}(B). \)

(IEPA)_4: \( \text{let } A \subset B \rightarrow B^c \subset A^c \rightarrow \text{iep}(B^c) \subset \text{iep}(A^c) \rightarrow \text{iep}(B) \subset \text{iep}(A). \)

(IEPA)_5: \( \text{iep}(A \cap B) = \text{iep}(A \cap B)^c = \text{iep}(A^c \cup B^c) \subset \text{iep}(A^c) \cup \text{iep}(B^c) = \text{iep}(A) \cup \text{iep}(B). \)
Theorem 3.6: Let \((X, \tau_{iX})\) be an (ITS) and \(A \subset X\). The Infra-Boundary Points Axioms (IBPA) given by:

\[
\begin{align*}
\text{(IBPA)}_1 & : \ i\beta(X) = i\beta(\emptyset) = \emptyset. \\
\text{(IBPA)}_2 & : \ i\beta(A \cap B) = i\beta(A) \cup i\beta(B).
\end{align*}
\]

Proof:

\(\text{(IBPA)}_1: i\beta(X) = X \setminus i\epsilon(X) \cup i\epsilon(X) = X \setminus \emptyset = X = \emptyset.\)

\(i\beta(\emptyset) = X \setminus i\epsilon(\emptyset) \cup i\epsilon(\emptyset) = X \setminus \emptyset \cup X = X \setminus \emptyset = \emptyset.\)

\(\text{(IBPA)}_2: i\beta(A \cap B) = X \setminus i\epsilon(A \cap B) \cup i\epsilon(A \cap B) = X \setminus i\epsilon(A) \cup X \setminus i\epsilon(B) \cup i\epsilon(A \cap B) = i\beta(A) \cup i\beta(B).\)

The following theorem illustrate the relations between \(\text{id}(A), \text{i}(A), i\epsilon(A), i\epsilon(A)\) and \(i\beta(A)\).

Theorem 3.7: Let \((X, \tau_{iX})\) be an (ITS) and \(A \subset X\). then:

1. \(A \subset i\epsilon(A) \rightarrow \text{id}(A) \subset \text{id}(i\epsilon(A)).\)
2. \(i\epsilon(A) \subset A \rightarrow \text{id}(i\epsilon(A)) \subset \text{id}(A).\)
3. If \(A\) is infra-closed, then \(\text{id}(A) \subset A.\)
4. \(i\epsilon(A) = A \cup \text{id}(A).\)
5. \(i\beta(A) = i\epsilon(A) \cup i\epsilon(A).\)
6. \(i\epsilon(A) = i\beta(A) \cup i\epsilon(A).\)
7. \(i\beta(A) \subset i\epsilon(A).\)
8. \(i\epsilon(A) \cap i\beta(A) = \emptyset.\)

Proof:

1. Let \(A \subset i\epsilon(A).\) By (IDSA) \(i\epsilon(A) \subset \text{id}(i\epsilon(A)).\)
2. Let \(i\epsilon(A) \subset A.\) By (IDSA) \(i\epsilon(A) \subset \text{id}(A).\)
3. Let \(A\) be a infra closed set and \(x \in \text{id}(A),\) then \(\forall \emptyset \ni x, A \cap (O - (x)) \neq \emptyset.\) Hence \(x \in A\) and \(\text{id}(A) \subset A.\)
4. Since \(A \subset i\epsilon(A)\) and \(\text{id}(A) \subset \text{id}(i\epsilon(A)) \subset i\epsilon(A).\) we have \(A \cup \text{id}(A) \subset i\epsilon(A).\) Another direction, To show that \(i\epsilon(A) \subset A \cup \text{id}(A).\)

Let \(x \in i\epsilon(A),\) but \(A \subset i\epsilon(A),\) then \(x \in A or x \notin A.\)

Probability 1) If \(x \in A,\) then \(x \in A \cup \text{id}(A).\)

Probability 2) if \(x \notin A,\) Let \(x \notin \text{id}(A) \rightarrow \exists \emptyset \ni x, A \cap O \setminus \{x\} \neq \emptyset.\) but \(x \notin A,\) that is contradiction, therefore \(x \in \text{id}(A)\) and \(x \in A \cup \text{id}(A).\)

So \(i\epsilon(A) = A \cup \text{id}(A).\)

5. By defn \(i\beta(A) = X \setminus i\epsilon(A) \cup i\epsilon(A) = X \setminus i\epsilon(A) \cap X \setminus i\epsilon(A) = X \setminus i\epsilon(A) \cap i\epsilon(A) = i\epsilon(A) \setminus i\epsilon(A).\)

6. By (1) \(i\beta(A) = i\epsilon(A) \cap i\epsilon(A) \rightarrow i\beta(A) = i\epsilon(A) \cup i\epsilon(A) = i\epsilon(A) \setminus i\epsilon(A) \cup i\epsilon(A) = i\epsilon(A).

7. By (2) it is clear that \(i\beta(A) \subset i\epsilon(A).\)

8. \(i\epsilon(A) \cap i\beta(A) = i\epsilon(A) \cap i\epsilon(A) \cap i\beta(A) = \emptyset.\)

Theorem 3.8: Let \(X\) be any finite set and \(A \subset X\) and order \((o(A) = 1).\) The collection \(\tau_{iX} = \{\emptyset, X\} \cup \{A \subset X\} such that o A=1\) is infra topological space.

Proof: Since \(\emptyset, X \in \{\emptyset, X\},\) so \(\emptyset, X \in \tau_{iX}\) and Ax-1 is hold. Now, Assume that \(A_i \in \tau_{iX}, 1 \leq i \leq n.\) And \(o(A) = 1)\) Then \(A_i \cap \emptyset = \emptyset, \forall i \text{ and } A_i \cap X = A_i, \forall i,\) so that Ax-2 is hold.

The pair \((X, \tau_{iX})\) is called the Particular singleton set of infra-topological space on \(X.\)
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