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FIXED POINT THEOREMS IN FUZZY METRIC SPACES WITH INTEGRAL TYPE INEQUALITY

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ABSTRACT

This paper is to present common fixed point theorem in fuzzy metric spaces for occasionally weakly compatible mappings with integral type inequality.

Keywords: *Fuzzy metric space, occasionally weakly compatible (owc) mappings, common fixed point.*

1. INTRODUCTION

Prior to 1968 all work involving fixed poins used the Banach contraction principle. The concept of fuzzy set was initially introduced by Zadeh [27]. Kramosil and Michalek [15] introduced fuzzy metric space, many authors extend their views, Grorge and Veermani [6] modified the notion of fuzzy metric spaces with the help of continuous t-norms Grabiec[9], Subramanyam[29], Vasuki[26], Pant and Jha, [21] obtained some analogous results proved by Balasubramaniam *et al.* Subsequently, it was developed extensively by many authors and used in various fields, Jungck [11] introduced the notion of compatible maps for a pair of self maps. Several papers have come up involving compatible mapping proving the existence of common fixed points both in the classical and fuzzy metric spaces. The theory of fixed point equations is one of the basic tools to handle various physical formulations. Fixed point theorems in fuzzy mathematics has got a direction of vigorous hope and vital trust with the study of Kramosil and Michalek [15], who introduced the conce of fuzzy metric space. Later on this concept of fuzzy metric space was modified by George and Veermani [7] Sessa [28] initiated the tradition of improving commutative condition in fixed point theorems by introducing the notion of weak commuting property. Further Juncgk [11] gave a more generalized condition defined as compatibility in metric spaces. Jungck and Rhoades [12] introduced the concept of weakly compatible maps which were found to be more generalized than compatible maps Grabiec [8] obtained fuzzy version of Banach contraction principle. Singh M.S. Chauhan [30] brought forward the concept of compatibility in fuzzy metric space. Pan [19, 20, 21] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam *et al.* [5], have shown that Rhoade [23] open problem on the existence of contractive definition which generates a fixed poin but does not force the mappings to be continuous at the fixed point, posses an affirmativ answer. Recent literature in fixed point in fuzzy metric space can be viewed in [2, 3, 10, 17, 25, 31]. This paper proves the fixed point theorems on fuzzy metric space which generalize extend and fuzz if y several known fixed point theorems for occasionally compatible maps on metric space by making use of integral type inequality. introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam *et al.* [5], have shown that Rhoades [23] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Recent literature in fixed point in fuzzy metric space can be viewed in [2, 3, 10, 17, 25,31].This paper proves the fixed point theorems on fuzzy metric spaces which generalize extend and fuzz if y several known fixed point theorems for occasionally compatible maps on metric space by making use of integral type inequality. [30] brought forward the concept of compatibility in fuzzy metric space. Pant [19, 20, 21] introduced the new concept reciprocally continuous mappings and established some commonfixed point theorems. Balasubramaniam *et al.* [5], have shown that Rhoades [23] open problem on the existence of contractive definition which generates a fixed point

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but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Recent literature in fixed point in fuzzy metric space can be viewed in [2, 3, 10, 17, 25,31]. This paper proves the fixed point theorems on fuzzy metric spaces which generalize extend and fuzz if y several known fixed point theorems for occasionally compatible maps on metric space by making use of integral type inequality.

2. PRELIMINARIES

Definition 2.1: [27] A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2: [24] A binary operation *: [0, 1] x [0, 1] — [0, 1] is a continuous t-norms if it satisfies the following conditions:
(i) *is associative and commutative
(ii) *is continuous
(iii) $a * 1 = a$ for all a $\in$ [0,1];
(iv) *b $\leq$ c*d whenever a $\leq$ c and b $\leq$ d, and a, b, c, d $\in$ [0,1].

Definition 2.3: [7] A 3-tuples (X, M, *****) is said to be a fuzzy metric space (shortly FM Space) if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on X^2x[0, ∞) satisfying the following conditions, for all x, y, z $\in$ X and s, t > 0;
(FM 1): M(x, y, t) > 0
(FM 2): M(x, y, t) = 1 for all t > 0 if and only if x = y
(FM3): M(x, y, t) = M(y, x, t)
(FM 4): M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s)
(FM 5): M(x, y, .) : [0,∞) →(0,1] is left continuous. (X, M, *) denotes a fuzzy metric space, (x, y, t) can be thought of as degree of nearness between x and y with respect to t We identify x = y with M(x, y, t) = 1for all t > 0. In the following example every metric induces a fuzzy metric.

Example 2.4: Let $X = [0,1]$, t-norm defined by $a * b = min\{a, b\}$ where a, b∈[0, 1] and M is the fuzzy set on X^2 x (0, o) defined by M(x, y, t) = **[exp**$\left[\frac{|x-y|}{t}\right]$} $]^{-1}$for all $x, y \in X, t > 0$. Then (X, M, *****) is a fuzzy metric space.

Example 2.5: (Induced fuzzy metric [6]) Let (X, d) be a metric space, denote a * b = a.b & for all $a, b \in [0,1]$ and let M_d be fuzzy sets on X^2 x(0,∞). Defined as follows

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

Then (X, M, *****) is a fuzzy metric space. We call this fuzzy metric induced by a metric **d** as the standard intuitionistic fuzzy metric.

Definition 2.6: [12] Two self mappings f and g of a fuzzy metric space (X, M, *) are called compatible if $lim_{n\to\infty} M(fgx_n, gfx_n)$ = 1 wherever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n$ = x for some x in X

Definition 2.7: [6] Two self maps f and g of a fuzzy metric space (X, M,*) are called reciprocally continuous on X if $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n$ *and* $lim_{n\to\infty} gfx_n = gx$ wherever $\{x_n\}$ is sequence in X, such that
$lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = x$ for some x in X..

Definition 2.8: [7] Let (X, M, *****) be a fuzzy metric space.
(a). Then A sequence $\{x_n\}$in X is said
(i) to converges to x in X if for each ε> 0 and each t > 0, there exist $n_o \in$ N such that M $(x_n, x, t) > 1 - \varepsilon$ for all n≥,n_o
(ii) to be Cauchy if for each ε> 0 and each t > 0, there exist n_o εN such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all n, m $\geq n_0$,
(b) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.9: Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute..

Definition 2.10: Let (X, d) be a compatible metric space, a∈[0,1], f:X→X a mapping such that for each x, y ∈X

$$\int_o^{d(fx,fy)} \varphi(t)dt \leq \alpha \int_0^{d(x,y)} \varphi(t)dt$$

where $\psi : R^+ \to R$ is lebesgue integral mapping which is summable
$\varepsilon \geq, 0, \int_0^{\varepsilon} \varphi(t)dt > 0$
nonnegative and such that, for each. Then f has a unique common fixed z ∈X such that for each x∈ $\boldsymbol{x}$, $\lim_{n\to\infty} f^n x = z$

Rhodes [30], extended this result by replacing the above condition by the following:

$\int_0^{d(fx,fy)} \varphi(t)dt \leq \alpha \int_0^{\max\{d(x,y),d(y,fx),d(y,fy)\frac{1}{2}[d(x,fy)+d(x,fx)\}} \varphi(t)dt$Ojha *et al.* (2010). Let (X, d) be a metric space and let $f: X \to X$, $F: X \to CB(X)$ be single and a multivalued map respectively, suppose that f and F are occasionally weakly commutative (owc) and satisfy the inequality

$$\int_0^{d((Fx,Fy)^p} \varphi(t)dt \leq \int_0^{max\{d(fx,fy)d^{p-a}(fx,Fx),as(fx,fy)d^{p-1}(fy,Fy),ad(fx,Fx)d^{p-1}(fx,Fy)d(fy,Fx)\}^p} \varphi(t)dt$$

For all x, y in X, where $P \geq 2$ is an integer a≥ 0 and 0 < c < 1 then f and F have unique common fixed point in X.

Lemma 2.1: [13] Let X be a set, f, g owc self maps of X. Iff and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point off and g

Lemma 2.2: Let (X, M, *) be a fuzzy metric space. If there exist q∈(0, 1) such that
M (x, y, qt) $\geq$ M (x, y, t) for all x, y $\in$ X & t > 0 then x = y.

3. MAIN RESULTS

Theorem 3.1: Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T are self-mapping of X. Let the pairs {P,S} and {Q ,T} be owc . If there exists q **E** (0, 1) such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)dt \geq \int_0^{\min\ M(Sx.Ty,t)*M(Ax.Sx,t)*M(By,Ty,t)*M(Ax,Ty,t)} \varphi(t)dt$$

For all x, y∈X and for all t > 0, then there exists a unique point w∈X such that Aw = Bw = w and a unique point z **E** X such that Qz = Tz = z Moreover, z = w, so that there is a unique common fixed point of A,B,S and T.

Proof: Since AX ⊂ TX and BX ⊂ SX, for any x0 ∈ X, there exists x_1 ∈ X such that $Ax_0 = Tx_1$ and for this x_1 ∈ X, there exists x_2 ∈ X such that $Bx_1 = Sx_2$.Inductively, we can find a sequence $\{y_n\}$ in X as follows:

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ for } n = 1, 2 \cdots .$$

From (iii),

$$\int_0^{M\left(y_{2n+1,}y_{2n+2,}qt\right)} \varphi(t)dt$$

$$= \int_0^{Ax_{2n},Bx_{2n+1},qt)} \varphi(t)dt$$

$$\geq \int_0^{M(sx_{2n},Tx_{2n+1},t)*M(Ax_{2n},sx_{2n+1}t)*M(Bx_{2n+1,}Tx_{2n+1,},t)*M(Ax_{2n},Tx_{2n+1}t)} \varphi(t)dt$$

$$= \int_0^{M(y_{2n},y_{2n+1,},t)*M(y_{2n+1,}y_{2n},t)*M(y_{2n+2,}y_{2n},i)*M(y_{2n+1,}y_{2n+1},t)} \varphi(t)dt$$

$$\geq \int_0^{M(y_{2n},y_{2n+1,},t)*M(y_{2n+1},y_{2n+2},t)} \varphi(t)dt$$

We have$\int_0^{M\left(y_{n+1,}y_{n+2,}qt\right)} \varphi(t)dt \geq \int_0^{M(y_n,y_{n-1},t)} \varphi(t)dt$

$$\int_0^{M(y_n,y_{n+1},t)} \varphi(t)dt \geq \int_0^{M(y_n,y_{n-1},t/q)} \varphi(t)dt \geq \int_0^{M(y_{n-2},y_{n-1},t/q^2)} \varphi(t)dt \geq \cdots \int_0^{M(y_n,y_{n+1},t/q^n)} \varphi(t)dt \to 1\ as\ n \to \infty$$

$$\int_0^{M(y_n,y_{n+1},t)} \varphi(t)dt \to 1\ as\ n \to \infty\ for\ any\ t > 0$$

For each ε>0 and each t>0,we can choose n_0 ε N such that $M(y_n,Y_{n+1},t) > 1-\varepsilon$ for all $n > n_0$.

For m, n ε N, we suppose m≥n. Then we have that

$M(y_n, y_m, t) \geq M(y_n,y_{n+1},t/m-n)*M(y_{n+1},y_{n+2},t/m-n)*\ldots\ldots*M(y_{m-1},y_m,t/m-n)$
$> (1-\varepsilon)*(1-\varepsilon)*\ldots\ldots*(1-\varepsilon) \geq (1-\varepsilon)$

and hence $\{ y_n\}$ is a Cauchy sequence in X.

Since (X. M, *) is complete,$\{y_n\}$ converges to some point z∈X, and so $\{Ax_{2n-2}\}$,$\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z.,

$ASx_{2n} \to Sz$ and $BTx_{2n-1} \to TZ$ From we get

$$\int_0^{M(ASx_{2n},BTx_{2n-1},qt)} \varphi(t)dt \geq \int_0^{M(SSx_{2n},TTx_{2n-1},t)*M(ASx_{2n},SSx_{2n},t)*M(BTx_{2n-1},TTx_{2n-1},t)*M(ASx_{2n},TTx_{2n-1}t)} \varphi(t)dt$$

Taking limit as n→∞

$$\int_0^{M(Sz,Tz,qt)} \varphi(t)dt \geq \int_0^{M(Sz,Tz,t)*M(Sz,Sz,t)*M(Tz,Tz,t)*M(Sz,Tz,t)} \varphi(t)dt$$
$$\geq \int_0^{M(Sz,Tz,t)*1*M(Sz,Tz,t)} \varphi(t)dt$$
$$\geq \int_0^{M(Sz,Tz,t)} \varphi(t)dt$$

Thus we have

$\int_0^{M(Sz,Tz,qt)} \varphi(t)dt \geq \int_0^{M(Sz,Tz,t} \varphi(t)dt$ and hence Sz=Tz

Now,

$\int_0^{M(Az,BTx2n-1,qt)} \varphi(t)dt \geq \int_0^{M(Sz,TTx2n-1,t)*M(Az,Sz,t)*M(BTx2n-1,TTx2n-1,t)*M(Az,TTx2n-1,t)} \varphi(t)dt$

which implies that

taking limit as n→ ∞

$$\int_0^{M(Az,Bz,qt)} \varphi(t)dt \geq \int_0^{M(Sz,Tz,t)*M(Az,Sz,t)*M(Bz,Tz,t)*M(Az,Tz,t)} \varphi(t)dt$$
$$\geq \int_0^{M(Az,Tz,t)} \varphi(t)dt$$

And hence Az=Tz

$$\int_0^{M(Az,Bz,qt)} \varphi(t)dt \geq \int_0^{M(Sz,Tz,t)*M(Az,Sz,t)*M(Bz,Tz,t)*M(Az,Tz,t)} \varphi(t)dt$$
$$= \int_0^{M(Az,Az,t)*M(Az,Az,t)*M(Bz,Az,t)*M(Az,Az,t)} \varphi(t)dt$$
$$= \int_0^{M(Az,Bz,t)} \varphi(t)dt$$

And so AZ=Bz

It gives that AZ=Bz=Tz=SZ

Now, we show that Bz=Z

We get $\int_0^{AX_{2n},Bz,qt)} \varphi(t)dt \geq \int_0^{M(Sx_{2n},Tz,t)*M(Ax_{2n},Sx_{2n},t)*M(BZ,Tz,t)*M(Ax_{2n},Tz,t)} \varphi(t)dt$

Which implies that taking as n→ ∞

And

$$\int_0^{M(z,Bz,qt)} \varphi(t)dt \geq \int_0^{M(z,Tz,t)*M(z,z,t)*M(Bz,Tz,t)*M(z,Tz,t)} \varphi(t)dt$$
$$\geq \int_0^{M(z,Bz,t)*1*M(Az,Az,t)*M(z,Bz,t)} \varphi(t)dt$$
$$\geq \int_0^{M(z,Bz,t)} \varphi(t)dt$$

And hence Bz=z thus from Z=Az=Bz=Tz=Sz and z is a common fixed point of A, B, S, and T

For uniqueness, let w be another common fixed point of A, B, S, and T then

$$\int_0^{M(z,w,qt)} \varphi(t)dt = \int_0^{M(AZ,Bw,qt)} \varphi(t)dt$$
$$\geq \int_0^{M(Sz,Tw,t)*M(Az,Sz,t)*M(Bw,Tw,t)*M(Az,Tw,t)} \varphi(t)dt$$
$$\geq \int_0^{M(z,w,t)} \varphi(t)dt$$

Hence z=w. This complete the proof of the theorem.

Corollary 3.1: Let (X, M, $*$) be a complete fuzzy metric space and let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T}are owc. If $M(Sx, Ty, t)$ there exists a point q∈ $(0,1)$ for all x, y∈X and t.> 0

$$\int_0^{m(Ax,By,qt)} \varphi(t)dt \geq \int_0^{M(Sx,Ty,t)*M(Ax,Sy,t)M(Bx,Ty,t)*M(By,Sy,t)*M(By,Sx,2t)*M(Ax,Ty,t)} \varphi(t)dt$$

For every x, y∈X and t>0.Then A, B, S and T have unique fixed point in X.

Corollary 3.2.: Let (X, M, $*$) be a complete fuzzy metric space and let A, B, S and T be self mappings of X. Let the pairs {A, S}and {B,T} are owc. If $M(Sx, Ty, t)$ there exists a point q∈ (0,1)for all x, y∈ X and t.> 0

$$\int_0^{m(Ax,By,qt)} \varphi(t)dt \geq \int_0^{M(Sx,Ty,t)} \varphi(t)dt$$

For every x, y∈X and t > 0.Then A, B, S and T have unique fixed point in X.

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