International Journal of Mathematical Archive-6(11), 2015, 195-205
Available online through www.ijma.info ISSN 2229 – 5046

BOUND FOR THE COMPLEX GROWTH RATE IN SORET DRIVEN DOUBLE DIFFUSIVE CONVECTION IN RIVLIN-ERICKSEN VISCOELASTIC FLUIDIN POROUS MEDIUM

MEGH RAJ GOYAL*[1], ANUBHAV GOYAL[2]

[1]**Department of Mathematics, D. A. V. College, Malout152107, Punjab, India.**

[2]**Department of Mechanical Engineering, MIMIT, Malout 152107, Punjab, India.**

(Received On: 24-10-15; Revised & Accepted On: 30-11-15)

ABSTRACT

In the present paper, Double diffusive convection in a porous layer of Rivlin- Ericksen Viscoelastic fluid of Veronis type in the presence of Soret effectis investigated. For the porous medium, the Darcy model is employed and Rivlin-Ericksenfluid model is used to chracterize the rheological behavior of the Viscoelastic fluid. Following the linear stability theory based upon normal mode technique, the paper through mathematical analysis of the governing equations of the problem for any combination of free and rigid boundaries estaiblishs that the complex growth rate $\sigma = \sigma_r + i\sigma_i$ *of an arbitrary oscillatory perturbation of neutral or growing amplitude* ($\sigma_r \geq 0$) *in thermosolutal convection in a porous layer of Rivlin- Ericksen Viscoelastic fluid of Veronis type in the presence of Soret effect must lies inside a semi circle in the right half of the* $\sigma_r\sigma_i$*-plane whose centre is at origin and whose radius is:*

$$\frac{1}{EP_r}\frac{R_T}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J^2-1}.\ \textit{where}\quad J=\frac{R_T}{\left\{\frac{4\pi^2}{P_l}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{27\pi^4}{4}\right\}}$$

and R_T *is the modified thermal Rayleigh number,* E, E' *are thermal andsolutal capacity ratio, F is the Visco-elasticity parameter,* P_r *is the thermal Prandtl number,* S_c *is the Schamditnumber,* P_l *is the medium permeability and* ε *is theporosity. Further, in the limiting cases some of the important results have been recovered.*

Keywords: *Double diffusive convection; Rivlin-Ericksen Fluid; Porous medium; Soret effect; Oscillatory motion; Rigid boundaries; Rayleigh number.*

INTRODUCTION

The problem of convection driven by buoyancy that is contributed by two different diffusive components, namely, temperature and solutalconcentration, with differing rates of diffusion is widely known as ‘‘double diffusive convection’’ or ‘‘thermosolutalconvection’’ has generated considerable interest during the last few decades. If gradients of two stratifying agencies having different diffusivities are simultaneously present in a fluid layer, a variety of interesting convective phenomena can occur that are not possible in single component fluids. The double-diffusive convection was observed first by Stern, [1] and studied the convection in a layer of fluid heated and salted from above. The problem of thermosolutal convection in a layer of fluid heated from below and subjected to a stable salinity gradient is considered by Veronis [2]. The mechanism of instability at the onset of thermohaline convection in a porous medium have been described by Nield[3]. The double diffusive convection in porous media has also become important in recent years because of its many applications in geophysics, soil sciences, ground water hydrology, astrophysics, food processing, limnology and engineering etc. Excellent review of the literature concerning double diffusive convection in a binary fluid saturated porous medium may be found in the book by Nield and Bejan [4], Mojtabi and Charrier-Mojtabi [5].

Corresponding Author: Megh Raj Goyal*[1]
[1]***Department of Mathematics, D. A. V. College, Malout152107, Punjab, India.***

Since in a double diffusive system the fluid density depends on heat and solute concentration, it leads to a competition between thermal and compositional gradients. When two transport processes take place simultaneously, they interfere with each other, producing cross-diffusion effects (Soret and Dufour effects). The flux of concentration caused by temperature gradient and the flux of heat caused byconcentration gradient are known as Soret and Dufour effects respectively (De Groot and Mazur [6] and Hurle & Jakeman [7]. McDougall [8] observed that the spatiotemporal properties of convection in binary mixture show quite different trends from those of the double-diffusive systems without these cross diffusioneffects.We shall presently take into account the Soreteffect only, as the in fluence of Dufour effect is negligible $(10^{-3^{o}}\mathrm{C})$ in liquid mixtures and hence generally neglected. Dufoureffect is important in gaseous mixtures only (Schechter *et al.* [9]). Dhiman and Goyal [10] recently studied the stability of Soret driven double-diffusive convection problem for the case of rigid, impervious and thermally perfectly conducting boundary conditions using variational principle.

The major available literature onthe phenomena of double diffusive convection with or without cross diffusion effects are mainly concerned with Newtonian fluids and the medium in general has been considered to be non-porous. In recent years, non-Newtonian fluids in porous medium have attracted the attention of several scholars because of their practical and fundamental importance associated with many industrial applications. The study of non-Newtonian fluids with non-linear constitutive equations was first studied by Rivlin [11] for incompressible materials and broadened and elaborated by Ericksen [12]. There exist many different types of non-Newtonian fluids. The common liquids such as polymer solution, some organic liquids, paints and many new materials of industrial importance which exhibit both viscous and viscoelastic properties are called visco-elastic fluid. The study of viscoelastic fluids in a porous media has important applications in various branches of science and technology particularly in the field of petroleum technology for the flow of oil through porous rocks, chemical and nuclear industries, material processing, manufacturing of foods and paper and many other similar activities. The wide range of industrial and technology applications of these fluids attracted the attention of various researchers. Fredricksen [13] has given a good review of non–Newtonian fluids whereas Joseph [14] has also considered the stability of viscoelastic fluids. In recent years, considerable interest has been evinced in the study of Rivlin-Ericksen viscoelastic fluid having relevance and importance in agriculture, chemical technology and biomedical applications. Rivlin and Ericksen [15] have proposed a theoretical model for such viscoelastic fluid. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water. When a fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Rivlin-Ericksen fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$, Where μ and μ' are the viscosity and the viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. A good account of thermal and thermosolutal instabilities problems in a Rivlin-Ericksenelastico-viscous fluid in a porous medium is given by Sharma *et al.* [16, 17], Chand and Rana [18], and many others. Wang and Tan [19] also studied the stability analysis of a soret-driven double-diffusive convection of Maxwell fluid in a porous medium using linear and non-linear stability analysis.

Banerjee *et al.* [20] formulated a new scheme for combining the governing equations and boundary conditions of Veronisthermosolutal configuration [2], which lead to a circle theorem prescribing the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries. Banerjee *et al.* [21] established a characterization theorem for thermohaline convection of Veronis type that disallow the existence of oscillatory motions of neutralor unstable in an initially bottom heavy configuration for the certain parameter regime. Gupta *et al.* [22] derived a sufficient conditions for the validity of the principle of exchange of stabilities (PES) in Veronis thermosolutal configuration [2]. Mohan [23] extend the results of Banerjee *et al.* [20, 21] by including the effect of Cross diffusion coefficients for the case of Newtonian fluid. However no such result to our knowledge exist for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid in the presence of Cross diffusion effect.

Copious literature is available on the thermosolutal instability of a viscoelastic fluidsaturated porous layer with Soret effect. In most of the studies that related to the convection in viscoelastic fluids referred above, it has been noticed that either the inuflence of Cross diffusion effect are neglected on the basis first that they are of smaller order of magnitude in liquid mixture and their effect on the stability criteria is negligible. The cross diffusion effects, however small they may be, are present in double diffusive convections and are equally important and they have a large influence on hydrodynamic stability relative to their contributions to the buoyancy of the fluid. Further, the Soret effect introduces a coupling between concentration transport and the local temperature gradient in the mixture, and this causes a concentration gradient to develop when a temperature gradient is imposed on the fluid layer. Therefore one cannot ignore the role of Soret effect chiefly in liquids.

Keeping in mind the importance of cross diffusion effect and the growing importance of Visco-elastic fluids in modern technology, the investigations of such fluids are desirable. Hence, the aim of present paper is to extend the results of Banerjee *et al.* [20, 21] to the thermosolutal convection in Rivlin-Ericksen viscoelastic fluid in porous media in the presence of Soreteffect and to derive a semi-circle theorem that prescribes upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude in the problem for any arbitrary combinations of dynamically free and rigid boundaries. However, the governing equations in the present configuration are not amenable to the analysis followed by Banerjee *et al.* [20] on account of some mathematical complexities arises due to introduction of Soret effect term. Therefore, an attempt is made mathematicallytotackle the problems, the governing equations of the present problem are transformed to mathematically tractable forms using some linear relations and derived the required results as per the scheme of Banerjee *et al.* [20, 21].

2. MATHEMATICAL FORMULATION AND ANALYSIS

An infinite, horizontal porous layer of in compressible Rivlin-Ericksen viscoelastic fluid is statically confined between two parallel horizontal planes $z=0$ and $z=d$ maintained at uniform temperature T_0 and T_1 $(T_0 > T_1)$ and solute concentrations C_0 and C_1 $(C_0 > C_1)$ at the lower and upper boundaries respectively. A cartesian frame of reference is chosen with origin in the lower boundary $z=0$ and the z-axis vertically upward. The layer of fluid mixture is heated and salted from below in the the force field of gravity $\vec{g}$ (0, 0, -g). The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and the permeability k_1. The Boussinesq [24] approximation is assumed to be hold, which states that the variation in density is negligible everywhere except in its association with the external force.

Under these assumptions, the basic equations (*i.e.* the equations of continuity, motion, heat conduction, mass diffusion and the equation of state) that govern the thermosolutal Rivlin-Ericksen viscoelastic fluid in the presence of Soret effect (Following Rivlin and Ericksen[15]; Sharma *et al* [17]; De Groot and Mazur[6] and McDougall [8] are given by

$$\nabla.\vec{q}=0 \tag{1}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)q\right)=-\frac{1}{\rho_0}\nabla p+g\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)q \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T \tag{3}$$

$$E'\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C=\kappa'\nabla^2 C+D_{21}\nabla^2 T \tag{4}$$

And $\rho=\rho_0\left[1-\alpha\left(T-T_0\right)+\alpha'\left(C-C_0\right)\right]$ (5)

Where $u_i=(u,v,w)$ is the Darcy velocity ; *p* is the pressure, μ is the coefficient of viscosity and $\upsilon=\frac{\mu}{\rho_0}$ is the coefficient of kinematic viscosity; *g* is gravity; D_{21} is the Soret coefficient; *T* is the temperature, *C* concentration, κ is the thermal conductivity and κ' is the solutal diffusivity, E is thermal capacity ratio; $E=\frac{(\rho c_p)_m}{(\rho c_p)_f}$,where $(\rho c_p)_f$ is the volumetric heat capacity of the fluid; E' is constant analogous to E but corresponding to solute rather than heat sand $(\rho c_p)_m=\varepsilon(\rho c_p)_f+(1-\varepsilon)(\rho c_p)_s$ is the volumetric heat capacity of the saturated medium as a whole, with the subscripts *f, s* and *m* denoting the properties of the fluid, solid, and porous matrix, respectively; α and α' are respectively thermal and concentration expansion coefficient, ρ is the density.

2.1 Basic State and its Solutions

The basic state of the system is assumed to be a quiescent state whose stability we want to examine is characterized by,

$$\vec{q}=(0,0,0),\, C=C_b(z),\quad p=p_b(z),\quad \rho=\rho_b(z) \tag{6}$$

Thus the basic state solution on the basis of the basic state is obtained by using equation (6) in (1)- (5) we get,

$$\vec{q} = (0,0,0),$$
$$T_b = T_0 - \beta z, C_b = C_0 - \beta' z,$$
$$\rho_b = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)] = \rho_0[1 + \alpha\beta z - \alpha'\beta' z]$$
$$p = p_0 - \rho_0 g\left[z + \frac{1}{2}(\alpha\beta - \alpha'\beta')z^2\right] \tag{7}$$

where, $\beta = \frac{T_0 - T_1}{d} > 0$ and $\beta' = \frac{C_0 - C_1}{d} > 0$ are maintained uniform temperature and concentration gradients respectively and p_0 the is the value of p at z = 0.

2.2 The Perturbation solutions

The initial state described by (7) be slightly perturbed so that perturbed state is given by

$$\vec{q} = (0 + u, 0 + v, 0 + w), \quad T = T_b + \theta$$
$$C = C_b + \varphi, \quad \rho = \rho_0\left[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0) - \alpha\theta + \alpha'\varphi\right] \tag{8}$$
$$p = p_b + \delta p$$

Where $(u, v, w), \delta\rho, \delta p, \theta, \phi$ denote respectively the perturbations in velocity (0, 0, 0), density, pressure p, temperature T, solute concentration C. The change in density $\delta\rho$ caused by perturbation θ and ϕ in temperature and solute concentration is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\phi) \tag{9}$$

Then the linearized perturbation equations of continuity, momentum, heat conduction and mass diffusion (1)-(5) using equation (9) give

$$\nabla.\vec{q} = 0 \tag{10}$$
$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla\delta p) - \vec{g}(\alpha\theta - \alpha'\phi) - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\vec{q} \tag{11}$$
$$E\frac{\partial\theta}{\partial t} - \beta w = \kappa\nabla^2\theta \tag{12}$$
$$E'\frac{\partial\phi}{\partial t} - \beta' w = \kappa'\nabla^2\phi + D_{21}\nabla^2\theta \tag{13}$$

3. NORMAL MODE AND STABILITY ANALYSIS

Analyze an arbitrary perturbation into a complete set of normal modes and assume that the perturbed quantities are of the form

$$[w, \theta, \phi] = [W(z), \Theta(z), \Gamma(z)]\exp[i(k_x x + k_y y) + nt] \tag{14}$$

Where $a = \sqrt{k_x^2 + k_y^2}$ is the resultant wave member of the perturbation, k_x and k_y are wave numbers along x and y directions respectively and *n* is the time constant (which is complex in general). Using equation (14), the linearized perturbation equations (10) – (13) in the non-dimensional form becomes

$$\left(\frac{\sigma}{\varepsilon} + \frac{1 + F\sigma}{P_l}\right)(D^2 - a^2)W = -Ra^2\Theta + R'a^2\Gamma \tag{15}$$
$$(D^2 - a^2 - EP_r\sigma)\Theta = -W \tag{16}$$
$$[\tau(D^2 - a^2) - E'P_r\sigma]\Gamma + S_T(D^2 - a^2)\Theta = -W, \tag{17}$$

Where $P_r = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number; $S_c = \frac{P_r}{\tau}$ is the Schamidt number; $P_l = \frac{k_1}{d^2}$ is permeability parameter; $F = \frac{\upsilon'}{d^2}$ is the viscoelastic parameter; $\tau = \frac{\kappa'}{\kappa}$ is the Lewis number; $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the thermal Rayleigh number; $R' = \frac{g\alpha'\beta' d^4}{\nu\kappa}$ is the solutal Rayleigh number; $S_T = \frac{D_{21}\beta}{\kappa\beta'}$ is the soret parameter. We have put the coordinates x, y, z in the new unit of length d and D = d/dz. Also we have used $a_* = kd;\ D_* = dD;\ \ \sigma_* = \frac{nd^2}{\upsilon};$

$$W = W_*;\ \ \Theta = \frac{\beta d^2}{\kappa}\Theta_*;\ \ \Gamma = \frac{\beta' d^2}{\kappa}\Gamma_*; \tag{18}$$

In the resulting equations omitting the asterisks for simplicity.

The appropriate boundary conditions with respect to which equations (15)-(17), must possess a solution are

$$W = 0 = \Theta = \Gamma = DW \quad at \ z = 0, z = 1 \tag{19}$$

(both the boundaries rigid)

$$or \qquad W = 0 = \Theta = \Gamma = D^2W \quad at \ z = 0, z = 1 \tag{20}$$

(both the boundaries dynamically free)

$$\begin{pmatrix} or \ \ W = 0 = \Theta = \Gamma = D^2W \quad at \quad z = 0, \qquad z = 1 \\ and \quad or \ \ W = 0 = \Theta = \Gamma = DW \quad at \quad z = 1, \qquad z = 0 \end{pmatrix} \tag{21}$$

(one dynamically free and other is rigid)

It may further be noted that the system of equations (15)-(17) together with one of the boundary conditions (19)-(21) constitutes an *eigenvalue problem* for σ that govern Soret driven Double diffusive convection in Rivlin- Ericksen Viscoelastic fluid of Veronis type in a porous medium for any combination of dynamically free and rigid boundaries. we wish to characterize σ_i when $\sigma_r \geq 0$

Remarks

(i) A given state of the system is *stable*, *neutral* or *unstable* according as; $\sigma_r < 0$, $\sigma_r = 0$ or $\sigma_r > 0$ for all wave numbers a^2 (where σ_r and σ_i are the real and imaginary parts of σ)

(ii) The system of equations (15)-(17) together with one of the boundary conditions (19)-(21) when $S_T = 0$ yields the non dimensional linear perturbation equations governing Veronis Type Double diffusive *convection in*Rivlin- Ericksen Viscoelastic fluid in a porous medium for any combination of dynamically free and rigid boundaries..

Theorem 1: If $(\sigma, W, \Theta, \Gamma)$), $\sigma = \sigma_r + i\sigma_i, \sigma_r \geq 0\ \sigma_i \neq 0$ is a non-trivial solution of equation (15)-(17) together with one of the boundary conditions (19)-(21) with $R_T > 0$, $R_S > 0$ and $F > 0, P_r > 0, P_l > 0$, then

$$|\sigma| < \frac{1}{EP_r} \frac{R_T}{\left\{\frac{1}{P_l} + \frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right)\right\}} \sqrt{J^2 - 1}$$

Where $J = \dfrac{R_T}{\left\{\frac{4\pi^2}{P_l} + \frac{1}{E'S_c}\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right)\frac{27\pi^4}{4}\right\}}$

Proof: Using linear transformations

$$W = \hat{W}, \Theta = \hat{\Theta}, \ \hat{\Gamma} = A\Theta + B\Gamma \tag{22}$$

Equations (15)-(17) upon using the transformation (22), assumes the following form:

$$\left(\frac{\sigma}{\varepsilon}+\frac{1+F\sigma}{P_l}\right)\left(D^2-a^2\right)W=-R_T a^2\Theta+R_S a^2\Gamma \tag{23}$$

$$\left(D^2-a^2-EP_r\sigma\right)\Theta=-W \tag{24}$$

$$\left[\tau(D^2-a^2)-E'P_r\sigma\right]\Gamma=-W \tag{25}$$

together with

$$W=0=\Theta=\Gamma=DW \quad at \;\; z=0, z=1 \tag{26}$$

(both the boundaries rigid)

$$or \quad W=0=\Theta=\Gamma=D^2W \quad at \;\; z=0, z=1 \tag{27}$$

(both the boundaries dynamically free)

$$\left(\begin{array}{l} or \quad W=0=\Theta=\Gamma=D^2W \quad at \quad z=0, \qquad z=1 \\ and \quad or \quad W=0=\Theta=\Gamma=DW \quad at \quad z=1, \qquad z=0 \end{array}\right] \tag{28}$$

(one dynamically free and other is rigid)

Where $R_T=R+\left(\frac{ES_T}{\tau E-E'}\right)R'$ and $R_S=\left(\frac{\tau E-(1-S_T)E'}{\tau E-E'}\right)R'$ (29)

and the sign of cap has been ignored for the sake of convenience in writing. It is important to note here that in the absence of Soret effect $(S_T=0)$, we have $R_T=R$ and $R_S=R'$ hence, in the absence of Soret effect the equations (23)-(25) yield Doublediffusive *convection in* Rivlin- Ericksen Viscoelastic fluid of Veronis type in a porous medium.

Multiplying equation (23) by W *(complex conjugate of W) and integrating the resulting equation over the vertical range of z, we get

$$\left(\frac{\sigma}{\varepsilon}+\frac{1+F\sigma}{P_l}\right)\int_0^1 W^*\left(D^2-a^2\right)W\,dz=-R_T a^2\int_0^1 W^*\Theta dz+R_s a^2\int_0^1 W^*\Gamma dz \tag{30}$$

Making use of (24) and (25) and the fact that $W(0)=0=W(1)$, we have

$$-R_T a^2\int_0^1 W^*\Theta dz=R_T a^2\int_0^1 \Theta\left(D^2-a^2-EP_r\sigma^*\right)\Theta^* dz \tag{31}$$

$$R_S a^2\int_0^1 W^*\Gamma dz=-R_S a^2\int_0^1 \Gamma\left[\tau(D^2-a^2)-E'P_r\sigma^*\right]\Gamma^* dz \tag{32}$$

Combining Equations (30)-(32), we get

$$\left(\frac{\sigma}{\varepsilon}+\frac{1+F\sigma}{P_l}\right)\int_0^1 W^*\left(D^2-a^2\right)Wdz=R_T a^2\int_0^1 \Theta\left(D^2-a^2-EP_r\sigma^*\right)\Theta^* dz-R_S a^2\int_0^1 \Gamma\left[\tau(D^2-a^2)-E'P_r\sigma^*\right]\Gamma^* dz \tag{33}$$

Integrating by parts, the various terms of equation (33) for an appropriate number of times by using either of the boundary conditions (26)-(28), it follows that

$$\begin{aligned}&\left(\frac{\sigma}{\varepsilon}+\frac{1+F\sigma}{P_l}\right)\int_0^1\left[|DW|^2+a^2|W|^2\right]dz-R_T a^2\int_0^1\left[|D\Theta|^2+a^2|\Theta|^2\right]dz-R_T a^2 EP_r\sigma^*\int_0^1|\Theta|^2 dz\\&+R_s a^2\tau\int_0^1\left[|D\Gamma|^2+a^2|\Gamma|^2\right]dz+R_s a^2E'P_r\sigma^*\int_0^1|\Gamma|^2 dz=0\end{aligned} \tag{34}$$

Equating real and imaginary parts of equation (34) and cancelling $\sigma_i (\neq 0)$ throughout from the imaginary part, we get

$$\left[\sigma_r\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)+\frac{1}{P_l}\right]\int_0^1\left[|DW|^2+a^2|W|^2\right]dz-R_Ta^2\int_0^1\left[|D\Theta|^2+a^2|\Theta|^2\right]-R_Ta^2EP_r\sigma_r\int_0^1|\Theta|^2dz$$
$$+R_sa^2\tau\int_0^1\left[|D\Gamma|^2+a^2|\Gamma|^2\right]dz+R_sa^2E'P_r\sigma_r\int_0^1|\Gamma|^2dz=0 \tag{35}$$

$$\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\int_0^1\left\{|DW|^2+a^2|W|^2\right\}+R_Ta^2EP_r\int_0^1|\Theta|^2dz-R_Sa^2E'P_r\int_0^1|\Gamma|^2dz=0 \tag{36}$$

Multiplying equation (36) by σ_r and adding the resulting equation to the equation (35), we get

$$\left[2\sigma_r\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)+\frac{1}{P_l}\right]\int_0^1\left[|DW|^2+a^2|W|^2\right]dz-R_Ta^2\int_0^1\left[|D\Theta|^2+a^2|\Theta|^2\right]dz+R_sa^2\tau\int_0^1\left[|D\Gamma|^2+a^2|\Gamma|^2\right]dz=0 \tag{37}$$

Since W,Θ,Γ vanishes at $z=0$ and $z=1$, therefore Rayleigh-Ritz inequality (Schultz [25]) gives

$$\int_0^1|DW|^2dz\geq\pi^2\int_0^1|W|^2dz \tag{38}$$

$$\int_0^1|D\Theta|^2dz\geq\pi^2\int_0^1|\Theta|^2dz \tag{39}$$

$$\int_0^1|D\Gamma|^2dz\geq\pi^2\int_0^1|\Gamma|^2dz \tag{40}$$

Equation (36) implies

$$\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\int_0^1\left\{|DW|^2+a^2|W|^2\right\}dz\leq R_Sa^2E'P_r\int_0^1\ \int|\Gamma|^2dz \tag{41}$$

Combining equation (38) and (41), we get

$$\frac{1}{E'P_r}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)(\pi^2+a^2)\int_0^1|W|^2dz\leq R_Sa^2\int_0^1|\Gamma|^2dz \tag{42}$$

$$\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)(\pi^2+a^2)\int_0^1|W|^2dz\leq R_Sa^2E'P_r\int_0^1|\Gamma|^2dz \tag{43}$$

Also upon using inequality (40), we have

$$R_sa^2\int_0^1\left[|D\Gamma|^2+a^2|\Gamma|^2\right]dz\geq(\pi^2+a^2)R_Sa^2\int_0^1\ \int|\Gamma|^2dz \tag{44}$$

Combining (42) and (44), we get

$$R_sa^2\int_0^1\left[|D\Gamma|^2+a^2|\Gamma|^2\right]dz\geq\frac{1}{E'P_r}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)(\pi^2+a^2)^2\int_0^1|W|^2dz \tag{45}$$

Multiplying equation (24) by its complex conjugate and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1\left[\left(D^2-a^2-EP_r\sigma\right)\Theta\left(D^2-a^2-EP_r\sigma^*\right)\Theta^*\right]dz=\int_0^1WW^*dz \tag{46}$$

Integrating by parts, the equation (46) for an appropriate number of times by using either of the boundary conditions (26)-(28), it follows that

$$\int_0^1 \left|\left(D^2-a^2\right)\Theta\right|^2 dz + 2EP_r\sigma_r\int_0^1\left(\left|D\Theta\right|^2+a^2\left|\Theta\right|^2\right)dz + E^2\left|\sigma\right|^2 P_r^2\int_0^1\left|\Theta\right|^2 dz = \int_0^1 \left|W\right|^2 dz \quad (47)$$

Since $\sigma_r \geq 0$, it follow from inequality (47) that

$$\int_0^1 \left|W\right|^2 dz \geq \int_0^1 \left|\left(D^2-a^2\right)\Theta\right|^2 dz + E^2\left|\sigma\right|^2 P_r^2\int_0^1\left|\Theta\right|^2 dz \quad (48)$$

And $\int_0^1 \left|W\right|^2 dz \geq \int_0^1 \left|\left(D^2-a^2\right)\Theta\right|^2 dz$ (49)

Further, utilizing the Schwartz inequality (Fitts [26]), we have

$$\left(\int_0^1 \left|\Theta\right|^2 dz\right)^{\frac{1}{2}}\left(\int_0^1 \left|D^2\Theta\right|^2 dz\right)^{\frac{1}{2}} \geq \left|-\int_0^1 \Theta^* D^2\Theta dz\right| = \int_0^1 \left|D\Theta\right|^2 dz \geq \pi^2\int_0^1\left|\Theta\right|^2 dz \quad (50)$$

Consequently,

$$\int_0^1 \left|D^2\Theta\right|^2 dz \geq \pi^4\int_0^1 \left|\Theta\right|^2 dz \quad (51)$$

Thus we have

$$\int_0^1 \left|(D^2-a^2)\Theta\right|^2 dz = \int_0^1 (\left|D^2\Theta\right|^2 + 2a^2\left|D\Theta\right|^2 + a^4\left|\Theta\right|^2)dz$$

$$\geq (\pi^2+a^2)^2\int_0^1\left|\Theta\right|^2 dz \quad (52)$$

Combining inequality (48) and (52), we obtain

$$\int_0^1 \left|W\right|^2 dz \geq \left((\pi^2+a^2)^2 + E^2P_r^2\left|\sigma\right|^2\right)\int_0^1\left|\Theta\right|^2 dz \quad (53)$$

$$\int_0^1 \left|W\right|^2 dz \geq (\pi^2+a^2)^2\left[1+\frac{E^2P_r^2\left|\sigma\right|^2}{(\pi^2+a^2)^2}\right]\int_0^1\left|\Theta\right|^2 dz \quad (54)$$

$$\int_0^1 \left|W\right|^2 dz = \left(\int_0^1 \left|W\right|^2 dz\right)^{\frac{1}{2}}\left(\int_0^1 \left|W\right|^2 dz\right)^{\frac{1}{2}} \quad (55)$$

Making use of equations (49) and (54), the equation (55) yields

$$\int_0^1 \left|W\right|^2 dz \geq (\pi^2+a^2)\left[1+\frac{E^2P_r^2\left|\sigma\right|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}\left(\int_0^1\left|\Theta\right|^2 dz\right)^{\frac{1}{2}}\left(\int_0^1\left|\left(D^2-a^2\right)\Theta\right|^2 dz\right)^{\frac{1}{2}}$$

$$\geq (\pi^2+a^2)\left[1+\frac{E^2P_r^2\left|\sigma\right|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}\left|-\int_0^1\Theta^*\left(D^2-a^2\right)\Theta dz\right|$$

$$= (\pi^2+a^2)\left[1+\frac{E^2P_r^2\left|\sigma\right|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}\int_0^1\left(\left|D\Theta\right|^2+a^2\left|\Theta\right|^2\right)dz \quad (56)$$

Using inequality (38), (45), (56) in equation (37) we obtain

$$\left\{\frac{1}{P_l}(\pi^2+a^2)+\frac{\tau}{E'P_r}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)(\pi^2+a^2)^2\right\}\int_0^1|W|^2\,dz<\frac{R_T a^2}{(\pi^2+a^2)}\left[1+\frac{E^2P_r^2|\sigma|^2}{(\pi^2+a^2)^2}\right]^{\frac{-1}{2}}\int_0^1|W|^2\,dz \tag{57}$$

Or

$$\left\{\frac{1}{P_l}\frac{(\pi^2+a^2)^2}{a^2}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{(\pi^2+a^2)^3}{a^2}\right\}\int_0^1|W|^2dz<R_T\left[1+\frac{E^2P_r^2|\sigma|^2}{(\pi^2+a^2)^2}\right]^{\frac{-1}{2}}\int_0^1|W|^2dz \tag{58}$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ is $4\pi^2$ (for $a^2=\pi^2$) and the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$ at $a^2=\frac{\pi^2}{2}$, therefore it follows from inequality (58) that

$$\left\{\frac{4\pi^2}{P_l}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{27\pi^4}{4}\right\}\left[1+\frac{E^2P_r^2|\sigma|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}<R_T$$

Or

$$\left[1+\frac{E^2P_r^2|\sigma|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}<\frac{R_T}{\left\{\frac{4\pi^2}{P_l}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{27\pi^4}{4}\right\}}=J \tag{59}$$

$$|\sigma|<\frac{(\pi^2+a^2)}{EP_r}\sqrt{J^2-1} \tag{60}$$

Further $\left[1+\frac{E^2p_1^2|\sigma|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}>1,\frac{(\pi^2+a^2)}{a^2}>1.$ it follows from equation (57) that

$$(\pi^2+a^2)\left\{\frac{1}{P_l}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{(\pi^2+a^2)^2}{a^2}\right\}<R_T \tag{61}$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ is $4\pi^2$ therefore it follows from inequality (61) that

$$(\pi^2+a^2)<\frac{R_T}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}} \tag{62}$$

Combining inequalities (60) and (62), we get

$$|\sigma|<\frac{1}{EP_r}\frac{R_T}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J^2-1} \tag{63}$$

Which complete the proof of the theorem.

Theorem 1, from the physical point of view of hydrodynamic stability theory , may be stated as: the complex growth rate $\sigma=\sigma_r+i\sigma_i$ of an arbitrary oscillatory perturbation of neutral or growing amplitude of thermosolutal convection in a porous layer of Rivlin- Ericksenviscoelastic fluid of Veronis type in the presence of Soret effect lies inside a semi circlein the right half of the $\sigma_r\sigma_i$-plane whose centre is origin and whose radius is

$$\frac{1}{EP_r}\frac{R_T}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J^2-1}.$$

Corollary 1: If ($\sigma, W, \Theta, \Gamma$), $\sigma=\sigma_r+i\sigma_i$, $\sigma_i\neq 0$ is a non trivial solution of equation (15)-(17) together with one of the boundary conditions (19)-(21) with $R_T>0$, $R_S>0$ and $J\leq 1$, then $\sigma_r<0$

Proof: Follows from Theorem 1.

Corollary 1 yields that oscillatory motion of growing amplitude are not allowed in thermosolutal convection in a porous layer of Rivlin- Ericksenviscoelastic fluid of Veronis type in the presence of Soret effect if $J\leq 1$

Corollary 2: In the absence of Soret effect ($S_T=0$), the complex growth rate $\sigma=\sigma_r+i\sigma_i$ of an arbitrary oscillatory perturbation of neutral or growing amplitude in thermosolutal convection in a porous layer of Rivlin-Ericksenviscoelastic fluid of Veronis type must lies inside a semi circle in the right half of the $\sigma_r\sigma_i$-plane whose centre is origin and whose radius is $\frac{1}{EP_r}\frac{R}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J'^2-1}$.

Where $J'=\frac{R}{\left\{\frac{4\pi^2}{P_l}+\frac{1}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\frac{27\pi^4}{4}\right\}}$

Proof: In the absence of Soret effect ($S_T=0$), we have $R_T=R$ hence on replacing R_T by R in Theorem 1, we obtain the required result.

CONCLUSIONS

In the present analysis firstly, the eigenvalue problem governing the Soret-driven doublediffusive convection in a porous layer of Rivlin- Ericksenviscoelastic fluid of Veronis type has been transformed into an eigenvalue problem (using some linear relations) which behaves nicely as per the scheme of Banerjee *et al.* [20,21] for the treatment of the problem. In the second part, using linear stability theory based upon normal mode technique, our analysis of the governing equations of the problem for any combination of free and rigid boundaries leads to the following important conclusions.

(i) The complex growth rate $\sigma=\sigma_r+i\sigma_i$ of an arbitrary oscillatory perturbation of neutral or growing amplitude($\sigma_r\geq 0$) in thermosolutal convection in a porous layer of Rivlin- Ericksenviscoelastic fluid of Veronis type in the presence of Soret effectmust lies inside a semi circle in the right half of the $\sigma_r\sigma_i$-plane whose centre is at origin and whose radius is $\frac{1}{EP_r}\frac{R_T}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J^2-1}$.

(ii) The oscillatory motion of growing amplitude are not allowed in thermosolutal convection in a porous layer of Rivlin- Ericksenviscoelastic fluid of Veronis type in the presence of Soret effect if $J\leq 1$

(iii) In the absence of Soret effect ($S_T=0$), the complex growth rate $\sigma=\sigma_r+i\sigma_i$ of an arbitrary oscillatory perturbation of neutral or growing amplitude in thermosolutal convection in a porous layer of Rivlin-Ericksenviscoelastic fluid of Veronis typemust lies inside a semi circle in the right half of the $\sigma_r\sigma_i$-plane whose centre is origin and whose radius is $\frac{1}{EP_r}\frac{R}{\left\{\frac{1}{P_l}+\frac{4\pi^2}{E'S_c}\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\right\}}\sqrt{J'^2-1}$, where $J' = \dfrac{R}{\left\{\dfrac{4\pi^2}{P_l} + \dfrac{1}{E'S_c}\left(\dfrac{1}{\varepsilon} + \dfrac{F}{P_l}\right)\dfrac{27\pi^4}{4}\right\}}$.

Further, utilizing the remarks contained in Banerjee *et al.* [20,21] Gupta *et al.* [22] for obtaining the eigenvalue equations for the present problem but of Stern type and then following the present analysis, one can easily obtain the analogous results for case of Stern type configuration.

REFERENCES

1. M. E. Stern, Tellus, 12, 172 (1960).
2. G.Veronis: On finite amplitude instability in the thermohaline convection, J. Marine. Res., 23(1965), pp. 1-170.
3. D.A Nield : Onset of thermohaline convection in a porous medium. Water Resour. Res. (1968) 4: 553–560.
4. D.A Nield, A. Bejan.: Convection in Porous Media, 3rd edn. Springer, New York, (2006)
5. M.C Cr-Mojtabi, B. Elhajjah, A. Mohtabi, Analytical and numerical stability analysis of Soret-driven convection in a horizontal porous layer. Phys. Fluids (2007), 19, 124104.
6. S. R. de Groot and P. Mazur, ‘‘Nonequilibrium Thermodynamics,’’ North-Holland, Amsterdam, (1962). p. 273
7. Hurle DTJ and Jakeman E: Soret-driven thermosolutal convection. Journal of. Fluid Mechanics, (1971) 47(4) pp 667-687.
8. McDougall TJ: Double-diffusive convection caused by coupled molecular diffusion. Journal of Fluid Mechanics, (1983), 126 pp 379.
9. R.S Schechter, Prigogine I and JR Hamm: Thermal Diffusion and Convective Stability. Physics of Fluids, (1972) 15 pp 379.
10. J.S Dhiman JS and M.R Goyal: Stability Analysis of Soret driven Double diffusive Steady Convection Problems for Rigid and Impervious Boundaries, Int. J. of Appl. Math. and Mech. (2013) 2(1)pp 11-20.
11. RS Rivlin:. The hydrodynamics of non-Newtonian fluids, I.Proceedings of Royal Society (London) A193, (1948) pp. 260 -281.
12. J.LEricksen: Characterstic surfaces of equations of motion of non-Newtonian fluids. Zeitschrift fur angewandte Mathematik und Physik (ZAMP) (1953) 4, pp. 260-267.
13. A.G Fredricksen : Principles and applications of Rheology. Prentice-Hall Inc., New Jersey.(1964)
14. D.D Joseph, Stability of Fluid Motions, Vol. 2, Springer-Verlag, Berlin, 1976.
15. R.SRivlin, and J.L Ericksen,: Stress deformation relations for isotropic materials, J. Rat. Mech. Anal. 4 (1955), 323.
16. R.C Sharma and P. Kumar: Effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid, Zeitschrift fur Naturforschung 51a (1996), 821-824.
17. R.C Sharma, Sunil and M. Pal: Thermosolutal convection in Rivlin-Ericksen rotating fluid in porous medium in hydromagnetics, indian J. pure appl. Math., 32(1) (2001), pp. 143-156.
18. R. Chand and G. C. Rana, “Dufour and soret effects on the thermosolutal instability of rivlin-ericksen elastico-viscous fluid in porous medium,” *Zeitschrift fur Naturforschung A*, vol.67, no. 12, pp. 685–691, 2012.
19. S.W. Wang and W.C. Tan: Stability analysis of Soret-driven double-diffusive convection of Maxwell fluid in a porous medium. Int. J. Heat Fluid Flow (2011) 32, 88–94.
20. M. B. Banerjee, D. C. Katoch, G. S. Dube, and K. Banerjee, (1981) Bounds for growth rate of a perturbation in thermohaline convection, *Proc. Roy. Soc. London Ser. A* 378-301.
21. M. B. Banerjee, J. R. Gupta, and Jyoti Parkash, On theormohaline convection of the Veronis, type, *J.Math. Anal. Appl.* (1993), 179. 327.
22. J. R. Gupta, S. K. Sood, and U. D. Bhardwaj: On characterization of nonoscillatory motions in rotatory hydromagnetic thermohaline convection, *Indian J. Pure Appl. Math.* (1986)17, 100.
23. H. Mohan: Bound for complex Growth Rate in Thermosolutal convection with cross-diffusions. Appl. App.Math. (2010), 05(10) pp1428-1441.
24. JE Boussinesq: ‘Theorie analytique de la chaleur’, Paris: Gauthier Villars, 2 (1903), 172.
25. M. H. Schultz, ‘‘Sapline Analysis,’’ Prentice Hall, Englewood Cliffs, New Jersey, 1973.
26. D. D. Fitts, ‘‘Non-equilibrium Thermodynamics,’’ McGraw] Hill, New York, 1962.

Source of support: Nil, Conflict of interest: None Declared