RELATIONS ON GENERALIZED FUZZY SOFT SETS

MANASH JYOTI BORAH*

Department of Mathematics, Bahona College, Jorhat, Assam-785101, India.

(Received On: 05-12-15; Revised & Accepted On: 30-12-15)

ABSTRACT

In this paper, some properties of generalized fuzzy soft equivalence relation have been discussed and finally applications of generalized fuzzy soft relation in decision making problem have been shown. Also we present the definition of union and intersection between two generalized fuzzy soft relations.

Keywords: Fuzzy soft sets, Generalized fuzzy soft set, union, intersection, generalized fuzzy soft relations.

2010 Mathematics Subject Classification: 03E72; 03B52.

1. INTRODUCTION


In this paper we give the proof of some propositions of soft relation and support them with examples. We also discussed irreflexive, equivalence relation, complement, union, intersection, Identity and some algebraic properties of fuzzy soft relation.

2. PRELIMINARIES

Definition 2.1[7]: A pair \((F, E)\) is called a Soft set over \(U\) if and only if \(F\) is a mapping of \(E\) into the set of all subsets of the set \(U\).

In other words, the soft set is a parameterized family of subsets of the set \(U\). Every set \(F(\varepsilon)\), \(\varepsilon \in E\), from this family may be considered as the set of \(\varepsilon\)-approximate elements of the soft set.

Definition 2.2[5]: A pair \((F, A)\) is called a fuzzy soft set over \(U\) where \(F: A \rightarrow \mathcal{P}(U)\) is a mapping from \(A\) into \(\mathcal{P}(U)\).

Definition 2.3 [8]: A binary operation \(\ast: [0,1] \times [0,1] \rightarrow [0,1]\) is continuous \(t\)-norm if \(\ast\) satisfies the following conditions.

(i) \(\ast\) is commutative and associative
(ii) \(\ast\) is continuous
(iii) \(a \ast 1 = a \ \forall a \in [0,1]\)
(iv) \(a \ast b \leq c \ast d\) whenever \(a \leq c, b \leq d\) and \(a,b,c,d \in [0,1]\)

An example of continuous \(t\)-norm is \(a \ast b = ab\).
Definition 2.4 [8]: A binary operation $\diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative and associative
(ii) $\diamond$ is continuous
(iii) $a \diamond 0 = a \ \forall a \in [0, 1]$
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0, 1]$

An example of continuous t-conorm is $a * b = a + b - ab$.

Definition 2.5 [5]: For two fuzzy soft sets $(F, A)$ and $(G, B)$ in a fuzzy soft class $(U, E)$, we say that $(F, A)$ is a fuzzy soft subset of $(G, B)$ if

(i) $A \subseteq B$
(ii) For all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \sqsubseteq (G, B)$

Definition 2.6 [5]: Union of two fuzzy soft sets $(F, A)$ and $(G, B)$ in a fuzzy soft class $(U, E)$ is a fuzzy soft set $(H, C)$ where $C = A \cup B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$

and is written as $(F, A) \cup (G, B) = (H, C)$.

Definition 2.7 [5]: The complement of a fuzzy soft set $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \rightarrow \mathcal{P}(U)$ is a mapping given by $F^c(\sigma) = (F(\neg \sigma))^c$ for all $\sigma \in \neg A$

Definition 2.8 [6]: Let $F_\mu$ be a GFSS over $(U, E)$. Then the complement of $F_\mu$, denoted by $F_\mu^c$ and is defined by $F_\mu^c = G_\delta$, where $\delta(\varepsilon) = \mu(\varepsilon^c)$ and $G(\varepsilon) = F^c(\varepsilon), \forall \varepsilon \in E$.

Definition 2.9 [3]: Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of parameters. Let $A \subseteq E$ and $F : A \rightarrow I^U$ and $\lambda$ be a fuzzy subset of $A$ i.e. $\lambda : A \rightarrow [0, 1]$, where $I^U$ is the collection of all fuzzy subsets of $U$. Let $F : A \rightarrow I^U \times I$ be a function defined as follows:

$F_\lambda^A(\varepsilon) = (F(e), \lambda(e))$, where $F(e) \in I^U$. Then $F_\lambda^A$ is called a generalized fuzzy soft set (GFSS) over $(U, E)$.

Here for each parameter $e_i, F_\lambda^A(e_i)$ indicates not only degree of belongingness of the elements of $U$ in $F(e_i)$ but also degree of preference of such belongingness which is represented by $\lambda(e_i)$.

Definition 2.10 [4]: The union of two GFSS $F_\lambda^A$ and $G_\mu^B$ over $(U, E)$ is denoted by $F_\lambda^A \cup G_\mu^B$ and defined by GFSS $H_\delta^{A \cup B} : A \cup B \rightarrow I^U \times I$ such that for each $e \in A \cup B$ and $A, B \subseteq E$

$H_\delta^{A \cup B} (e) = \begin{cases} (F(e), \lambda(e)), & \text{if } e \in A - B \\ (G(e), \mu(e)), & \text{if } e \in B - A \\ (F(e) \cup G(e), \lambda(e) * \mu(e)), & \text{if } e \in A \cap B \end{cases}$

Definition 2.11 [4]: The intersection of two GFSS $F_\lambda^A$ and $G_\mu^B$ over $(U, E)$ is denoted by $F_\lambda^A \cap G_\mu^B$ and defined by GFSS $K_\delta^{A \cap B} : A \cap B \rightarrow I^U \times I$ such that for each $e \in A \cap B$ and $A, B \subseteq E$

$K_\delta^{A \cap B} (e) = (K(e), \delta(e))$.

Whence $K(e) = F(e)^* G(e), \delta(e) = \lambda(e) \circ \mu(e)$. In order to avoid degenerate case, we assume here that $A \cap B \neq \varnothing$.

3. MAIN RESULTS

Definition 3.1: Let $F_\lambda^A, G_\mu^B$ be a GFSS over $(U, E)$. Then generalized fuzzy soft relation (in short GFSR) $R$ from $F_\lambda^A$ to $G_\mu^B$ is a function $R : A \times B \rightarrow I^U \times I$ defined by $R(e_a, e_b) \subseteq F_2^A(e_a) \cap G_2^B(e_b), \forall (e_a, e_b) \in A \times B$. 

© 2015, IJMA. All Rights Reserved
Definition 3.2: If \( R \) is a GFSR from \( F^A_{\lambda} \) to \( G^B_{\mu} \) then \( R^{-1} \) is defined as \( R^{-1}(e_a, e_b) = R(e_b, e_a), \forall (e_a, e_b) \in A \times B \)

Definition 3.3: Let \( R \) and \( S \) be two generalized fuzzy soft relations from \( F^A_{\lambda} \) to \( G^B_{\mu} \) and \( G^B_{\mu} \) to \( H^C_{\delta} \) respectively. Then the composition \( \circ \) of \( R \) and \( S \) is defined by 
\[
(R \circ S)(e_a, e_c) = R(e_a, e_b) \cap S(e_b, e_c)
\]

Definition 3.4: A generalized fuzzy soft relation \( R \) on \( F^A_{\lambda} \) is said to be generalized fuzzy soft reflexive relation if
\[
R(e_a, e_a) \subseteq R(e_a, e_a), \forall e_a, e_b \in A
\]

Definition 3.5: A generalized fuzzy soft relation \( R \) on \( F^A_{\lambda} \) is said to be generalized fuzzy soft symmetric relation if
\[
R(e_a, e_b) = R(e_b, e_a), \forall e_a, e_b \in A
\]

Definition 3.6: A generalized fuzzy soft relation \( R \) on \( F^A_{\lambda} \) is said to be generalized fuzzy soft transitive relation if
\[
R \circ R \subseteq R
\]

Definition 3.7: A generalized fuzzy soft relation \( R \) on \( F^A_{\lambda} \) is said to be generalized fuzzy soft irreflexive relation if
\[
R(e_a, e_b) \not\subseteq R(e_a, e_a), \forall e_a, e_b \in A
\]

Definition 3.8: A generalized fuzzy soft relation \( R \) on \( F^A_{\lambda} \) is said to be generalized fuzzy soft equivalence relation if it is reflexive, symmetric and transitive.

Proposition 3.1: If a generalized fuzzy soft relation \( R \) is reflexive then \( R^{-1} \) is also reflexive.

Proof: \( R^{-1}(e_a, e_b) = R(e_b, e_a) \subseteq R(e_a, e_a) = R^{-1}(e_a, e_a) \)
And \( R^{-1}(e_b, e_a) = R(e_a, e_b) \subseteq R(e_a, e_a) = R^{-1}(e_a, e_a) \)
Hence \( R^{-1} \) is reflexive.

Proposition 3.2: A generalized fuzzy soft relation \( R \) is Symmetric if and only if \( R = R^{-1} \).

Proof: Let \( R \) is symmetric, then 
\[
R^{-1}(e_a, e_b) = R(e_b, e_a) = R(e_a, e_b)
\]
So, \( R = R^{-1} \)
Conversely, let \( R = R^{-1} \) then 
\[
R(e_a, e_b) = R^{-1}(e_b, e_a) = R(e_b, e_a)
\]
So \( R \) is Symmetric.

Proposition 3.3: A generalized fuzzy soft relation \( R \) is Symmetric if and only if the \( R^{-1} \) is symmetric.

Proof: Let \( R \) is symmetric, then 
\[
R^{-1}(e_a, e_b) = R(e_b, e_a) = R(e_a, e_b) = R^{-1}(e_b, e_a)
\]
So \( R \) is Symmetric.
Conversely, let \( R^{-1} \) is symmetric. Then 
\[
R^{-1}(e_a, e_b) = (R^{-1})^{-1}(e_a, e_b) = R^{-1}(e_b, e_a) = R^{-1}(e_a, e_b) = R(e_a, e_b)
\]
So \( R \) is Symmetric.
**Proposition 3.4.** If a generalized fuzzy soft relation $R$ is transitive then $R^{-1}$ is also transitive.

**Proof:**

$R^{-1}(e_a, e_a) = R(e_b, e_a) \supseteq (R \circ R)(e_a, e_a) = R(e_b, e_a) \cap R(e_a, e_a) \cap R(e_b, e_a) = R^{-1}(e_a, e_a) \cap R^{-1}(e_a, e_a) \cap R^{-1}(e_a, e_a)$

Therefore $R^{-1} \circ R^{-1} \subseteq R^{-1}$.

**Proposition 3.5:** If a generalized fuzzy soft relation $R$ is transitive then $R \circ R$ is also transitive.

$(R \circ R)(e_a, e_a) = R(e_a, e_a) \cap R(e_a, e_a) \supseteq (R \circ R)(e_a, e_a) \cap (R \circ R)(e_a, e_a) = R(e_a, e_a) \cap (R \circ R)(e_a, e_a) = R(e_a, e_a) \cap (R \circ R)(e_a, e_a)$

Therefore $R \circ R \circ R \subseteq R \circ R$.

**Proposition 3.6:** If $R$ and $S$ are on symmetric generalized fuzzy soft relation $F_\lambda$ then $R \circ S$ is symmetric on $F_\lambda$ and then if and only if $R \circ S = S \circ R$.

**Proof:** Since $R$ and $S$ are symmetric. Therefore $R^{-1} = S^{-1} = S$.

Now $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$. So $R \circ S$ is symmetric.

Conversely, $(R \circ S)^{-1} = S^{-1} \circ R^{-1} = S \circ R = R \circ S$.

Hence $R \circ S$ is symmetric.

**Proposition 3.7:** If $R$ is symmetric and transitive generalized fuzzy soft relation then $R$ is reflexive.

**Proof:**

$R(e_a, e_a) \supseteq (R \circ R)(e_a, e_a)$, Since $R$ is transitive.

$= R(e_a, e_b) \cap R(e_a, e_b) = R(e_a, e_b) \cap R(e_b, e_b) = R(e_a, e_b)$

Similarly, $R(e_a, e_a) \supseteq R(e_b, e_a)$

Hence the proof.

**Proposition 3.8:** If $S$ be a generalized fuzzy soft equivalence relation on $F_\lambda$ then $S \circ S$ is also equivalence relation.

**Proof:** Since $S$ is a generalized fuzzy soft equivalence relation therefore $S$ is reflexive, symmetric and transitive.

Now $(S \circ S)(e_a, e_b) \subseteq S(e_a, e_a)$

$\therefore S \circ S$ is reflexive.

Again

$(S \circ S)(e_a, e_b) = S(e_a, e_b) \cap S(e_a, e_b) = S(e_a, e_b) \cap S(e_a, e_b)$ since $S$ is symmetric

$= S(e_b, e_a) \cap S(e_a, e_a) = (S \circ S)(e_a, e_a)$

$\therefore S \circ S$ is symmetric.

$(S \circ S)(e_a, e_b) \supseteq (S \circ S)(e_a, e_b) \cap (S \circ S)(e_a, e_b)$ since $S$ is symmetric

$= (S \circ S \circ S \circ S)(e_a, e_b)$

$\therefore S \circ S \circ S \circ S \subseteq S \circ S$

$\therefore S \circ S$ is transitive.

Hence $S \circ S$ is equivalent relation.

© 2015, IUMA. All Rights Reserved
Proposition 3.9: If $S$ be a generalized fuzzy soft equivalence relation on $F^A_A$ then $S^{-1}$ is also equivalence relation.

Proof:

$S^{-1}(e_a, e_b) = S(e_b, e_a) \subseteq S(e_a, e_a) = S^{-1}(e_a, e_a)$, since $S$ is reflexive

$S^{-1}(e_a, e_b) = S(e_a, e_b) \subseteq S(e_a, e_a) = S^{-1}(e_a, e_a)$

$S^{-1}$ is reflexive

$S^{-1}(e_a, e_b) = S(e_b, e_a) = S(e_a, e_b) = S^{-1}(e_b, e_a)$, since $S$ is symmetric

$S^{-1}$ is symmetric.

$S^{-1}(e_a, e_b) = S(e_a, e_b) \supseteq (S \circ S)(e_a, e_a) = S(e_b, e_c) \cap S(e_c, e_a)$

$= S(e_a, e_c) \cap S(e_c, e_a) = S^{-1}(e_a, e_c) \cap S^{-1}(e_c, e_a)$

$= (S^{-1} \circ S^{-1})(e_a, e_b)$

$S^{-1} \circ S^{-1} \subseteq S^{-1}$

$\therefore S^{-1}$ is transitive.

Thus $S^{-1}$ is equivalence relation.

Definition 3.9: Let $R$ and $S$ be two generalized fuzzy soft relations from $A$ to $B$. Then $SR$ is defined as follows

$R(e_a, e_b) \subseteq S(e_a, e_b) \iff (F_R(e_a, e_b) \leq G_S(e_a, e_b), \lambda_R(e_a, e_b) \geq \mu_S(e_a, e_b)), \forall (e_a, e_b) \in A \times B$

Definition 3.10: Let $S$ be a generalized fuzzy soft relation from $A$ to $B$. Then complement of $S$ is denoted by $S^C$ and it is defined as follows

$S^C(e_a, e_b) = (F_{S^C}(e_a, e_b), \lambda_{S^C}(e_a, e_b)), \forall (e_a, e_b) \in A \times B$

Definition 3.11: Let $R$ and $S$ be two generalized fuzzy soft relations from $A$ to $B$. Then $R \cup S, R \cap S$ are defined as follows

$(R \cup S)(e_a, e_b) = (F_R(e_a, e_b) \cup G_S(e_a, e_b), \lambda_R(e_a, e_b) \cup \mu_S(e_a, e_b)), \forall (e_a, e_b) \in A \times B$

Proposition 3.10: Let $R$ and $S$ be two elements of $GFSR(A \times B)$

(i) $R \leq R \cup S, P \leq R \cup S$

(ii) $R \geq R \cap S, P \geq R \cap S$

(iii) $R \leq S \Rightarrow R^{-1} \leq S^{-1}$

(iv) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

(v) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

Proof:

(i) We have

$(R \cup S)(e_a, e_b) = (F_R(e_a, e_b) \cup G_S(e_a, e_b), \lambda_R(e_a, e_b) \cup \mu_S(e_a, e_b))$

By definition of t-norm and t-co norm

$F_R(e_a, e_b) \cup G_S(e_a, e_b) \geq F_R(e_a, e_b)$

$\lambda_R(e_a, e_b) \cup \mu_S(e_a, e_b) \leq \lambda_R(e_a, e_b)$

Therefore, by definition of $\leq$

$(R \cup S)(e_a, e_b) \geq R(e_a, e_b)$

Hence

$R \leq R \cup S$
(ii) similar to (i)
(iii) \[ R(e_a,e_b) \leq S(e_a,e_b) \Rightarrow (F_R(e_a,e_b) \leq G_S(e_a,e_b)) \]
\[ \Rightarrow \left( \lambda^R_R(e_a,e_b) \geq \mu^S_S(e_a,e_b) \right) \]
\[ \Rightarrow R^{-1}(e_b,e_a) \leq S^{-1}(e_b,e_a) \Rightarrow R^{-1} \leq S^{-1} \]

HENCE \[ R \leq S \Rightarrow R^{-1} \leq S^{-1} \]

(iv) \[ (R \cap S)^{-1}(e_a,e_b) = (R \cap S)(e_b,e_a) = (F_R(e_b,e_a) \ast G_S(e_b,e_a)) \]
\[ = (F_R^{-1}(e_a,e_b) \ast G_S^{-1}(e_a,e_b)) \]
\[ = (R^{-1} \cap S^{-1})(e_a,e_b) \]
Hence \[ (R \cap S)^{-1} = R^{-1} \cap S^{-1} \].

(v) Similar to (iv).

**Proposition 4.11:** Let \( P, R \) and \( S \) be three elements of \( GFSR(A \times B) \)

(i) If \( R \leq S \) then \( R \circ P \leq S \circ P \)

(ii) If \( R \leq S \) then \( R \circ R \leq S \circ R \)

**Proof:**

(i) It is obvious.

(ii) \[ R(e_a,e_b) \leq S(e_a,e_b) \Rightarrow (F_R(e_a,e_b) \leq G_S(e_a,e_b)) \]
\[ \Rightarrow (R \circ P)(e_a,e_c) \leq (S \circ P)(e_a,e_c) \Rightarrow R \circ P \leq S \circ P \]

HENCE \( R \leq S \) then \( R \circ P \leq S \circ P \). (ii) Similar to (i).

**Definition 3.12:** A generalized fuzzy soft relation is said to be a generalized identity fuzzy soft relation, denoted by \( \Delta_I \),

\[ \Delta_I(e) = (F(e), I(e)) \text{ where } F(e) = \bar{1}, I(e) = 0, \forall e \in A \subseteq E \]

It is clear from our definition that the generalized fuzzy soft absolute set is also not unique in our way, it would depend upon the set of parameters under consideration.

**Proposition 3.12:**

(i) \[ \Delta_I = \Delta_I^{-1} \]

(ii) \[ R \circ \Delta_I = \Delta_I \circ R = R \]

**Proof:**

(i) It is obvious.

(ii) \[ (R \circ \Delta_I)(e_a,e_c) \]
\[ = R(e_a,e_b) \cap \Delta_I(e_b,e_c) \]
\[ = (F_R(e_a,e_b) \ast \bar{1}, \lambda_R(e_a,e_b) \ominus \bar{0}) \]
\[ = (F_R(e_a,e_b), \lambda_R(e_a,e_b)) \]
\[ = R(e_a,e_c) \]
\[ R \circ \Delta_I = R \]

Similarly we prove that \( R \circ \Delta_I = \Delta_I \circ R = R \).
4. AN APPLICATION OF GENERALIZED FUZZY SOFT RELATION IN A DECISION MAKING PROBLEM

In this section, we present an application of generalized fuzzy soft relation in a candidate’s selection of an interview.

4.1: Algorithm:

**Step-1:** Construct the generalized fuzzy soft sets for each set of parameters.

**Step-2:** Construct the generalized fuzzy soft relation.

**Step-3:** Compute $F(e) \ast \lambda(e)$

**Step-4:** Compute the total Score of each Candidate.

**Step-5:** Find the lowest Score.

Suppose $U = \{c_1, c_2, c_3, c_4, c_5\}$ be the five candidates appearing in an interview and $E = \{e_1(enterprising), e_2(confident), e_3(wiling to take risk)\}$ be the set of parameters. Suppose two experts X and Y take interview of the five candidates and they consider different set of parameters and the following generalized fuzzy soft sets are constructed accordingly,

$$F \lambda^A(e_1) = \{(C_1 / 0.3, C_2 / 0.7, C_3 / 0.5, C_4 / 0.2, C_5 / 0.1), 0.6\),
$$

$$F \lambda^A(e_2) = \{(C_1 / 0.8, C_2 / 0.3, C_3 / 0.1, C_4 / 0.9, C_5 / 0.6), 0.2\)
$$

And

$$G \mu^B(e_1) = \{(C_1 / 0.4, C_2 / 0.2, C_3 / 0.3, C_4 / 0.7, C_5 / 0.3), 0.4\),
$$

$$G \mu^B(e_3) = \{(C_1 / 0.6, C_2 / 0.4, C_3 / 0.3, C_4 / 0.4, C_5 / 0.6), 0.7\)
$$

Now, the following generalized fuzzy soft relations are constructing

$$R^A \cap B(e_1, e_1) = \{(C_1 / 0.12, C_2 / 0.14, C_3 / 0.15, C_4 / 0.14, C_5 / 0.03), 0.76\)
$$

$$R^A \cap B(e_1, e_3) = \{(C_1 / 0.18, C_2 / 0.28, C_3 / 0.15, C_4 / 0.08, C_5 / 0.06), 0.88\).
$$

$$R^A \cap B(e_3, e_1) = \{(C_1 / 0.32, C_2 / 0.06, C_3 / 0.03, C_4 / 0.63, C_5 / 0.18), 0.52\)
$$

$$R^A \cap B(e_3, e_3) = \{(C_1 / 0.48, C_2 / 0.12, C_3 / 0.03, C_4 / 0.36, C_5 / 0.36), 0.76\)
$$

|      | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$
|------|------|------|------|------|------
| $e_1, e_1$ | 0.0912 | 0.1064 | 0.114 | 0.1064 | 0.0228
| $e_1, e_3$ | 0.1584 | 0.2464 | 0.132 | 0.0704 | 0.0528
| $e_2, e_1$ | 0.1664 | 0.0312 | 0.0156 | 0.3276 | 0.0936
| $e_2, e_3$ | 0.3648 | 0.0912 | 0.0228 | 0.2736 | 0.2736

Table-1

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.7808</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.4752</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.2844</td>
</tr>
<tr>
<td>$c_4$</td>
<td>0.778</td>
</tr>
<tr>
<td>$c_5$</td>
<td>0.4428</td>
</tr>
</tbody>
</table>

Table-2

It is clear that the lowest score 0.2844, scored by $c_3$ and the decision is in favor of selecting $c_3$. 

© 2015, IUMA. All Rights Reserved
5. CONCLUSION

In this paper we discussed the fuzzy soft set and extend the concept of relation in fuzzy soft set theory. We also establish some properties of fuzzy soft relation such as symmetric, transitive, reflexive, irreflexive, equivalence etc. Also we present the definition of union and intersection between two fuzzy soft sets with a new approach.

6. ACKNOWLEDGEMENT

The author would like to thank the fund for University Grant Commission (UGC), India, for funding the research elaborated on in this paper. [Grant No. F.5-60/2013-14/MRP/NERO/17424]

7. REFERENCES


Source of support: University Grant Commission (UGC), India. Conflict of interest: None Declared

[Copy right © 2015. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]