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ON STRONGER FORMS OF IRRESOLUTE FUNCTIONS, CONTINUOUS FUNCTIONS AND QUOTIENT MAPPINGS

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ABSTRACT

In this paper we introduce stronger forms of continuous maps and irresolute maps and study the relationship between them. We also introduce a new class of quotient map namely $\beta^*g\alpha$-quotient map and investigate the properties in terms of composition and restriction. Further, we introduce strongly $\beta^*g\alpha$-quotient maps and study the relationship with weak and strong forms of open maps.

Keywords*: strongly-$\beta^*g\alpha$-irresolute, almost $\beta^*g\alpha$-continuous, almost $\beta^*g\alpha$-irresolute, completely $\beta^*g\alpha$-continuous, completely $\beta^*g\alpha$-irresolute, perfectly $\beta^*g\alpha$-irresolute, perfectly contra $\beta^*g\alpha$-irresolute, $\beta^*g\alpha$-quotient map, almost $\beta^*g\alpha$-quotient map and strongly $\beta^*g\alpha$-quotient map.*

1. INTRODUCTION

D.Andrijevic introduced and studied the properties of Semi-pre open (β -open) [1] sets. R.Devi *et.al* introduced generalized -alpha (briefly gα)-closed sets [11], and derive their properties. R.Devi *et.al* introduced $^*g\alpha$-closed sets [20] in topological spaces. M.Vigneshwaran *et.al* introduce $\beta^*g\alpha$-closed sets [21] in topological spaces and using this set, the spaces ${}_cT_{\beta^*g\alpha}$ and ${}_{1/2}T_{\beta^*g\alpha}$ were introduced. β -continuity [15] was investigated by Popa and Noiri and the concept of contra continuity [4] was introduced by J.Dontchev which is the stronger form of locally continuous function introduced by Ganster and Reilly[5]. The notion of super continuous and δ-continuous [18] was introduced by T.Noiri.

M. Lellis Thivagar introduce the concept of Quotient mapping [9] and contra Quotient mapping as being stronger forms of continuous mapping. Consequently, quotient mapping finds its application in the study of relations between various spaces. Although it is classified as pure mathematics, when converted into Bitopology, Fuzzy topology and Digital topology, it becomes application oriented around.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) and (Z,η) represent topological spaces. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively. Let us recall the following definitions.

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Definition 2.1: A subset A of a space (X, τ) is called
1. semi-open set [8] if $A \subseteq$ cl(int(A)) and semi-closed set if int(cl(A)) $\subseteq A$,
2. α-open set [10] if $A \subseteq$ int(cl(int((A))) and α-closed set if cl(int(cl(A))) $\subseteq A$,
3. β-open set [1] if $A \subseteq$ cl(int(cl(A))) and β-closed set if int(cl(int(A))) $\subseteq A$,
4. regular-open [17] if A = int(cl(A)), and regular closed if A = cl(int(A)),
5. The δ -interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\text{int}_\delta(A)$. The subset A is called δ-open if $A = \text{int}_\delta(A)$ [18] i.e. a set is δ -open if it is the union of regular open sets, the complement of δ-open is called δ -closed,
6. generalized closed (briefly g-closed) set [7] if cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of a g-closed set is called a g-open set,
7. generalized α-closed (briefly gα-closed) set [11] if αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ),
8. *gα-closed set [20] if cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is gα-open in (X, τ),
9. β^*gα-closed set [21] if βcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is *gα-open in (X, τ).

Definition 2.2: A space (X, τ) is said to be
(i) ${}_cT_{\beta^*g\alpha}$-space [21] if every β^*gα-closed set is closed.
(ii) ${}_{1/2}T_{\beta^*g\alpha}$-space if every β^*gα-closed set is semi pre-closed.

Definition 2.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a
1. δ-continuous [18] if $f^{-1}(U)$ is δ-open in (X, τ) for every open set U in (Y, σ),
2. perfectly continuous [18] if $f^{-1}(U)$ is clopen in (X, τ) for every open set U in (Y, σ),
3. completely continuous [18] if $f^{-1}(U)$ is regular-open in (X, τ) for every open set U in (Y, σ),
4. quotient map [9], provided a subset U of (Y, σ) is open in (Y, σ) if and only if $f^{-1}(U)$ is open in(X, τ),
5. semi pre-continuous (β-continuous) [15] if $f^{-1}(U)$ is β-open in (X, τ) for every open set U in (Y, σ).

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be
1. β^*gα-continuous if $f^{-1}(U)$ is β^*gα-open in (X, τ) for every open set U in (Y, σ),
2. β^*gα-irresolute if $f^{-1}(U)$is β^*gα-open in (X, τ) for every β^*gα-open set U in (Y, σ),
3. β^*gα-closed if for every open set U in (X, τ), $f(U)$ is β^*gα-open in (Y, σ),
4. almost β^*gα-continuous if for every semi pre-open set V in (Y, σ), $f^{-1}(V)$ is β^*gα-open in (X, τ),
5. strongly-β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is open in (X, τ),
6. strongly semi-β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is semi-open in (X, τ),
7. almost β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is semi pre-open in (X, τ),
8. completely β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is regular-open in (X, τ),
9. completely β^*gα-continuous if for every regular-open set V in (Y, σ), $f^{-1}(V)$ is β^*gα-open in (X, τ),
10. perfectly β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is clopen in (X, τ),
11. contra β^*gα-irresolute if for every β^*gα-open set V in (Y, σ), $f^{-1}(V)$ is β^*gα - closed in (X, τ),
12. approximately- β-irresolute (briefly ap- β-irresolute) if βcl(A) $\subseteq f^{-1}(V)$ whenever V is β-open subset of (Y, σ), A is β^*gα-closed subset of (X, τ) and $A \subseteq f^{-1}(V)$,
13. approximately-β-closed (briefly ap- β-closed) if $f(A) \subseteq$ βint(V) whenever V is β^*gα-open subset of (Y, σ), A is β-closed subset of (X, τ) and $A \subseteq f^{-1}(V)$,
14. Perfectly contra β^*gα-irresolute if for every β^*gα-closed set V in (Y, σ), $f^{-1}(V)$ is β^*gα-clopen in (X, τ).

3. Strongly forms of β^*gα-irresolute and β^*gα-continuous functions, completely β^*gα-irresolute and β^*gα-continuous functions.

Theorem 3.1: Every almost β^*gα-irresolute map is β^*gα-irresolute.

Proof: Let V be β^*gα-open in (Y, σ). Since f is almost β^*gα-irresolute map, $f^{-1}(V)$ is semi pre-open in (X, τ). But every semi pre-open set is β^*gα-open. Hence $f^{-1}(V)$ is β^*gα-open in (X, τ). Therefore f is β^*gα-irresolute.

Converse of the above theorem need not be true by the following example.

Example 3.2: Let X = {a, b, c} = Y with τ ={ϕ, X, {a}, {a, b}} and σ ={ ϕ, Y, {a}, {b}, {a, b}, {a, c}}.

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
Semi pre open set of (X, τ) ={ ϕ, X , {a}, {a, b}, {a, c}},
β^*gα-open set of (X, τ) = {ϕ, X , {a}, {b}, {a, b}, {a, c}},
β^*gα-open set of (Y, σ) = {ϕ, Y , {a}, {b}, {a, b}, {a, c}}.

Here the function f is β^*gα-irresolute but not almost β^*gα-irresolute map, since the inverse image of β^*gα-open set {b} in (Y, σ) is not semi pre open in (X, τ).

Theorem 3.3: Every completely β^*gα-irresolute function is strongly-β^*gα-irresolute and β^*gα-irresolute.

Proof: Let V be β^*gα-open subset of (Y, σ). By hypothesis, $f^{-1}(V)$ is regular open in (X, τ). Since every regular open set is open, $f^{-1}(V)$ is open in (X, τ) and hence β^*gα-open. Thus f is strongly-β^*gα-irresolute and β^*gα-irresolute.

Converse of the above theorem need not be true by the following example.

Example 3.4: Let X ={a, b, c} = Y with τ ={ϕ, X , {a}, {b}, {a, b}, {a, c}} and σ = {ϕ, Y , {a}, {a, b}, {a, c}}.

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
β^*gα-open sets of (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}
Regular-open sets of (X, τ) = {ϕ,, {b}, {a, c}}
β^*gα-open sets of (X, τ) = {ϕ, X, {a}, {b}, {a, b}, {a, c}}

Here f is strongly-β^*gα-irresolute and β^*gα-irresolute but not completely β^*gα-irresolute since the inverse image of β^*gα-open sets {a} and {a, b} in (Y, σ) are not regular open (X, τ).

Theorem 3.5: Every almost β^*gα-irresolute map is almost β^*gα-continuous.

Proof: Let V be semi pre-open set in (Y, σ). Since every semi pre-open set is β^*gα-open, V is β^*gα-open. Since f is almost β^*gα-irresolute, $f^{-1}(V)$ is semi pre-open in (X, τ) and hence β^*gα-open. Therefore f is almost β*gα-continuous.

Example 3.6: Let X = {a, b, c} = Y with τ = {ϕ, X, {a}, {a, b}} and σ ={ ϕ, Y , {a}, {b}, {a, b}, {a, c}}.

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
Semi pre open set of (X, τ) = {ϕ,, {a}, {a, b}, {a, c}},
β^*gα-open sets of (X, τ) = {ϕ, X, {a},{b}, {a, b}, {a, c}},
β^*gα-open sets of (Y, σ) = {ϕ, Y , {a},{b}, {a, b}, {a, c}},
Semi pre-open sets of (Y, σ) = {ϕ, Y, {a},{b}, {a, b}, {a, c}}.

Here the function f is almost β^*gα-continuous but not almost β^*gα-irresolute map, since the inverse image of β^*gα-open set {b} in (Y, σ) is not semi pre open in (X, τ).

Theorem 3.7: Every β^*gα-irresolute map is almost β^*gα-continuous.

Proof: Let V be semi pre-open set in (Y, σ). Since every semi pre-open set is β^*gα-open, V is β^*gα-open. Since f is β^*gα-irresolute, $f^{-1}(V)$ is β^*gα-open in (X, τ). Therefore f is almost β*gα-continuous.

Example 3.8: Let X = {a, b, c}= Y with τ = {ϕ, X , {a}, {a, b}, {a, c}} and σ ={ ϕ, Y , {a}, {a, b}}.

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
Semi pre open set of (X, τ) = {ϕ, , {a}, {a, b}, {a, c}},
β^*gα-open sets of (X, τ) = {ϕ, X , {a}, {a, b}, {a, c}},
β^*gα-open sets of (Y, σ) = {ϕ, Y, {a},{b}, {a, b}, {a, c}},
Semi pre open sets of (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}.

Here the function f is almost β*gα-continuous but not β^*gα-irresolute map, since the inverse image of β^*gα-open set {b} in (Y, σ) is not β^*gα-open in (X, τ).

Theorem 3.9: Every strongly-β^*gα-irresolute map is almost β^*gα-irresolute.

Proof: Let V be β^*gα-open in (Y, σ). Since f is strongly-β^*gα -irresolute, $f^{-1}(V)$ is open in (X, τ) and hence semi pre-open. Therefore f is almost β^*gα-irresolute.

Example 3.10: Let X = {a, b, c}= Y with τ ={ϕ, X , {a}, {a, b}} and σ = { ϕ, Y , {a}, {a, b}, {a, c}}.

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
Semi pre open sets of (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}},
β^*gα-open sets of (X, τ) = {ϕ, X, {a}, {b},{a, b}, {a, c}},
β^*gα-open sets of (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}},
Semi pre-open sets of (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}.

Here the function f is almost β^*gα-irresolute but not strongly β^*gα-irresolute map, since the inverse image of β^*gα-open set {a, c} in (Y,σ) is not open in (X,τ).

Theorem 3.11: If $f: (X,\tau) \to (Y,\sigma)$ is continuous and Y is ${}_cT_{\beta^*g\alpha}$-space, then f is almost β^*gα-irresolute.

Proof: Let V be β^*gα-open in (Y,σ). Since Y is ${}_cT_{\beta^*g\alpha}$-space, V is open in Y. Since f is continuous, $f^{-1}(V)$ is open in (X,τ) and hence semi pre-open. Therefore f is almost β^*gα-irresolute.

Theorem 3.12: If $f: (X,\tau) \to (Y,\sigma)$ is β^*gα-irresolute and X is ${}_{1/2}T_{\beta^*g\alpha}$-space, then f is almost β^*gα-irresolute.

Proof: Let V be β^*gα-open in (Y,σ). Since f is β^*gα-irresolute, $f^{-1}(V)$ is β^*gα-open in (X, τ). Since X is ${}_{1/2}T_{\beta^*g\alpha}$-space, V is semi pre-open in X. Therefore f is almost β^*gα-irresolute.

Theorem 3.13: If $f: (X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\eta)$, then $g \circ f: (X,\tau) \to (Z,\eta)$ is

(i) almost β^*gα-irresolute if f is strongly-β^*gα-irresolute and g is β^*gα-irresolute.
(ii) strongly-β^*gα-irresolute if f is completely β^*gα-irresolute and g is β^*gα-irresolute.
(iii) almost β^*gα-irresolute if f is semi pre-continuous and g is strongly-β^*gα-irresolute.
(iv) completely β^*gα-irresolute if f is completely-continuous and g is strongly-β^*gα-irresolute.

Proof:

(i) Let V be β^*gα-open in (Z,η). Since g is β^*gα-irresolute, $g^{-1}(V)$ is β^*gα -open in (Y,σ). Since f is Strongly-β^*gα-irresolute, $f^{-1}(g^{-1}(V))$ is open in (X,τ) and hence semi pre-open. Therefore $g \circ f$ is Almost β^*gα-irresolute.
(ii) Let V be β^*gα-open in (Z,η). Since g is β^*gα-irresolute, $g^{-1}(V)$ is β^*gα-open in (Y,σ). Since f is Completely β^*gα-irresolute, $f^{-1}(g^{-1}(V))$ is regular-open in (X,τ) and hence open. Therefore $g \circ f$ is Strongly-β^*gα-irresolute.
(iii) Let V be β^*gα-open in (Z,η). Since g is strongly-β^*gα-irresolute, $g^{-1}(V)$ is open in (Y,σ). Since f is semi pre-continuous, $f^{-1}(g^{-1}(V))$ is semi pre-open in (X,τ). Therefore $g \circ f$ is almost β^*gα-irresolute.
(iv) Let V be β^*gα-open in (Z,η). Since g is strongly-β^*gα-irresolute, $g^{-1}(V)$ is open in (Y,σ). Since f is completely continuous, $f^{-1}(g^{-1}(V))$ is regular-open in (X,τ). Therefore $g \circ f$ is completely β^*gα-irresolute.

Theorem 3.14: If $f: (X,\tau) \to (Y,\sigma)$ is completely β^*gα-irresolute and A is regular open subset of X, then $f/A: A \to (Y,\sigma)$ is completely β^*gα-irresolute.

Proof: Let V be β^*gα-open in (Y,σ). Since f is completely β^*gα-irresolute, $f^{-1}(V)$ is regular-open in (X,τ). But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is regular-open in A. Hence $f/A: A \to (Y,\sigma)$ is completely β^*gα-irresolute.

Theorem 3.15: Let $f: (X,\tau) \to (Y,\sigma)$ and $\{A_\lambda: \lambda \in \wedge\}$ be a regular-open cover of (X,τ). Then f is completely β^*gα-irresolute if $f/A_\lambda: A_\lambda \to (Y,\sigma)$is completely β^*gα-irresolute for each $\lambda \in \wedge$.

Proof: Let V be β^*gα-open in (Y,σ). Since f/A_λ is completely β^*gα-irresolute, $(f/A_\lambda)^{-1}(V)$ is regular-open in (X,τ). Since A_λ is regular-open in X, $(f/A_\lambda)^{-1}(V) = A_\lambda \cap f^{-1}(V)$ is regular-open in X for each $\lambda \in \wedge$. Hence $f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_\lambda \cap f^{-1}(V): \lambda \in \wedge\} = \cup\{(f/A_\lambda)^{-1}(V) : \lambda \in \wedge\}$ is regular open in X. Therefore f is completely β^*gα-irresolute.

Theorem 3.16: If a function $f: (X,\tau) \to (Y,\sigma)$ is almost β^*gα-irresolute, then $f^{-1}(B)$ is semi pre-closed in X for any nowhere dense set B of Y.

Proof: Since B is nowhere dense subset of Y, $(Y - B)$ is regular-open in Y. Since every regular-open set is open, $(Y - B)$ is open and hence β^*gα-open. Since f is almost β^*gα-irresolute, $f^{-1}(Y - B)$ semi pre-open in X. Hence $f^{-1}(B)$ is semi pre-closed in X.

Theorem 3.17: Let X be submaximal and extremely disconnected space. Then the following are equivalent.

(i) f is strongly-semi β^*gα-irresolute
(ii) f is almost β^*gα-irresolute
(iii) f is strongly-β^*gα-irresolute.

Proof: Since X is submaximal and extremely disconnected, then open sets of (X,τ) = semi-open sets of (X,τ) = semi pre-open sets of (X,τ) and hence the proof.

Example 3.18: Let X = {a, b, c} with τ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and

τ^c = { ϕ, X, {b}, {c},{ b, c}, {a, c}}
semi-open sets of (X, τ) = {ϕ, X, {a}, {b},{a, b}, {a, c}}
semi pre-open sets of (X, τ) = {ϕ, X, {a}, {b},{a, b}, {a, c}}.

Theorem 3.19: The following are equivalent for a function $f: (X, \tau) \to (\mathrm{Y}, \sigma)$

(i) f is completely β^*gα-irresolute.
(ii) For each $x \in X$ and each β^*gα-open V of Y containing $f(x)$, there exists a regular open set U in X containing x such that $f(U) \subset V$.
(iii) $f^{-1}(V) \subset$ cl(int($f^{-1}(V)$)) for each β^*gα-open set V of Y.
(iv) $f^{-1}(F)$ is regular closed in X for every β^*gα-closed set F of Y .

Proof:
(i) ⇒(ii): Let $x \in X$ and V be β^*gα-open set of Y containing $f(x)$. Since f is completely β^*gα-irresolute, $f^{-1}(V)$ is regular open in X containing x. Let $U = f^{-1}(V)$. Then U is open in X and $f(U) \subset V$.

(ii) ⇒(iii): Let V be β^*gα-open set in Y and $x \in f^{-1}(V)$. By assumption, there exists an regular open set U in X containing x such that $f(U) \subset V$. Then $x \in U \subset$ int(U) $\subset$ int($f^{-1}(V)$) $\subset$ cl(int($f^{-1}(V)$)). Then $f^{-1}(V) \subset$ cl(int($f^{-1}(V)$)).

(iii) ⇒(iv): Let F be β^*gα-closed in Y . Set V = Y − F. Then V is β^*gα-open set in Y. By (iii), $f^{-1}(V) \subset$ cl(int($f^{-1}(V)$)) which implies $f^{-1}(V)$ is regular-open in X. Hence $f^{-1}(F)$is regular closed in X.

(iv) ⇒(i):. Let V be β^*gα-open set in Y. Let F = Y − V. That is F is β^*gα-closed in Y. Then $f^{-1}(F)$ is regular closed in X, (by iv). Then $f^{-1}(V)$ is regular open in X. Hence f is completely β^*gα-irresolute.

Theorem 3.20: For a function $f: (X, \tau) \to (\mathrm{Y}, \sigma)$ the following are equivalent

(i) f is perfectly contra- β^*gα-irresolute map,
(ii) f is contra- β^*gα-irresolute map and β^*gα-irresolute.

Proof:
(i) ⇒ (ii): Let V be β^*gα-open set in (Y, σ). Since f is perfectly contra- β^*gα-irresolute map, $f^{-1}(V)$ is β^*gα-closed and β^*gα-open in (X, τ). Hence f is contra- β^*gα-irresolute map and β^*gα-irresolute.

(ii) ⇒ (i): follows directly from definition.

Example 3.21: Let X ={a, b, c} with τ = { ϕ, X, {a}, {b, c}} and σ ={ ϕ, Y, {a}, {a, b}}

A function $f: (X, \tau) \to (\mathrm{Y}, \sigma)$ is defined by f(a) = a, f(b) = b, and f(c) = c,
β^*gα-open sets of (Y, σ) = { ϕ, Y, {a}, {b}, {a, b}, {a, c}}
β^*gα-open sets of (X, τ) = { ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
β^*gα-closed sets of (X, τ) = { ϕ, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}

Hence every perfectly contra-β^*gα-irresolute map is contra-β^*gα-irresolute map and β^*gα-irresolute and every contra-β^*gα-irresolute map and β^*gα-irresolute map is perfectly contra- β^*gα-irresolute map**.**

Theorem 3.22:

(i) If $f: (X, \tau) \to (\mathrm{Y}, \sigma)$ is ap-β-closed and A is a semi pre-closed subset of (X, τ), then its restriction $f_A: (A, \tau_A) \to (\mathrm{Y}, \sigma)$ is ap-β-closed.
(ii) If $f: (X, \tau) \to (\mathrm{Y}, \sigma)$ is ap- β -irresolute and A is an open, β-closed subset of (X, τ), then $f_A: (A, \tau_A) \to (\mathrm{Y}, \sigma)$ is ap-β -irresolute.

Proof:

(i) Suppose B is any arbitrary semi pre-closed subset of (A, τ_A) and U be β^*gα-open subset of (Y, σ) for which $f_A(B) \subseteq U$. Now we have B is semi pre-closed of (X, τ)) because A is semi pre-closed subset of (X, τ). Then $f_A(B) = f(B) \subseteq U$. Using Definition 2.4 (12), we have $f_A(B) \subseteq$ βint(U). Thus f_A is an ap- β-closed map.
(i) Assume that F is a β^*gα -closed subset relative to A, i.e., β^*gα-closed in(A, τ_A), and G is a β -open subset of (Y, σ) for which $F \subseteq {f_A}^{-1}(G)$. Then $F \subseteq f^{-1}(G) \cap A$. Since f is ap- β- irresolute βCl(F) $\subseteq f^{-1}(G)$. Then βCl(F) $\cap A \subseteq f^{-1}(G) \cap A$. Using the fact that βCl(F) $\cap A$ = $\beta\mathrm{Cl}_A(F)$ for every β-open subset, we have $\beta\mathrm{Cl}_A(\mathrm{F}) \subseteq {f_A}^{-1}(G)$. Thus $f_A: (A, \tau_A) \to (\mathrm{Y}, \sigma)$ is ap-β irresolute.

Theorem 3.23: For a function $f: (X,\tau) \to (\mathrm{Y},\sigma)$, the following are equivalent.

(i) (X,τ) is a $_{1/2}T_{\beta^*g\alpha}$- space.

(ii) f is ap- β-irresolute, for every space (Y,σ).

Proof:

(i) ⇒ (ii): Let V be β^*gα-closed set in (X,τ) and $V \subseteq f^{-1}(A)$, where $A \in \beta^*g\alpha O(\mathrm{Y},\sigma)$. Since (X,τ) is $_{1/2}T_{\beta^*g\alpha}$-space, V is β-closed. Then $V = \beta cl(V)$. Therefore $\beta cl(V) \subseteq f^{-1}(A)$ and hence f is ap- β^*gα-irresolute.

(i) ⇒ (ii): Let V be β^*gα-closed set in (X,τ) and B be β-open in (Y,σ), $V \subseteq f^{-1}(B)$, it follows that $\beta cl(V) \subseteq f^{-1}(B) = B$. Hence B is β-closed in (X,τ). Therefore (X,τ) is $_{1/2}T_{\beta^*g\alpha}$ -space.

Definition 3.24: A bijection $f: (X,\tau) \to (\mathrm{Y},\sigma)$ is,

(i) β^*gα-homeomorphism if both f and f^{-1} are β^*gα-continuous,

(ii) β^*gαc-homeomorphism if both f and f^{-1} are β^*gα-irresolute,

(iii) strongly-β^*gαc-homeomorphism if both f and f^{-1} are strongly-β^*gα-irresolute,

(iv) completely β^*gαc-homeomorphism if both f and f^{-1} are completely β^*gα-irresolute,

(v) almost β^*gαc-homeomorphism if both f and f^{-1} are almost β^*gα-irresolute.

Theorem 3.25: If a bijective function $f: (X,\tau) \to (\mathrm{Y},\sigma)$ is completely β^*gαc-homeomorphism, then

(i) f is β^*gαc-homeomorphism,

(ii) f is strongly-β^*gαc-homeomorphism.

Proof:

(i) Since a bijection f is completely β^*gαc-homeomorphism, f and f^{-1} are completely β^*gα-irresolute. Since every completely β^*gα-irresolute function is β^*gα-irresolute function, f and f^{-1} are β^*gα-irresolute functions and hence f is β^*gαc-homeomorphism.

(ii) Proof is obvious since every completely β^*gα-irresolute function is strongly- β^*gα-irresolute.

Remark 3.26: Every completely β^*gαc-homeomorphism is strongly-β^*gαc-homeomorphism and almost β^*gαc-homeomorphism.

Theorem 3.27: If $f: (X,\tau) \to (\mathrm{Y},\sigma)$ and $g: (\mathrm{Y},\sigma) \to (Z,\eta)$ are completely β^*gαc-homeomorphism then $g \circ f: (X,\tau) \to (Z,\eta)$ is completely β^*gαc-homeomorphism.

Proof: Let V be a β^*gα-open in (Z,η). Since g is completely β^*gαc-homeomorphism, g is completely β^*gα-irresolute and $g^{-1}(V)$ is regular-open in (Y,σ) for every β^*gα-open V in Z. Since every regular open is β^*gα-open, $g^{-1}(V)$ is β^*gα-open in (Y,σ). Also, since f is completely β^*gα-irresolute, $f^{-1}(U)$ is regular-open in (X,τ) and hence open in (X,τ). Therefore, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X,τ). Hence $g \circ f$ is completely β^*gα-irresolute.

Now, $(g \circ f)(A) = g\big(f(A)\big) = g(B)$, where $B = f(A)$. Since f is completely β^*gαc-homeomorphism, f^{-1} is completely β^*gα-irresolute. Therefore $f(A)$ is regular open in (Y,σ) and hence β^*gα-open. Now g is completely β^*gαc-homeomorphism, g^{-1} is completely β^*gα-irresolute. Therefore $g(B)$ is regular open in (Z,η) and hence open. Hence $(g \circ f)^{-1}$ is completely β^*gα-irresolute.

4. β^*gα-quotient map, Almost β^*gα-quotient map and Strongly β^*gα-quotient map.

Definition 4.1: A surjective map $f: (X,\tau) \to (\mathrm{Y},\sigma)$ is said to be a β^*gα-quotient map if f is β^*gα-continuous and $f^{-1}(V)$ is open in (X,τ) implies V is β^*gα-open in (Y,σ).

Definition 4.2: A surjective map $f: (X,\tau) \to (\mathrm{Y},\sigma)$ is said to be a almost β^*gα-quotient map if f is β^*gα-irresolute and $f^{-1}(V)$ is β^*gα-open in (X,τ) implies V is open in (Y,σ).

Definition 4.3: Let $f: (X,\tau) \to (\mathrm{Y},\sigma)$ be a surjective map. Then f is called strongly β^*gα-quotient map provided a set U of (Y,σ) is open in Y if and only if $f^{-1}(U)$ is a β^*gα-open in (X,τ).

Definition 4.4: Let $f: (X,\tau) \to (\mathrm{Y},\sigma)$ be a surjective map. Then f is called strongly β^*gα-open map provided a set U of (X,τ) is β^*gα-open in X if and only if $f(U)$ is a β^*gα-open in (Y,σ).

Proposition 4.5: If a map $f: (X,\tau) \to (\mathrm{Y},\sigma)$ is surjective, β^*gα-continuous and β^*gα-open, then f is a β^*gα-quotient map.

Proof: Let $f^{-1}(V)$ be open in (X, τ). Then $f(f^{-1}(V))$ is a $\beta^*g\alpha$-open set in (Y, σ), since f is $\beta^*g\alpha$-open. Hence V is a $\beta^*g\alpha$-open set, as f is surjective, $f(f^{-1}(V)) = V$. Thus, f is a $\beta^*g\alpha$-quotient map.

Proposition 4.6: If h: $(X, \tau) \to (Y, \sigma)$ is a $\beta^*g\alpha$ -quotient map and $g: (X, \tau) \to (Z, \eta)$ is a continuous map that is constant on each set $h^{-1}(y)$ for $y \in Y$, then g induces a $\beta^*g\alpha$ -quotient map f: $(Y, \sigma) \to (Z, \eta)$ such that $f \circ h = g$.

Proof: Since g is constant on $h^{-1}(y)$ for each $y \in Y$, the set $g(h^{-1}(y))$ is a one point set in (Z, η). If $f(y)$ denote this point, then it is clear that f is well defined and for each $x \in X$ $f(h(X)) = g(X)$.

We claim that f is $\beta^*g\alpha$ -continuous. For, let V be any open set in (Z, η), then $g^{-1}(V)$ is an open set in (X, τ) as g is continuous. But $g^{-1}(V) = h^{-1}(f^{-1}(V))$ is open in (X, τ). Since h is $\beta^*g\alpha$-quotient map, $f^{-1}(V)$ is a $\beta^*g\alpha$ -open set in (Y, σ). Hence f is $\beta^*g\alpha$-continuous.

Proposition 4.7: Let f: $(X, \tau) \to (Y, \sigma)$ be an strongly $\beta^*g\alpha$-open surjective and $\beta^*g\alpha$-irresolute map and g: $(Y, \sigma) \to (Z, \eta)$ be a strongly $\beta^*g\alpha$-quotient map then $g \circ f$: $(X, \tau) \to (Z, \eta)$ is a strongly $\beta^*g\alpha$-quotient map.

Proof: Let V be any open set in (Z, η). Since g is strongly $\beta^*g\alpha$-quotient, $g^{-1}(V)$ is a $\beta^*g\alpha$ -open set in (Y, σ). Since f is $\beta^*g\alpha$ -irresolute, $f^{-1}(g^{-1}(V))$ is a $\beta^*g\alpha$ -open set in (X, τ). Now, assume that $(g \circ f)^{-1}(V)) = f^{-1}(g^{-1}(V))$ is a $\beta^*g\alpha$ -open set in (X, τ) for open set V in (Z, η). Since f is strongly $\beta^*g\alpha$ -open, $f(f^{-1}(g^{-1}(V)))$ is a $\beta^*g\alpha$ -open set in (Y, σ). It follows that $g^{-1}(V)$ is a $\beta^*g\alpha$ -open set in (Y, σ). Since g is strongly $\beta^*g\alpha$ -quotient, V is an open set in (Z, η). Thus $g \circ f$ is strongly $\beta^*g\alpha$ -quotient map.

Theorem 4.8: Let f: $(X, \tau) \to (Y, \sigma)$ be a $\beta^*g\alpha$-quotient map where (X, τ) and (Y, σ) are ${}_cT_{\beta^*g\alpha}$-spaces. Then g: $(Y, \sigma) \to (Z, \eta)$ is a strongly-$\beta^*g\alpha$-irresolute if and only if the composite map $g \circ f$: $(X, \tau) \to (Z, \eta)$ is strongly-$\beta^*g\alpha$-irresolute.

Proof: Let g be strongly-$\beta^*g\alpha$-irresolute and U be any $\beta^*g\alpha$-open set in (Z, η). Then $g^{-1}(U)$ is open in (Y, σ). Since f: $(X, \tau) \to (Y, \sigma)$ be a $\beta^*g\alpha$-quotient map, $(g \circ f)^{-1}(U)) = f^{-1}(g^{-1}(U))$ is $\beta^*g\alpha$-open (X, τ). Since (X, τ) is a ${}_cT_{\beta^*g\alpha}$-space, $f^{-1}(g^{-1}(U))$ is open in (X, τ). Thus the composite map $g \circ f$ is strongly-$\beta^*g\alpha$ -irresolute.

Conversely let the composite map $g \circ f$ be strongly-$\beta^*g\alpha$ -irresolute. Then for any $\beta^*g\alpha$-open set U in(Z, η), $f^{-1}(g^{-1}(U))$ is open in (X, τ). Since f is a $\beta^*g\alpha$-quotient map, it implies that $g^{-1}(U)$ is $\beta^*g\alpha$-open in (Y, σ). Since (Y, σ) is a ${}_cT_{\beta^*g\alpha}$ -space, $g^{-1}(U)$ is open in (Y, σ). Hence g is strongly-$\beta^*g\alpha$ -irresolute.

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