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3-EQUITABLE AND 3-CORDIAL LABELING IN DUPLICATE GRAPH OF SOME GRAPHS

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ABSTRACT

In this paper, we prove that the duplicate graph of ladder graph, cycle graph, circular twig graph and extended duplicate graph of the twig graph are 3-equitble and 3-cordial.

Keywords: *Graph labeling, duplicate graph, extended duplicate graph of Twig graph, 3-cordial and 3-equitable.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [5]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 1100 papers [3]. Labeled graphs serve as useful models in a broad range of applications such as circuit design, communication network addressing, X-ray crystallography, radar, astronomy, data base management and coding theory. The concept of cordial labeling was introduced by I. Cahit[1]. The concept of 3-equitable labeling was introduced by Cahit [1] and he proved that an Eulerian graph with number of edges congruent to 3(mod 6) is not 3-equitable [1]. Hovey introduced simultaneous generalizations of harmonious and cordial labellings. He defines a graph G of vertex set V (G) and edge set E(G) to be k-cordial if there is a vertex labelling f from V (G) to Z_k, the group of integers modulo k, so that when each edge uv is assigned the label (f(u) + f(v)) (mod k), the number of vertices (respectively, edges) labelled with i and the number of vertices (respectively, edges) labelled with j differ by at most one for all i and j in Z_k [4]. K. Thirusangu, P.P. Ulaganathan and B. Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [7]. K. Thirusangu, P.P. Ulaganathan and P. Vijaya kumar have proved that the duplicate graph of Ladder graph L_m , $m \geq 2$, is cordial, total cordial and prime cordial[8]. P. Vijaya kumar, K. Thirusangu and P.P. Ulaganathan have proved that the duplicate graph of Ladder graph L_m , $m \geq 2$, is product cordial and E– cordial[10].

In this paper, we prove that the duplicate graph of the ladder graph L_m , $m \geq 3$, the duplicate graph of the cycle graph C_m, m > 2, the duplicate graph of the circular twig graph CT_m, $m \geq 3$ and the extended duplicate graph of the twig graph T_m, $m \geq 2$, are 3-equitable and 3-cordial.

II. PRELIMINARIES

In this section, we give the basic notions relevant to this paper.

Definition 2.1: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling).

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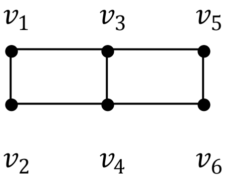
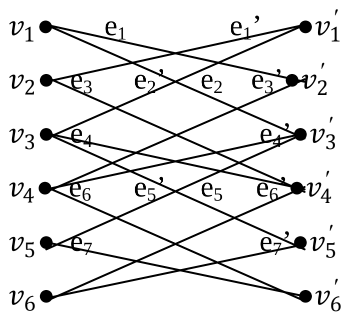
Definition 2.2: Let $G(V,E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V' \; and \; V \cap V' = \emptyset$ and $f : V \to V'$is bijective $(for\ v\ \in V, we\ write\ f(v) = v')$ and the edge set E_1 of DG is defined as : The edge uv is in E if and only if both uv' and $u'v$ are edges in E_1.

Definition 2.3: The ladder graph L_m is a planar undirected graph with 2m vertices and 3m – 2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: $L_{m,1} = P_m\ X\ P_1$, where m is the number of rungs in the ladder.

Example 2.1: The following figures show the Ladder graph L_3 and its duplicate graph.

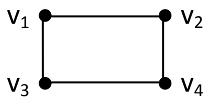




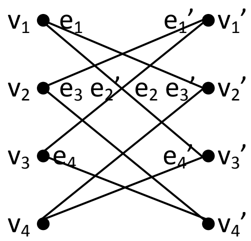
Clearly the duplicate graph of the ladder graph L_m contains 4m vertices and 6m – 4 edges.

Definition 2.4: The cycle graph C_m is a path graph of length m, in which the initial and the terminal vertices are the same. The cycle graph C_m has m vertices and m edges.

Example 2.2: The following figures show the cycle graph C_4 and its duplicate graph.



Cycle graph C_4



Duplicate graph of cycle graph C_4

Clearly the duplicate graph of the cycle graph C_m, have 2m vertices and 2m edges

Definition 2.4: A graph $G(V,E)$ obtained from a path by joining exactly two pendent edges to each internal vertices of the path is called a Twig graph, denoted by T_m. A Twig T_m with m internal vertices has 3m + 2 vertices and 3m + 1 edges. [6].

Definition 2.5: The extended duplicate graph of the Twig graph is obtained by adding an edge between any one vertex from V to any one vertex in V', except the terminal vertices of V and V'. Here we join v_2 and v_2' such that $v_2 v_2' = e_{3m+3}$. Clearly the extended duplicate graph of Twig graph has 6m + 4 vertices and 6m + 3 edges. [6].

Structure of extended duplicate graph of Twig graph

Fix $v_1 v_2' \leftarrow e_1$; $v_1' v_2 \leftarrow e_1'$; $v_2 v_2' \leftarrow e_{3m+2}$;

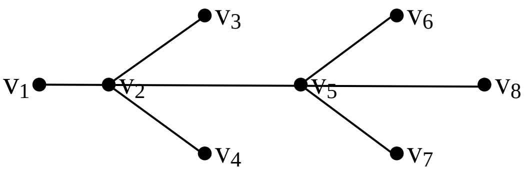
For $1 \le k \le m$

For $1 \le i \le 3$

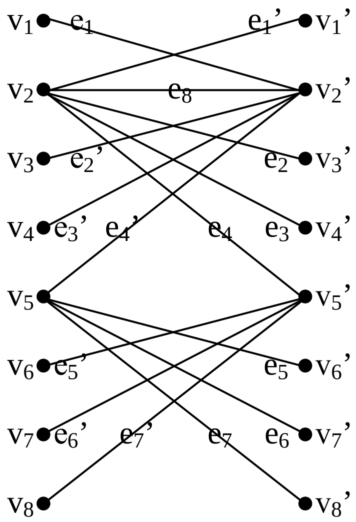
$v_{3k-1} v'_{(3k-1)+i} \leftarrow e_{(3k-1)+(i-1)}$;

$v'_{3k-1} v_{(3k-1)+i} \leftarrow e'_{(3k-1)+(i-1)}$;

Example 2.3: The following figures show the Twig graph T_2 and the extended duplicate graph of T_2



Twig graph T_2



Duplicate graph of Twig graph T_2

Definition 2.6: A graph $G(V,E)$ obtained from the cycle graph C_m by joining exactly two pendent edges to each of the vertices is called a circular twig. It is denoted by CT_m. and it has 3m vertices and 3m edges.

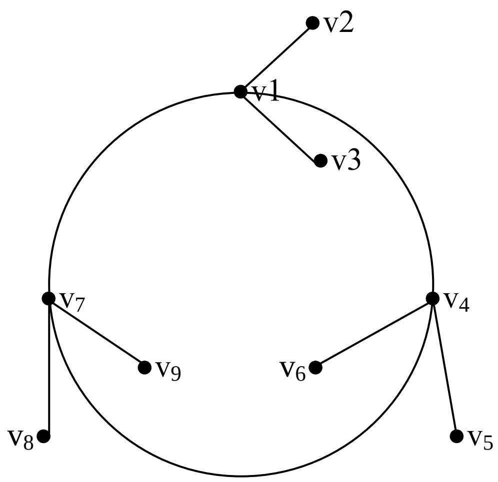
Structure of duplicate graph of Circular twig:

Fix $v_1 v'_{3m-2} \leftarrow e_{3m}$; $v'_1 v_{3m-2} \leftarrow e'_{3m}$;

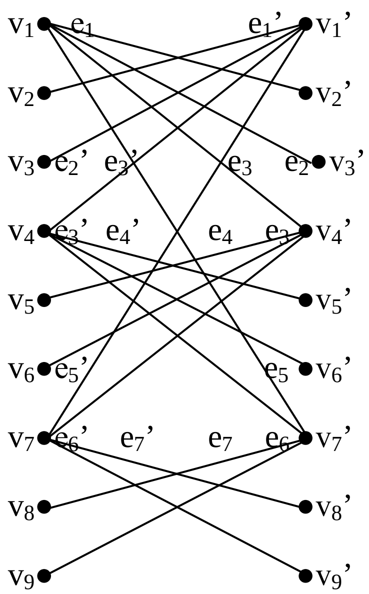
For $1 \le k \le m-1$
$v_{3k-2} v'_{3k-1} \leftarrow e_{3k-2}$; $v_{3k-2} v'_{3k} \leftarrow e_{3k-1}$; $v_{3k-2} v'_{3k+1} \leftarrow e_{3k}$;
$v'_{3k-2} v_{3k+1} \leftarrow e'_{3k}$; $v'_{3k-2} v_{3k-1} \leftarrow e'_{3k-2}$; $v'_{3k-2} v_{3k} \leftarrow e'_{3k-1}$;

For k = m
$v_{3k-2} v'_{3k-1} \leftarrow e_{3k-2}$; $v_{3k-2} v'_{3k} \leftarrow e_{3k-1}$;

Example 2.4: The following figures show the Circular Twig graph CT_3 and its duplicate graph.



Circular Twig CT_3



Duplicate graph of Circular Twig T_3

Remark: The duplicate graph of CT_m, has 6m vertices and 6m edges.

Definition 2.7: Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A mapping f: V (G) → {0, 1, 2} is called ternary vertex labeling of G and f(v) is called the label of the vertex v of G under f. For an edge uv, the induced edge labeling $f^*: E(G) \to \{0,1,2\}$ is given by $f^*(uv) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ *and* $v_f(2)$ be the number of vertices of G having labels 0,1 and 2 respectively under f and let $e_f(0), e_f(1)$ *and* $e_f(2)$ be the number of edges having labels 0,1 and 2 respectively under f^*.

Definition 2.8: A ternary vertex labeling of a graph G is called a 3-equitable labeling if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$. A graph G is 3-equitable if it admits 3-equitable labeling. [9], [11].

Definition 2.9: Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A vertex labeling $f: V(G) \to Z_k$ induces an edge labeling $f^*: E(G) \to Z_k$ defined by $f^*(uv) = [f(u) + f(v)]\ (mod\ k)$, for all edges $uv \in E(G)$. For $i \in Z_k$, let $n_i(f) = |\{v \in V(G) \mid f(v) = i\}|$ and $m_i(f) = |\{e \in E(G) \mid f^*(e) = i\}|$. A labeling f of a graph G is called k-cordial if $|n_i(f) - n_j(f)| \le 1$ and $|m_i(f) - m_j(f)| \le 1$ for all $i, j \in Z_k$. A graph G is called k-cordial if it admits a k-cordial labeling. [1], [4].

Definition 2.10: Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A vertex labeling $f: V(G) \to \{0,1,2\}$ induces an edge labeling $f^*: E(G) \to \{0,1,2\}$ defined by $f^*(uv) = [f(u) + f(v)]\ (mod\ 3)$, for all edges $uv \in E(G)$. For $i \in Z_3$, let $n_i(f) = |\{v \in V(G) \mid f(v) = i\}|$ and $m_i(f) = |\{e \in E(G) \mid f^*(e) = i\}|$. A labeling f of a graph G is called 3-cordial if $|n_i(f) - n_j(f)| \le 1$ and $|m_i(f) - m_j(f)| \le 1$ for all $i, j \in Z_3$. A graph G is called 3-cordial if it admits a 3-cordial labeling.

III. MAIN RESULTS

3 - Equitable labeling in duplicate graph of ladder graph L_m.

Algorithm:

// Assignment of labels to vertices//

Case-(i): When m ≡ 0 mod 3

For $1 \le k \le \frac{2m}{3}$

$v_{3k-2} \leftarrow 1;\ v_{3k-1} \leftarrow 2;\ v_{3k} \leftarrow 0;\ v'_{3k-2} \leftarrow 2;\ v'_{3k-1} \leftarrow 1;\ v'_{3k} \leftarrow 0$:

Case-(ii): When m ≡ 1 mod 3

Fix $v_1 = 2$, $v_{2m} = 0$, $v'_1 = 2$, $v'_{2m} = 1$.

For $1 \le k \le \frac{2(m-1)}{3}$

$v_{3k-1} \leftarrow 2;\ v_{3k} \leftarrow 0;\ v_{3k+1} \leftarrow 1;\ v'_{3k-1} \leftarrow 1;\ v'_{3k} \leftarrow 0;\ v'_{3k+1} \leftarrow 2$:

Case-(iii): When m ≡ 2 mod 3 and $m \ge 5$

Fix $v_1 = 2$, $v'_1 = 0$.

For $1 \le k \le \frac{2m-1}{3}$

$v_{3k-1} \leftarrow 2;\ v_{3k} \leftarrow 0;\ v_{3k+1} \leftarrow 1;\ v'_{3k-1} \leftarrow 1;\ v'_{3k} \leftarrow 0;\ v'_{3k+1} \leftarrow 2$:

Theorem 3.1: The duplicate graph of the ladder graph L_m, $m \ge 2$, admits 3 – equitable labeling.

Proof:

Case-(i): when m ≡ 0 mod 3.

The vertices are labeled using the algorithm in such a way that $\frac{4m}{3}$ vertices receive label 0, $\frac{4m}{3}$ vertices receive label 1 and $\frac{4m}{3}$ vertices receive label 2, so that the 4m vertices are labeled in such a way that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

Using the induced function f^* defined by $f^*(v_i v_j) = |f(v_i) - f(v_j)|$, the 2m – 2 edges namely $e_1, e_3, e_8, e_{10}, e_{12}, e_{17} \dots e_{3m-15}, e_{3m-10}, e_{3m-8}, e_{3m-6}, e_1', e_3', e_8', e_{10}', e_{12}', e_{17}' \dots e_{3m-15}', e_{3m-10}', e_{3m-8}'$, and e_{3m-6}' receive label 0, the 2m – 1 edges namely $e_2, e_5, e_6, e_9, e_{11}, e_{14}, e_{15}, e_{18} \dots e_{3m-13}, e_{3m-12}, e_{3m-9}, e_{3m-7}, e_{3m-4}, e_{3m-3}, e_4', e_7', e_{13}', e_{16}', \dots e_{3m-5}', e_{3m-2}'$ receive label 1 and the 2m – 1 edges namely $e_4, e_7, e_{13}, e_{16}, \dots e_{3m-5}, e_{3m-2}, e_2', e_5', e_6', e_9', e_{11}', e_{14}', e_{15}', e_{18}' \dots e_{3m-13}', e_{3m-12}', e_{3m-9}', e_{3m-7}', e_{3m-4}', e_{3m-3}'$ receive label 2. Hence the 6m – 4 edges are labeled such that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case-(ii): when m ≡ 1 mod 3.

The vertices are labeled using the algorithm in such a way that $\frac{4m-1}{3}$ vertices receive label 0, $\frac{4m-1}{3}$ vertices receive label 1 and $\frac{4m+2}{3}$ vertices receive label 2, so that the 4m vertices are labeled in such a way that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

Using the induced function as in case (i), the 2m – 1 edges namely $e_3, e_8, e_{10}, e_{12}, e_{17}, e_{19} \dots e_{3m-9}, e_{3m-4}, e_{3m-2}, e_1', e_3', e_8', e_{10}', e_{12}', e_{17}' \dots e_{3m-9}', e_{3m-4}', e_{3m-2}'$ receive label 0, the 2m – 2 edges namely $e_1, e_5, e_6, e_9, e_{11}, e_{15}, e_{18}, e_{20}, e_{23} \dots e_{3m-12}, e_{3m-7}, e_{3m-6}, e_{3m-3}, e_4', e_7', e_{13}', e_{16}' \dots e_{3m-8}', e_{3m-5}'$ receive label 1 and the 2m – 1 edges namely $e_2, e_4, e_7, e_{13}, e_{16} \dots e_{3m-8}, e_{3m-5}, e_2', e_5', e_6', e_9', e_{11}', e_{14}', e_{15}', e_{18}' \dots e_{3m-10}', e_{3m-7}', e_{3m-6}', e_{3m-3}'$ receive label 2. Hence the 6m – 4 edges are labeled such that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case-(iii): when m ≡ 2 mod 3 and m > 2.

The vertices are labeled using the algorithm in such a way that $\frac{4m+1}{3}$ vertices receive label 0, $\frac{4m-2}{3}$ vertices receive label 1 and $\frac{4m+1}{3}$ vertices receive label 2, so that the 4m vertices are labeled in such a way that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

Using the induced function as in case (i), the 2m – 1 edges namely $e_3, e_8, e_{10}, e_{12}, e_{17}, e_{19} \dots e_{3m-12}, e_{3m-7}, e_{3m-5}, e_{3m-3}, e_2', e_3', e_8', e_{10}', e_{12}' \dots e_{3m-12}', e_{3m-7}', e_{3m-5}', e_{3m-3}'$ receive label 0, the 2m – 2 edges namely $e_1, e_5, e_6, e_9, e_{11}, e_{14}, e_{15}, e_{18}, e_{20} \dots e_{3m-10}, e_{3m-9}, e_{3m-6}, e_{3m-4}, e_4', e_7', e_{13}', e_{16}' \dots e_{3m-11}', e_{3m-8}', e_{3m-2}'$ receive label 1 and the 2m – 1 edges namely $e_2, e_4, e_7, e_{13}, e_{16} \dots e_{3m-11}, e_{3m-8}, e_{3m-2}, e_1', e_5', e_6', e_9', e_{11}', e_{14}', e_{15}', e_{18}', e_{20}' \dots e_{3m-10}', e_{3m-9}', e_{3m-6}', e_{3m-4}'$ receive label 2. Hence the 6m – 4 edges are labeled such that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

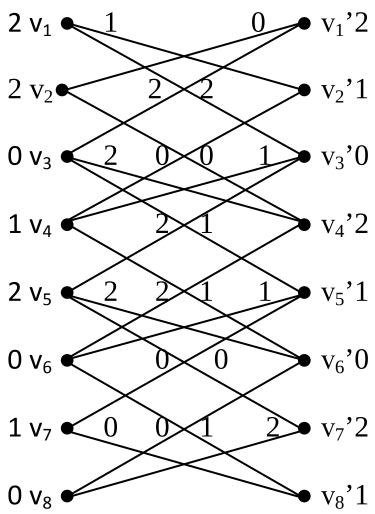
Case-(iv): when m = 2.

The vertices v_1, v_3, v_2' are labeled with 0, the vertices , v_2, v_4' are labeled with 1 and the vertices , v_4, v_1', , v_3' are labeled with 2. Using the induced function as in case (i), the edges , e_1, e_3, e_4' receive label 0, the edges e_4, e_1' receive label 1 and the edges e_2, e_2', e_3' receive label 2.

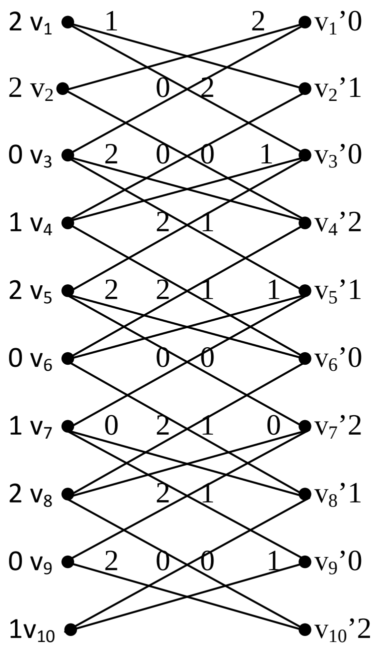
Hence the 6m – 4 edges are labeled such that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Hence the duplicate graph of the ladder graph L_m, m ≥ 2 admits 3 – equitable labeling.

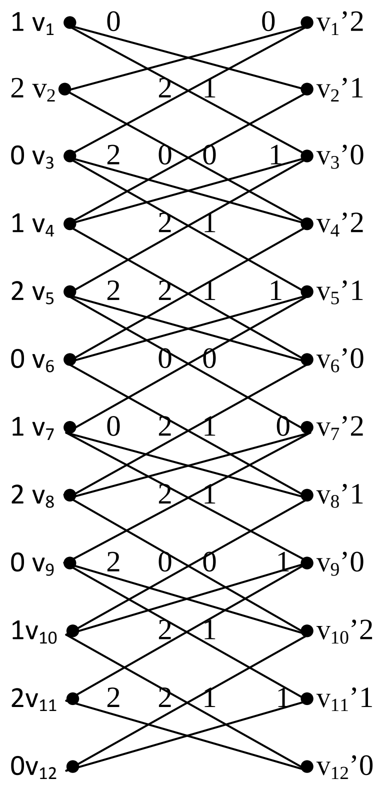
Illustration:



Duplicate graph of Ladder graph L_4



Duplicate graph of Ladder graph L_5



Duplicate graph of Ladder graph L_6

3 – Equitable labeling in duplicate graph of cycle graph C_m

// Assignment of labels to vertices//

Case-(i): $m \equiv 0 \bmod 3$

For $1 \leq k \leq \frac{m}{3}$;

$v_{3k-2} \leftarrow 0$; $v_{3k-1} \leftarrow 1$; $v_{3k} \leftarrow 2$: $v'_{3k-2} \leftarrow 1$; $v'_{3k-1} \leftarrow 0$; $v'_{3k} \leftarrow 2$:

Case-(ii): $m \equiv 1 \bmod 3$

For $1 \leq k \leq \frac{m-1}{3}$;

$v_{3k-2} \leftarrow 0$; $v_{3k-1} \leftarrow 1$; $v_{3k} \leftarrow 2$: $v'_{3k-2} \leftarrow 1$; $v'_{3k-1} \leftarrow 0$; $v'_{3k} \leftarrow 2$:

For $k = \frac{m+2}{3}$;

$v_{3k-2} \leftarrow 0$; $v'_{3k-2} \leftarrow 2$:

Case-(iii): $m \equiv 2 \bmod 3$

For $1 \leq k < \frac{m+1}{3}$;

$v_{3k-2} \leftarrow 0$; $v_{3k-1} \leftarrow 1$; $v'_{3k-2} \leftarrow 1$: $v_{3k} \leftarrow 2$; $v'_{3k-1} \leftarrow 0$; $v'_{3k} \leftarrow 2$:

For $k = \frac{m+1}{3}$;

$v_{3k-2} \leftarrow 0$; $v_{3k-1} \leftarrow 0$; $v'_{3k-1} \leftarrow 1$: $v'_{3k-2} \leftarrow 2$;

Theorem 3.2: The duplicate graph of the cycle C_m, $m \geq 3$ admits 3 – equitable labeling.

Proof:

Case-(i): when m ≡ 0 mod 3.

Using the algorithm, $\frac{2m}{3}$ vertices receive label 0, $\frac{2m}{3}$ vertices receive label 1, $\frac{2m}{3}$ vertices receive label 2, so that the number of vertices labeled with 0,1,2 mutually differ at most by one.

Using the induced function f^* defined by $f^*(v_i v_j) = |f(v_i) - f(v_j)|$, the $\frac{2m}{3}$ edges namely $e_1, e_4, e_7, e_{10} \dots e_{m-2}$, $e_1', e_4', e_7', e_{10}' \dots e_{m-2}'$ receive label 0, the $\frac{2m}{3}$ edges namely $e_2, e_3, e_5, e_6, e_8, e_9 \dots e_{m-4}, e_{m-3}, e_{m-1}$ receive label 1 and the $\frac{2m}{3}$ edges namely $e_2', e_3', e_5', e_6', e_8', e_9' \dots e_{m-4}', e_{m-3}', e_{m-1}'$ receive label 2. Hence the 2m edges are labeled so that the number of edges receive label 0, 1, 2 mutually differ at most by one.

Case-(ii): when m ≡ 1 mod 3.

Using the algorithm, $\frac{2m+1}{3}$ vertices receive label 0, $\frac{2m-1}{3}$ vertices receive label 1, $\frac{2m+1}{3}$ vertices receive label 2, so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

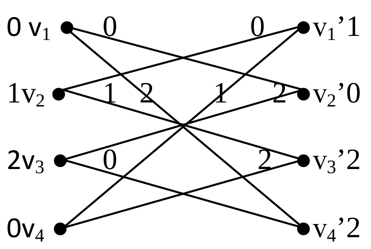
Using the induced function as in case (i), the , $\frac{2m+1}{3}$ edges namely $e_1, e_4, e_7, e_{10} \dots e_{m-3}, e_{m-1}$ $e_1', e_4', e_7', e_{10}' \dots e_{m-3}'$ receive label 0, the $\frac{2m-2}{3}$ edges namely $e_2, e_3, e_5, e_6, \dots e_{m-5}, e_{m-4}, e_{m-2}, e_m'$ receive label 1 and the $\frac{2m+1}{3}$ edges namely $e_m, e_2', e_3', e_5', e_6' \dots e_{m-5}', e_{m-4}', e_{m-2}', e_{m-1}'$ receive label 2. Hence the 2m edges are labeled so that the number of edges receive label 0, 1, 2 mutually differ at most by one.

Case-(iii): when m ≡ 1 mod 3.

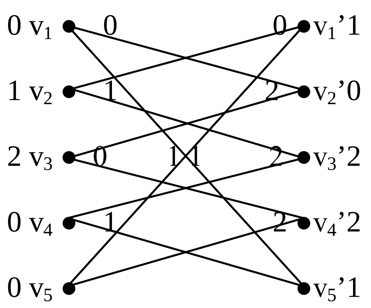
Using the algorithm, $\frac{2m+2}{3}$ vertices receive label 0, $\frac{2m-1}{3}$ vertices receive label 1, $\frac{2m-1}{3}$ vertices receive label 2, so that the number of vertices labeled with 0,1,2 mutually differ at most by one.

Using the induced function as in case (i), the , $\frac{2m-1}{3}$ edges namely $e_1, e_4, e_7 \dots e_{m-4}, e_{m-2}, e_1', e_4', e_7' \dots e_{m-4}'$ receive label 0, the $\frac{2m+2}{3}$ edges namely $e_2, e_3, e_5, e_6 \dots e_{m-6}, e_{m-5}, e_{m-3}, e_{m-1}, e_m, e_m'$ receive label 1 and the $\frac{2m-1}{3}$ edges namely $e_2', e_3', e_5', e_6', e_8', e_9' \dots e_{m-3}', e_{m-2}', e_{m-1}'$ receive label 2. Hence the 2m edges are labeled so that the number of edges receive label 0, 1, 2 mutually differ at most by one.

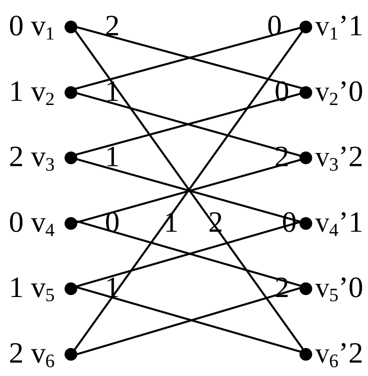
Illustration:



Duplicate graph of cycle C_4



Duplicate graph of cycle C_5



Duplicate graph of cycle C_6

3–equitable labeling in the extended duplicate graph of twig graph T_m

Algorithm:

// Assignment of labels to vertices//
For 1 ≤ k ≤ m + 1,
$v_{3k-2} \leftarrow 2$; $v_{3k-1} \leftarrow 0$; $v_{3k-2}' \leftarrow 1$; $v_{3k-1}' \leftarrow 2$;

For 1 ≤ k ≤ m,
$v_{3k} \leftarrow 1$; $v_{3k}' \leftarrow 0$;

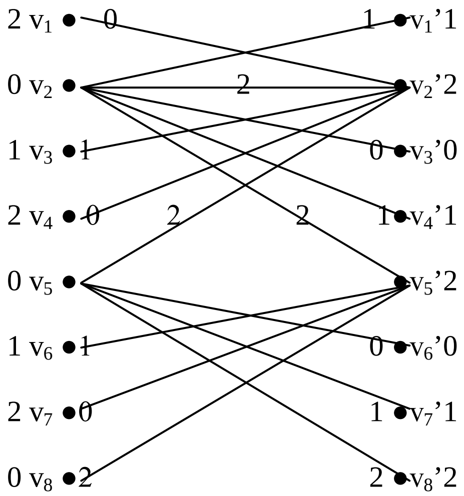
Theorem 3.3: The extended duplicate graph of the twig graph (T_m), m ≥ 2, is 3 – equitable.

Proof: Using the algorithm the 2m + 1 vertices are labeled with 0, 2m + 1 vertices are labeled with 1 and the 2m + 2 vertices are labeled with 2. Thus, the 6m + 4 vertices are labeled with 0, 1, 2 so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

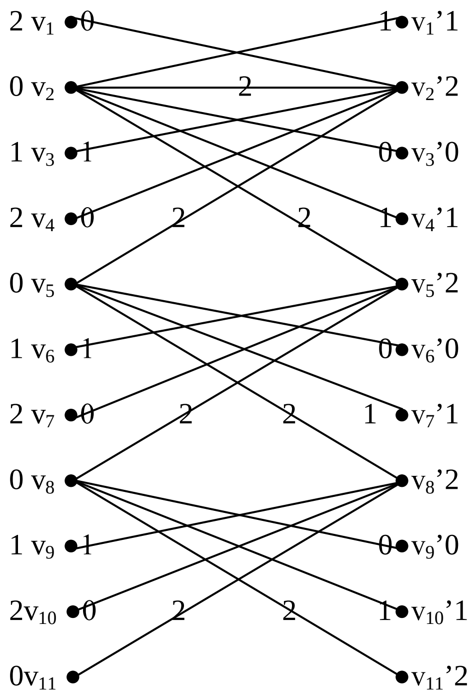
Using the induced function f^* defined by $f^*(v_i v_j) = |f(v_i) - f(v_j)|$, the 2m + 1 edges namely $e_1, e_2, e_5, e_8 \dots e_{3m-1}, e'_3, e'_6, e'_9 \dots e'_{3m}$ receive label 0, the 2m + 1 edges namely $e_3, e_6, e_9 \dots e_{3m}, e'_1, e'_2, e'_5, e'_8 \dots e'_{3m-1}$ receive label 1 and the 2m + 1 edges namely $e_4, e_7, e_{10} \dots e_{3m+1}, e_{3m+2}, e'_4, e'_7, e'_{10} \dots e'_{3m+1}$ receive label 2. Thus, the 6m + 3 edges are labeled so that the number of edges receive label 0, 1, 2 mutually differ at most by one.

Hence the extended duplicate graph of the twig graph (T_m), m ≥ 2, is 3 – equitable.

Illustration:



Extended duplicate graph of Twig graph (T_2)



Extended duplicate graph of Twig graph (T_3)

3 – equitable labeling in the duplicate graph of circular twig graph CT_m

Algorithm:
// Assignment of labels to vertices //

For 1 ≤ k ≤ m,
$v_{3k-2} \leftarrow 0;\ v_{3k-1} \leftarrow 1;\ v_{3k} \leftarrow 2;$
$v'_{3k-2} \leftarrow 0;\ v'_{3k-1} \leftarrow 1;\ v'_{3k} \leftarrow 2;$

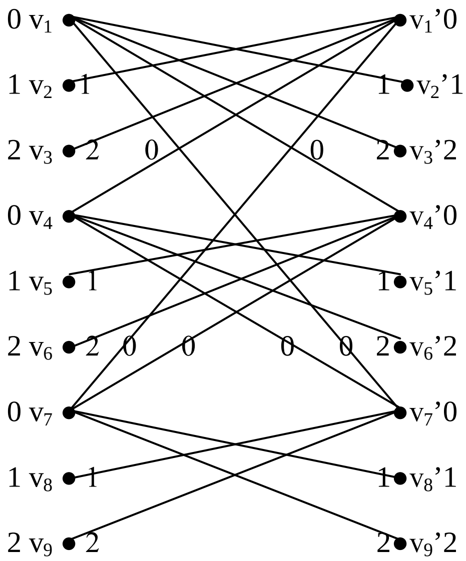
Theorem 3.4: The duplicate graph of the circular twig ($DGCT_m$), m ≥ 2, is 3 – equitable.

Proof: Using the algorithm, the 2m vertices are labeled with 0, the 2m vertices are labeled with 1, the 2m vertices are labeled with 2. Hence the 6m vertices are labeled with 0, 1, 2 so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

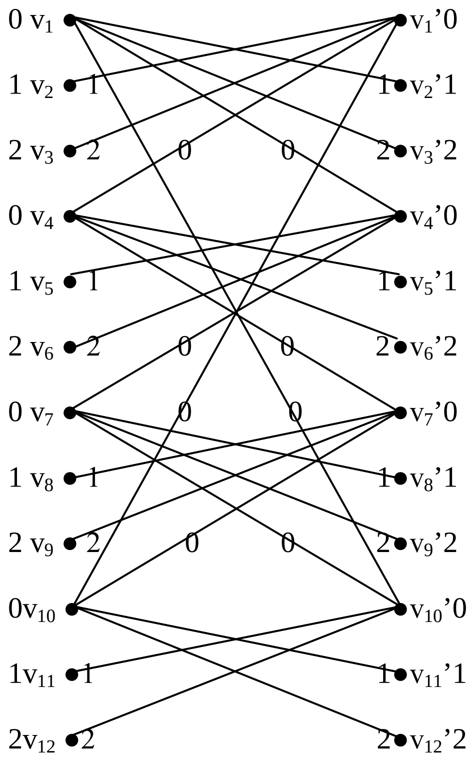
Using the induced function f^* defined by $f^*(v_i v_j) = |f(v_i) - f(v_j)|$, the 2m edges namely $e_3, e_6, e_9 \dots e_{3m-3}, e_{3m}, e'_3, e'_6, e'_9 \dots e'_{3m-3}, e'_{3m-2}$ receive label 0, the 2m edges namely $e_1, e_4, e_7, e_{10} \dots e_{3m-2}, e'_1, e'_4, e'_7 \dots e'_{3m-2}$ receive label 1 and the 2m edges namely $e_2, e_5, e_8 \dots e_{3k-1}, e'_2, e'_5, e'_8 \dots e'_{3k-1}$ receive label 2. Thus the 6m edges are labeled with 0, 1, 2 such that the number of edges with labels 0, 1, 2 mutually differ at most by one.

Hence, the duplicate graph of the circular twig ($DGCT_m$), m ≥ 2, is 3 – equitable.

Illustration:



Extended duplicate graph of Circular Twig graph (CT_3)



Extended duplicate graph of Circular Twig graph (CT_4)

3 - cordial labeling in duplicate graph of ladder graph L_m.

Algorithm:
// Assignment of labels to vertices//

Case-(i): when m ≡ 0 mod 3 and m > 3.

$v_{2m-1} \leftarrow 0$; $v_{2m} \leftarrow 2$; $v_1^{'} \leftarrow 2$; $v_2^{'} \leftarrow 0$:

For $k = \frac{m}{3}$
$v_{6k-1}^{'} \leftarrow 0$; $v_{6k}^{'} \leftarrow 0$;

For $1 \leq k \leq \frac{m}{3}$
$v_{6k-5} \leftarrow 2$; $v_{6k-4} \leftarrow 2$; $v_{6k-3} \leftarrow 1$; $v_{6k-2} \leftarrow 1$; $v_{6k-3}^{'} \leftarrow 1$; $v_{6k-2}^{'} \leftarrow 1$;

For $1 \leq k \leq \frac{m}{3}$ - 1
$v_{6k-1} \leftarrow 0$; $v_{6k} \leftarrow 0$; $v_{6k-1}^{'} \leftarrow 2$; $v_{6k}^{'} \leftarrow 2$;

Case-(ii): when m ≡ 1 mod 3.

Fix $v_1 \leftarrow 0$; $v_2 \leftarrow 1$; $v_1^{'} \leftarrow 2$; $v_2^{'} \leftarrow 0$;

For $1 \leq k \leq \frac{m-1}{3}$
$v_{6k-3} \leftarrow 1$; $v_{6k-2} \leftarrow 1$; $v_{6k-1} \leftarrow 0$; $v_{6k} \leftarrow 0$; $v_{6k+1} \leftarrow 2$; $v_{6k+2} \leftarrow 2$

$v_{6k-3}^{'} \leftarrow 1$; $v_{6k-2}^{'} \leftarrow 1$; $v_{6k-1}^{'} \leftarrow 2$; $v_{6k}^{'} \leftarrow 2$; $v_{6k+1}^{'} \leftarrow 0$; $v_{6k+2}^{'} \leftarrow 0$;

Case-(iii): when m ≡ 2 mod 3.

Fix $v'_{2m} \leftarrow 2$; $v'_{2m-1} \leftarrow 1$;

For $1 \leq k \leq \frac{m+1}{3}$
$v_{6k-5} \leftarrow 2$; $v_{6k-4} \leftarrow 2$; $v_{6k-3} \leftarrow 1$; $v_{6k-2} \leftarrow 1$; $v'_{6k-5} \leftarrow 0$; $v'_{6k-4} \leftarrow 0$;

For $1 \leq k \leq \frac{m-2}{3}$
$v'_{6k-3} \leftarrow 1$; $v'_{6k-2} \leftarrow 1$; $v'_{6k-1} \leftarrow 2$; $v'_{6k} \leftarrow 2$; $v_{6k-1} \leftarrow 0$; $v_{6k} \leftarrow 0$;

Theorem 3.5: The duplicate graph of the ladder graph L_m, m ≥ 3, is 3 – cordial.

Proof:

Case-(i): when m ≡ 0 mod 3 and m > 3.

Using the algorithm the $\frac{4m}{3}$ vertices receive label 0, the $\frac{4m}{3}$ vertices receive label 1 and the $\frac{4m}{3}$ vertices receive label 2. Thus the 4m vertices are labeled such that the number of vertices with label 0, 1, 2 mutually differs at most by one.

Using the induced function f^* defined by $f^*(uv) = [f(u) + f(v)]\ (mod\ 3)$, the 2m – 1 edges namely e_2, e_3, e_5, e_6, e_8 … e_{3m-7}, e_{3m-6}, e_{3m-2}, e'_2, e'_{3m-3} receive label 0, the 2m – 1 edges namely e_{3m-4}, e_{3m-3}, e'_1, e'_3, e'_5, e'_6, e'_8 … e'_{3m-8}, e'_{3m-6} receive label 1and the 2m – 2 edges namely e_1, e_4, e_7 … e_{3m-5}, e'_4, e'_7, e'_{10} … e'_{3m-5}, e^2_{3m-2} receive label 2. Thus the 6m – 4 edges are labeled with 0, 1, 2 so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case-(ii): when m ≡ 1 mod 3.

Using the algorithm the $\frac{4m+2}{3}$ vertices receive label 0, the $\frac{4m-1}{3}$ vertices receive label 1 and the $\frac{4m-1}{3}$ vertices receive label 2. Thus the 4m vertices are labeled such that the number of vertices with label 0, 1, 2 mutually differs at most by one.

Using the induced function as in case (i), the 2m – 1 edges namely e_1, e_5, e_6, e_8, e_9 … e_{3m-4}, e_{3m-3}, e'_1, e'_2 receive label 0, the 2m – 2 edges namely e_2, e'_3, e'_5, e'_6, e'_8 … e'_{3m-4}, e'_{3m-3} receive label 1 and the 2m – 1 edges namely e_3, e_4, e_7, e_{10}, e_{13} … e_{3m-5}, e_{3m-2}, e'_4, e'_7, e'_{10}… e'_{3m-2} receive label 2. Thus the 6m – 4 edges are labeled with 0, 1, 2 so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case-(iii): when m ≡ 2 mod 3.

Using the algorithm the $\frac{4m-2}{3}$ vertices receive label 0, the $\frac{4m+1}{3}$ vertices receive label 1 and the $\frac{4m+1}{3}$ vertices receive label 2. Thus the 4m vertices are labeled such that the number of vertices with label 0, 1, 2 mutually differs at most by one.

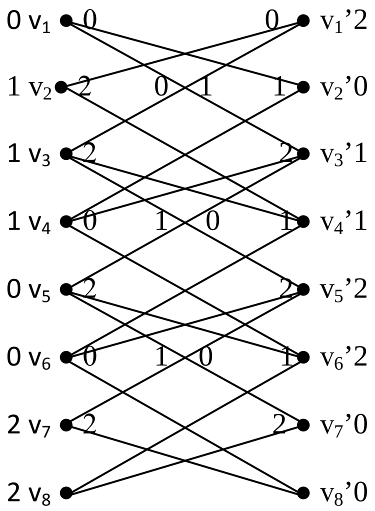
Using the induced function as in case (i), the 2m – 2 edges namely e_2, e_3, e_5, e_6 … e_{3m-6}, e_{3m-4}, e_{3m-2} receive label 0, the 2m – 1 edges namely e_{3m-3}, e'_2, e'_3, e'_5, e'_6, e'_8, e'_9 … e'_{3m-4}, e'_{3m-3} receive label 1 and the 2m – 1 edges namely e_1, e_4, e_7, e_{10} … e_{3m-8}, e_{3m-5}, e'_1, e'_4, e'_7 … e'_{3m-8}, e'_{3m-5}, e'_{3m-2} receive label 2. Thus the 6m – 4 edges are labeled with 0, 1, 2 so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case (iv): when m = 3.

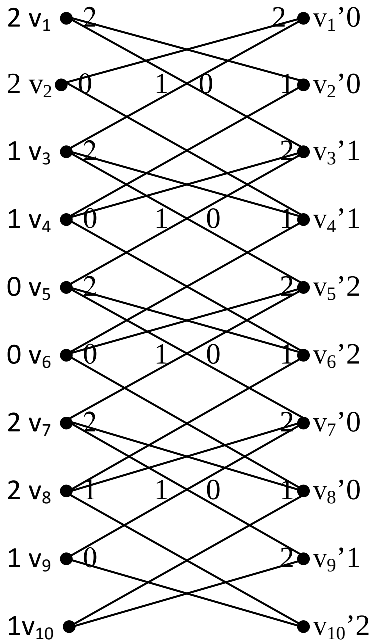
The vertices v_5, v_6, v'_2, v'_6 are labeled with 0, the vertices v_3, v_4, v'_3, v'_4 are labeled with 1 and the vertices v_1, v_2, v'_1, v'_5 are labeled with 2. Using the induced function as in case (i), the edges e_2, e_3, e_5, e_7, e'_2 receive label 0, the edges e_6, e'_1, e'_3, e'_5, e'_6 receive label 1 and the edges e_1, e_4, e'_4, e'_7 receive label 2. Thus the 6m – 4 edges are labeled with 0, 1, 2 so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Hence, the duplicate graph of the ladder graph L_m, m ≥ 3, is 3 – cordial.

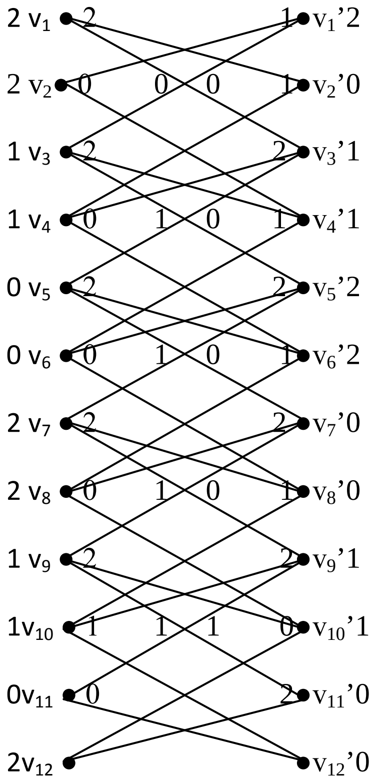
Illustration:



Duplicate graph of Ladder graph L_4



Duplicate graph of Ladder graph L_5



Duplicate graph of Ladder graph L_6

3 – Cordial labeling in duplicate graph of cycle graph C_m, m ≥ 3.

Algorithm:

// Assignment of labels to vertices //

Case (i): when m ≡ 0 mod 3.

For $1 \leq k \leq \frac{m}{3}$
$v_{3k-2} \leftarrow 0; v_{3k-1} \leftarrow 1; v_{3k} \leftarrow 2; v'_{3k-2} \leftarrow 1; v'_{3k-1} \leftarrow 2; v'_{3k} \leftarrow 0;$

Case-(ii): when m ≡ 1 mod 3.

For $1 \leq k \leq \frac{m-1}{3}$
$v_{3k-2} \leftarrow 0; v_{3k-1} \leftarrow 1; v_{3k} \leftarrow 2; v'_{3k-2} \leftarrow 1; v'_{3k-1} \leftarrow 2; v'_{3k} \leftarrow 0;$

For k = $\frac{m+2}{3}$
$v_{3k-2} \leftarrow 2; v'_{3k-2} \leftarrow 1;$

Case-(iii): when m ≡ 2 mod 3.

For $1 \leq k < \frac{m-2}{3}$
$v_{3k-2} \leftarrow 0; v_{3k-1} \leftarrow 1;\ v_{3k} \leftarrow 2; v'_{3k-2} \leftarrow 0; v'_{3k-1} \leftarrow 1; v'_{3k} \leftarrow 2;$

For k = $\frac{m+1}{3}$
$v_{3k-2} \leftarrow 0; v_{3k-1} \leftarrow 1: v'_{3k-1} \leftarrow 2; v'_{3k-2} \leftarrow 1;$

Theorem 3.6: The duplicate graph of the cycle graph C_m, $m \geq 3$, is 3 – cordial.

Proof:

Case-(i): when m ≡ 0 mod 3.

Using the algorithm the 2m vertices are labeled in such a way that $\frac{2m}{3}$ vertices receive label 0, $\frac{2m}{3}$ vertices receive label 1, $\frac{2m}{3}$ vertices receive label 2, so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

Using the induced function f^* defined by $f^*(uv) = [f(u) + f(v)]\ (mod\ 3)$, the $\frac{2m}{3}$ edges namely $e_3, e_6, e_9 \ldots\ e_m$, $e_3^{'}, e_6^{'}, e_9^{'} \ldots\ e_m^{'}$ receive label 0, the $\frac{2m}{3}$ edges namely $e_2, e_5, e_8 \ldots\ e_{m-1}, e_2^{'}, e_5^{'}, e_8^{'} \ldots\ e_{m-1}^{'}$ receive label 1and the $\frac{2m}{3}$ edges namely $e_1, e_4, e_7 \ldots\ e_{m-2}, e_1^{'}, e_4^{'}, e_7^{'} \ldots\ e_{m-2}^{'}$ receive label 2. Thus the 2m edges are labeled with 0, 1, 2, so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Case-(ii): when m ≡ 1 mod 3.

Using the algorithm the 2m vertices are labeled in such a way that $\frac{2m-2}{3}$ vertices receive label 0, $\frac{2m+1}{3}$ vertices receive label 1, $\frac{2m+1}{3}$ vertices receive label 2, so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

Using the induced function as in case(i), the $\frac{2m-2}{3}$ edges namely $e_3, e_6, e_9 \ldots\ e_{m-1}, e_3^{'}, e_6^{'}, e_9^{'} \ldots\ e_{m-4}^{'}, e_m^{'}$ receive label 0, the $\frac{2m+1}{3}$ edges namely $e_2, e_5, e_8 \ldots\ e_{m-2}, e_m, e_2^{'}, e_5^{'}, e_8^{'} \ldots\ e_{m-2}^{'}$ receive label 1and the $\frac{2m+1}{3}$ edges namely $e_1, e_4, e_7 \ldots\ e_{m-3}, e_1^{'}, e_4^{'}, e_7^{'} \ldots\ e_{m-3}^{'}, e_{m-1}^{'}$ receive label 2. Thus the 2m edges are labeled with 0, 1, 2, so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

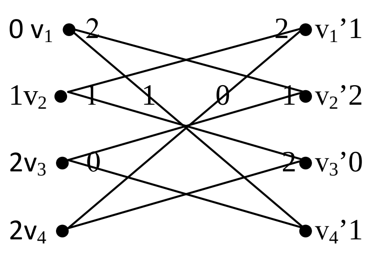
Case-(iii): when m ≡ 2 mod 3.

Using the algorithm the 2m vertices are labeled in such a way that $\frac{2m-1}{3}$ vertices receive label 0, $\frac{2m+2}{3}$ vertices receive label 1, $\frac{2m-1}{3}$ vertices receive label 2, so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

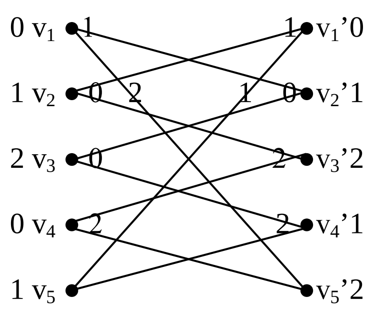
Using the induced function as in case(i), the $\frac{2m-1}{3}$ edges namely $e_2, e_5, e_8 \ldots\ e_{m-3}, e_{m-2}, e_2^{'}, e_5^{'}, e_8^{'} \ldots\ e_{m-3}^{'}$ receive label 0, the $\frac{2m-1}{3}$ edges namely $e_1, e_4, e_7 \ldots\ e_{m-4}, e_1^{'}, e_4^{'}, e_7^{'} \ldots\ e_{m-4}^{'},\ e_m^{'}$ receive label 1 and the $\frac{2m+2}{3}$ edges namely $e_3, e_6, e_9 \ldots\ e_{m-5},\ e_{m-1}, e_m, e_3^{'}, e_6^{'}, e_9^{'} \ldots\ e_{m-2}^{'}, e_{m-1}^{'}$ receive label 2. Thus the 2m edges are labeled with 0, 1, 2, so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Hence, the duplicate graph of the cycle graph C_m, $m \geq 3$, is 3 – cordial.

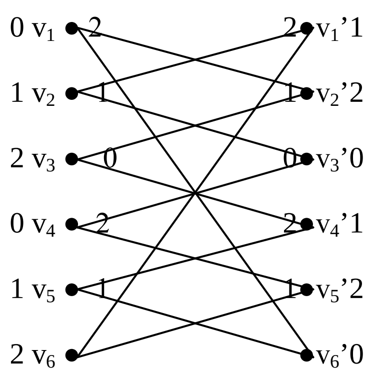
Illustration:



Duplicate graph of cycle C_4



Duplicate graph of cycle C_5



Duplicate graph of cycle C_6

3 – Cordial labeling in duplicate graph of Twig graph T_m, m ≥ 1.

Algorithm:

// Assignment of labels to vertices //

For 1 ≤ k ≤ m + 1
$v_{3k-2} \leftarrow 2$; $v_{3k-1} \leftarrow 1$;
$v_{3k-2}' \leftarrow 2$; $v_{3k-1}' \leftarrow 0$;

For 1 ≤ k ≤ m
$v_{3k} \leftarrow 0$; $v_{3k}' \leftarrow 1$;

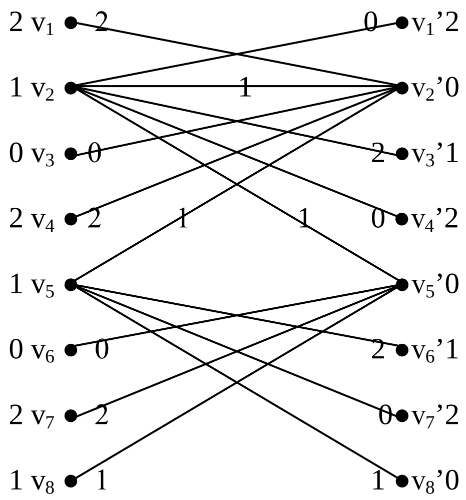
Theorem 3.7: The extended duplicate graph of twig graph T_m, m ≥ 1, is 3 – cordial.

Proof: Using the algorithm the 6m + 4 vertices are labeled in such a way that 2m + 1 vertices receive label 0, 2m + 1 vertices receive label 1 and 2m + 2 vertices receive label 2 so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

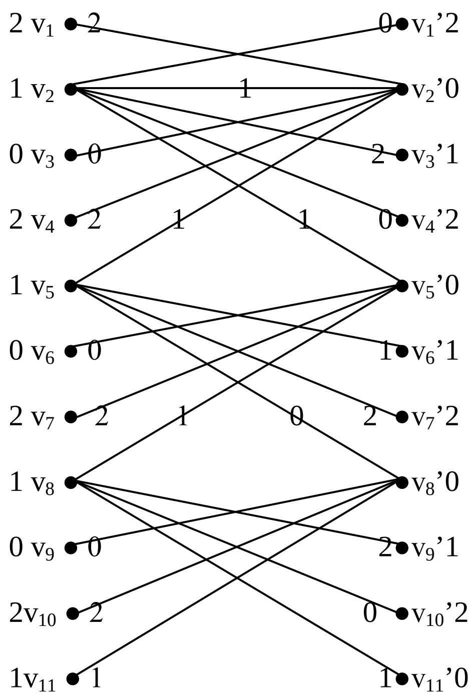
Using the induced function f^* defined by $f^*(uv) = [f(u) + f(v)] \ (mod\ 3)$, the 2m + 1 edges namely $e_3, e_6, e_9 \ldots e_{3m}, e_1', e_2', e_5', e_8' \ldots e_{m-1}'$ receive label 0, the 2m + 1 edges namely $e_4, e_7, e_{10}, \ldots e_{3m+1}, e_{3m+2}, e_4', e_7' \ldots e_{m+1}'$ receive label 1 and the 2m + 1 edges namely $e_1, e_2, e_5, e_8 \ldots e_{3m-1}, e_3', e_6', e_9' \ldots e_{3m}'$ receive label 2. Thus the 6m + 3 edges are labeled with 0, 1, 2, so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Hence the extended duplicate graph of twig graph is 3 – cordial.

Illustration:



Extended duplicate graph of Twig graph (T_2)



Extended duplicate graph of Twig graph (T_3)

3 – cordial labeling in duplicate graph of Circular Twig graph CT_m, m ≥ 3.

Algorithm:

// Assignment of labels to vertices //

For $1 \leq k \leq m$
$v_{3k-2} \leftarrow 2$; $v_{3k-1} \leftarrow 1$; $v_{3k} \leftarrow 0$;
$v'_{3k-2} \leftarrow 2$; $v'_{3k-1} \leftarrow 0$; $v'_{3k} \leftarrow 1$;

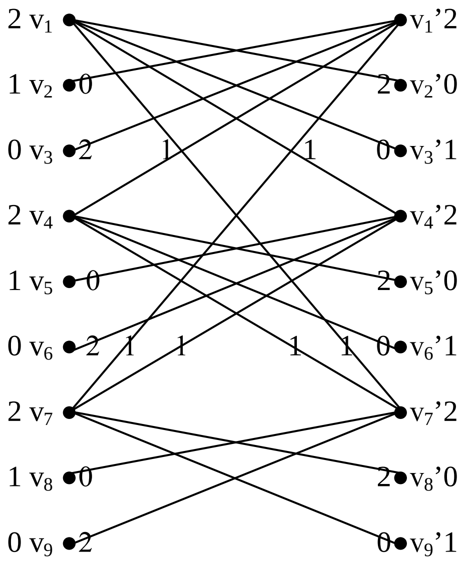
Theorem 3.8: The duplicate graph of the circular twig CT_m, $m \geq 3$, is 3 – cordial.

Proof: Using the algorithm the 6m vertices are labeled in such a way that 2m vertices receive label 0, 2m vertices receive label 1 and 2m vertices receive label 2 so that the number of vertices labeled with 0, 1, 2 mutually differ at most by one.

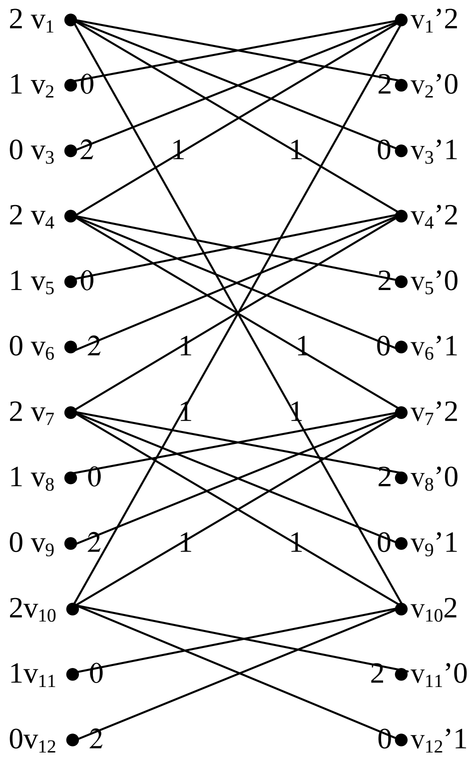
Using the induced function f^* defined by $f^*(uv) = [f(u) + f(v)]\,(mod\ 3)$, the 2m edges namely $e_2, e_5, e_8 \ldots e_{3m-1}, e'_1, e'_4, e'_7 \ldots e'_{3m-2}$ receive label 0, the 2m edges namely $e_3, e_6, e_9 \ldots e_{3m}, e'_3, e'_6, e'_9 \ldots e'_{3m}$ receive label 1 and the 2m edges namely $e_1, e_4, e_7, e_{10}, \ldots e_{3m-2}, e'_2, e'_5, e'_8 \ldots e'_{3m-1}$ receive label 2,. Thus the 6m edges are labeled with 0, 1, 2, so that the number of edges labeled with 0, 1, 2 mutually differ at most by one.

Hence, the duplicate graph of the circular twig CT_m, $m \geq 3$, is 3 – cordial.

Illustration:



Extended duplicate graph of Circular Twig graph (CT_3)



Extended duplicate graph of Circular Twig graph (CT_4)

IV. CONCLUSION

We proved that the duplicate graph of ladder graph, cycle graph, circular twig graph and the extended duplicate graph of the twig graph are 3-equitble and 3-cordial.

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