International Journal of Mathematical Archive-7(1), 2016, 1-7
Available online through www.ijma.info ISSN 2229 – 5046

ON (*m, k*)-REGULAR FUZZY GRAPHS

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(Received On: 06-12-15; Revised & Accepted On: 13-01-16)

ABSTRACT

In this paper, (m, k) - regular fuzzy graph and totally (m, k)-regular fuzzy graph are introduced and compared through various examples. Also, comparative study between (m, k)-regularity and totally (m, k)-regularity is done. A necessary and sufficient condition under which they are equivalent is provided. (m, k)-regularity on some fuzzy graphs whose underlying crisp graphs are a path and a cycle is studied with some specific membership functions.

Key Words: *degree of a vertex in fuzzy graph, regular fuzzy graph, total degree,* d_m *-degree of a vertex in fuzzy graph, total* d_m*- degree of a vertex in fuzzy graph, m- neighborly irregular fuzzy graphs.*

AMS Subject Code Classification 2010: *05C12, 03E72, 05C72.*

1. INTRODUCTION

In 1965, Lofti A. Zadeh introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situations. Azriel Rosenfeld introduced fuzzy graph in 1975[5]. It has been growing fast and has numerous applications in various fields. N. R. Santhi Maheswari and C.Sekar introduced d_2 *-degree* of a vertex in graphs and (2, *k*)-regular graphs [6]. A. Nagoor Gani and K. Radha introduced regular fuzzy graph, total degree and totally regular fuzzy graph [4]. Mini Tom and M.S.Sunitha introduced sum distance in fuzzy graphs [1]. M.S.Sunitha and Sunil Mathew discussed about growth of fuzzy graphs in fuzzy graph theory-A survey [11].

N. R. Santhi Maheswari and C.Sekar introduced d_2 *-degree, total* d_2*-degree* of a vertex in fuzzy graph, (2, *k*)-regular fuzzy graph and totally (2, *k*)-regular fuzzy graphs [7], (*r*, 2, *k*)-regular fuzzy graph and totally (*r*, 2, *k*)-regular fuzzy graphs [8]. Also they introduced d_m *-degree, total* d_m*-degree* of a vertex in fuzzy graph, *m*-Neighbourly Irregular fuzzy graph and totally *m*-Neighbourly totally irregular fuzzy graphs [9]. This is the background to define (*m*, *k*)-regular fuzzy graph and totally (*m*, *k*)-regular fuzzy graph [10]. Here, (*m*, *k*)-regularity on some fuzzy graphs whose underlying crisp graphs are a path and a cycle is studied with some specific membership functions.

2. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1: A Fuzzy graph denoted by G: (σ, μ) on graph G^*. (V,E) is a pair of functions (σ, μ) where σ: $V \rightarrow [0, 1]$ is a fuzzy subset of a non empty set V and μ: $V X V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv)= \sigma(u) \Lambda \sigma(v)$ is satisfied, where σ and μ are called membership function. A fuzzy graph G . (σ, μ) is complete if $\mu(u, v) = \mu(uv) = \sigma(u) \Lambda \sigma(v)$ for all $u, v \in V$, where uv denotes the edge between u and v. The graph G^*: (V, E) is called the underlying crisp graph of the fuzzy graph G. (σ, μ) [2].

Definition 2.2: The strength of connectedness between two vertices u and v is $\mu^{\infty}(u, v) = sup\{ \mu^k(u, v) : k = 1, 2, \ldots \}$ where $\gamma^k(u, v) = sup\{\mu(uu_1) \Lambda \mu(u_1u_2) \Lambda \ldots \Lambda \mu(u_{k-1}v) : u, u_1, u_2, \ldots, u_{k-1}, v$ is a path connecting u and v of length $k\}$ [3].

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Definition 2.3: Let G : (σ, μ) be a fuzzy graph on $G^*: (V,E)$. The degree of a vertex u is $d_G(u) = \Sigma\, \mu(uv)$ for $uv \in E$ and $\mu(uv) = 0$, for uv not in E, this is equivalent to $d_G(u) = \sum_{u \neq v\; uv \in E} \mu(uv)$[4].

Definition 2.4: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be an regular fuzzy graph of degree k[4].

Definition 2.5: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. The total degree of a vertex u is defined as $td(u) = \Sigma\, \mu(u, v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be a totally regular fuzzy graph of degree k or k-totally regular fuzzy graph [4].

Definition 2.6: Let G: (σ, μ) be a fuzzy graph. The *d_2-degree* of a vertex u in G is $d_2(u) = \Sigma\mu^2(u, v)$, where $\mu^2(uv) = \sup \{\mu(uu_1) \wedge \mu(u_1v) : u, u_1, v$ is the shortest path connecting u and v of length 2$\}$. Also, $\mu(uv) = 0$, for uv not in E.

The minimum d_2-degree of G is $\delta_2(G) = \Lambda\{d_2(v) : v \in V\}$.

The maximum d_2-degree of G is $\Delta_2(G) = \vee\{d_2(v) : v \in V\}$[7].

Definition 2.7: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. If $d_2(v) = k$ for all $v \in V$, then G is said to be a $(2, k)$-*regular* fuzzy graph[7].

Definition 2.8: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. If $d(v) = r$ and $d_2(v) = k$ for all $v \in V$, then G is said to be an $(r, 2, k)$-*regular* fuzzy graph[8].

Definition 2.9: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \Sigma\, \mu^2(u, v) + \sigma(u) = d_2(u) + \sigma(u)$.

The minimum td_2-degree of G is $t\delta_2(G) = \Lambda\{td_2(v) : v \in V\}$.

The maximum td_2-degree of G is $t\Delta_2(G) = \vee\{td_2(v) : v \in V\}$[7].

Definition 2.10: If each vertex of a fuzzy graph G has the same total d_2 - degree k, then G is said to be a totally $(2, k)$-regular fuzzy graph[7].

Definition 2.11: If each vertex of a fuzzy graph G has the same degree r and same total d_2 - degree k, then G is said to be a totally $(r, 2, k)$-regular fuzzy graph [8].

Definition 2.12: Let G: (σ, μ) be a fuzzy graph on $G^*: (V, E)$. The *d_m-degree* of a vertex u in G is $d_m(u) = \Sigma\, \mu^m(uv)$, where $\mu^m(uv) = \sup \{ \mu(uu_1) \wedge \mu(u_1u_2) \wedge \ldots \wedge \mu(u_{m-1}v) : u, u_1, u_2, \ldots, u_{m-1}, v$ is the shortest path connecting u and v of length $m\}$. Also, $\mu(uv) = 0$, for uv not in E.

The minimum d_m-degree of G is $\delta_m(G) = \Lambda\{ d_m(v) : v \in V \}$

The maximum d_m-degree of G is $\Delta_m(G) = \vee\{d_m(v) : v \in V \}$[9].

Definition 2.13: Let G: (σ, μ) be a fuzzy graph on $G^*:(V,E)$. The total d_m-degree of a vertex $u \in V$ is defined as $td_m(u) = \Sigma\, \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)$.

The minimum td_m-degree of G is $t\delta_m(G) = \Lambda\{ td_m(v): v \in V\}$.

The maximum td_m-degree of G is $t\Delta_m(G) = \vee\{td_m(v): v \in V\}$[9].

3. (m, k)-REGULAR FUZZY GRAPHS

In this section, we define (m, k)-Regular Fuzzy Graphs and illustrates this with $(3, k)$-regular graph.

Definition 3.1: Let G: (σ, μ) be a fuzzy graph on $G^*:(V,E)$. If $d_m(v) = k$ for all $v \in V$, then G is said to be an (m, k) - *regular* fuzzy graph.

Example 3.2: Consider G^*: (V, E) where $V = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_1\}$. Define G : (σ, μ) by $\sigma(u_1) = 0.5$, $\sigma(u_2) = 0.4$, $\sigma(u_3) = 0.4$, $\sigma(u_4) = 0.5$, $\sigma(u_5) = 0.4$, $\sigma(u_6) = 0.4$, $\sigma(u_7) = 0.5$ and $\mu(u_1u_2) = 0.2$, $\mu(u_2u_3) = 0.3$, $\mu(u_3u_4) = 0.2$, $\mu(u_4u_5) = 0.3$, $\mu(u_5u_6) = 0.2$, $\mu(u_6u_7) = 0.3$, $\mu(u_7u_1) = 0.2$

$d_3(u_1) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 = 0.4.$

$d_3(u_2) = \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.3\} = 0.2 + 0.2 = 0.4.$

$d_3(u_3) = \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 = 0.4.$

$d_3(u_4) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.3\} = 0.2 + 0.2 = 0.4.$

$d_3(u_5) = \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.2 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 = 0.4.$

$d_3(u_6) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.2\} = 0.2 + 0.2 = 0.4.$

$d_3(u_7) = \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.2 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 = 0.4.$

It is noted that $d_3(u_1) = 0.4$, $d_3(u_2) = 0.4$, $d_3(u_3) = 0.4$, $d_3(u_4) = 0.4$, $d_3(u_5) = 0.4$, $d_3(u_6) = 0.4$, $d_3(u_7) = 0.4$. Each vertex has the same d_3-degree 0.4. Hence G is a (3, 0.4)-regular fuzzy graph.

Example 3.3: Consider G^*: (V,E), where $V = \{u, v, w, x, y, z\}$ and $E = \{uv, vw, wx, xy, yz, zu\}$

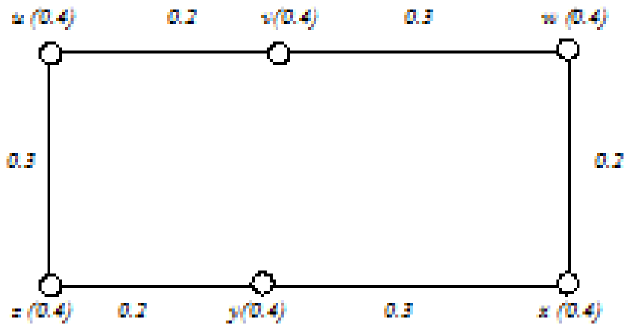

Figure-1

$d_3(u) = Sup\ \{0.2 \wedge 0.3 \wedge 0.2, 0.3 \wedge 0.2 \wedge 0.3\} = Sup\ \{0.2, 0.2\} = 0.2.$

$d_3(v) = Sup\ \{0.2 \wedge 0.3 \wedge 0.2, 0.3 \wedge 0.2 \wedge 0.3\} = Sup\ \{0.2, 0.2\} = 0.2.$

$d_3(w) = Sup\ \{0.3 \wedge 0.2 \wedge 0.3, 0.2 \wedge 0.3 \wedge 0.2\} = Sup\ \{0.2, 0.2\} = 0.2.$

$d_3(x) = Sup\ \{0.4 \wedge 0.3 \wedge 0.2, 0.4 \wedge 0.3 \wedge 0.2\} = Sup\ \{0.2, 0.2\} = 0.2.$

$d_3(y) = Sup\ \{0.4 \wedge 0.4 \wedge 0.3, 0.3 \wedge 0.2 \wedge 0.2\} = Sup\ \{0.2, 0.2\} = 0.2.$

$d_3(z) = Sup\ \{0.2 \wedge 0.2 \wedge 0.2, 0.3 \wedge 0.4 \wedge 0.4\} = Sup\ \{0.2, 0.2\} = 0.2.$

In Figure 1, $d_3(u) = 0.2$, $d_3(v) = 0.2$, $d_3(w) = 0.2$, $d_3(x) = 0.2$, $d_3(y) = 0.2$, $d_3(z) = 0.2$. Each vertex has the same d_3-degree 0.2. Hence G is a (3, 0.2)-regular fuzzy graph.

4. TOTALLY (m, k)-REGULAR FUZZY GRAPHS

In this section, we define totally (m, k)-regular fuzzy graphs and the necessary and sufficient condition under which they are equivalent is provided.

Definition 4.1: If each vertex of G has the same total d_m - degree k, then G is said to be totally (m, k)-regular fuzzy graph.

Example 4.2: From Figure 1, we have

$td_3(u) = d_3(u) + \sigma(u) = 0.2 + 0.4 = 0.6.$

$td_3(v) = d_3(v) + \sigma(v) = 0.2 + 0.4 = 0.6.$

$td_3(w) = d_3(w) + \sigma(w) = 0.2 + 0.4 = 0.6.$

$td_3(x) = d_3(x) + \sigma(x) = 0.2 + 0.4 = 0.6.$

$td_3(y) = d_3(y) + \sigma(y) = 0.2 + 0.4 = =0.6.$

$td_3(z) = d_3(z) + \sigma(z) = 0.2 + 0.4 = 0.6.$

It is noted that each vertex has the same total d_3-degree 0.6.

Hence G is a totally (3, 0.6)-regular fuzzy graph. This fuzzy graph is a (3, 0.2)-regular fuzzy graph which is a totally (3, 0.6)-regular fuzzy graph.

Remark 4.3: The (3, 0.4)-regular fuzzy graph given in example 3.2 is not totally $(3, k)$-regular graph. Since $td_3(u_1) = 0.9$, $td_3(u_2) = 0.8$, $td_3(u_3) = 0.8$, $td_3(u_4) = 0.9$, $td_3(u_5) = 0.8$, $td_3(u_6) = 0.8$, $td_3(u_7) = 0.9$. It is noted that $td_3(u_1) = td_3(u_4) = td_3(u_7) = 0.9$ and $td_3(u_2),= td_3(u3) = td_3(u_5) = td_3(u_6) = 0.8$.

Theorem 4.4: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). Then $\sigma(u) = c$, for all $u \in V$ iff the following conditions are equivalent.

1. G: (σ, μ) is an (m, k)-regular fuzzy graph.
2. G: (σ, μ) is a totally $(m, k + c)$-regular fuzzy graph.

Proof: Suppose that $\sigma(u) = c$, for all $u \in V$. Assume that G : (σ, μ) is (m, k)-regular fuzzy graph.

Then $d_m(u) = k$, for all $u \in V$. So, $td_m(u) = d_m(u) + \sigma(u)$, *for all* $u \in V$.
$\Longrightarrow td_m(u) = k + c$, *for all* $u \in V$.

Hence G: (σ, μ) is totally $(m, k + c)$-regular fuzzy graph.

Thus (1) $\Longrightarrow$ (2) is proved.

Suppose G: (σ, μ) is totally $(m, k + c)$-regular fuzzy graph.
$\Longrightarrow td_m(u) = k + c$, *for all* $u \in V$

$\Longrightarrow d_m(u) + \sigma(u) = k + c$, *for all* $u \in V$

$\Longrightarrow d_m(u) + c = k + c$, *for all* $u \in V$

$\Longrightarrow d_m(u) = k$, *for all* $u \in V$

Hence G: (σ, μ) is (m, k)-regular fuzzy graphs. Hence (1) and (2) are equivalent.

Conversely assume that (1) and (2) are equivalent. Let G: (σ, μ) is a totally $(m, k+c)$-regular fuzzy graph and an (m, k)-regular fuzzy graph.
$\Longrightarrow td_m(u) = k + c$ *and* $d_m(u) = k$, *for all* $u \in V$

$\Longrightarrow d_m(u) + \sigma(u) = k + c$ *and* $d_m(u) = k$, *for all* $u \in V$

$\Longrightarrow d_m(u) + \sigma(u) = k + c$ *and* $d_m(u) = k$, *for all* $u \in V \Longrightarrow \sigma(u) = c$, *for all* $u \in V$.

Hence $\sigma(u) = c$, *for all* $u \in V$.

Remark 4.5: Any graph with diameter less than m is $(m, 0)$ - regular fuzzy graph.

5. (m, k)-REGULARITY ON A PATH ON $2m$ VERTICES WITH SOME SPECIFIC MEMBERSHIP FUNCTIONS

In this section, (m, k)-regularity on a path is studied with some specific membership functions.

Theorem 5.1: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a path on $2m$ vertices. G: (σ, μ) is (m, k)-regular fuzzy graph if $\mu(uv) = k$, *for all* $uv \in E$.

Proof: Suppose that μ is constant function say $\mu(uv) = k$, for all $uv \in E$, then $d_m(v) = k$, for all $v \in V$. Hence G is an (m, k)-regular fuzzy graph.

Remark 5.2: Converse of the above theorem need not be true.

Theorem 5.3: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a path on $2m$ vertices. If the alternate edges have the same membership values, then G is an (m, k)-regular fuzzy graph, where $k = \min\{c_1, c_2\}$.

Proof: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a path on $2m$ vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of the path G^* in that order. If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ c_2 & if \quad i \quad is\ even \end{cases}$$

For, $i = 1, 2, 3, 4, 5, \ldots, m$.

$$\begin{aligned} d_m(v_i) &= \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\} \\ &= \{c_1 \wedge c_2 \wedge \ldots \wedge c_2 \wedge c_1\} \\ &= k, \textit{ where } k = \min\{c_1, c_2\} \end{aligned}$$

For, $i = 1, 2, 3, 4, 5, \ldots, m$.
$d_m(v_{m+i}) = \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\}$
$= \{c_1 \wedge c_2 \wedge \ldots \wedge c_2 \wedge c_1\}$
$= k$, where $k = \min\{c_1, c_2\}$

Hence G is an (m, k)- regular fuzzy graph.

Theorem 5.4: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a path on $2m$ vertices. If middle edge has membership value less than the membership values of the remaining edges, then G is an (m, k)-regular fuzzy graph, where k =membership value of the middle edge.

Proof: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a path on $2m$ vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of the path G^*: (V,E) in that order. Let the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ be $c_1, c_2, c_3, \ldots, c_{m-1}, c_{m+1}, \ldots, c_{2m-1}$ such that $c_m = k \leq c_1, c_2, c_3, \ldots, c_{2m-1}$.

For, $i = 1, 2, 3, 4, 5, \ldots, m$
$d_m(vi) = \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\}$
$= \{c_i \wedge c_{i+1} \wedge \ldots, \wedge c_{m-2+i} \wedge c_{m-1+i}\}$
$= c_m = k$.

For, $i = 1, 2, 3, 4, 5, \ldots, m$
$d_m(v_{m+i}) = \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\}$
$= \{c_i \wedge c_{i+1}, \ldots, \wedge c_{m-2+i} \wedge c_{m-1+i}\}$
$= c_m = k$.

Hence G is an (m, k)- regular fuzzy graph.

Remark 5.5: If σ is not constant function, then the (m, k)-regular fuzzy graph in the above theorems 6.1, 6.3 and 6.4 are not totally (m, k)-regular fuzzy graphs.

6. (m, k)-REGULARITY ON A CYCLE WITH SOME SPECIFIC MEMBERSHIP FUNCTIONS

In this section, (m, k)-regularity on a cycle is studied with some specific membership functions.

Theorem 6.1: For any $m \geq 2$, let G: (σ, μ) be a fuzzy graph on G^* :(V, E), a cycle of length $\geq 2m + 1$. If μ is a constant function, then G is an (m, k)-regular fuzzy graph, where $k = \mu(uv)$.

Theorem 6.2: For any $m \geq 1$, let G: (σ, μ) be a fuzzy graph on G^*: (V,E), a cycle of length $\geq 2m + 1$. If μ is a constant function, then G is an (m, k)-regular fuzzy graph, where $k = 2\mu(uv)$.

Theorem 6.3: For any $m \geq 1$, let G: (σ, μ) be a fuzzy graph on G^*: (V, E), an even cycle of length $\geq 2m + 2$. If the alternate edges have the same membership values, then G is an (m, k)-regular fuzzy graph.

Proof: If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ c_2 & if \quad i \quad is\ even \end{cases}$$

If $c_1 = c_2$, then μ is a constant function. So, G is an $(m, 2c_1)$-regular fuzzy graph. If $c_1 < c_2$, then
$d_m(v) = \min\{c_1, c_2\} + \min\{c_1, c_2\} = c_1 + c_1 = 2c_1$, for all $v \in V$.

Hence G is an $(m, 2c_1)$-regular fuzzy graph.

If $c_1 > c_2$, $d_m(v) = \min\{c_1, c_2\} + \min\{c_1, c_2\} = c_2 + c_2 = 2c_2$, for all $v \in V$.

Hence G is an $(m, 2c_2)$-regular fuzzy graph.

Remark 6.4: Even if the alternate edges of a fuzzy graph whose underlying graph is an even cycle of length $\geq 2m + 2$ have the same membership values, then G need not be totally (m, k)-regular fuzzy graph, since if σ is not constant function then G is not totally (m, k)-regular fuzzy graph, for any $m \geq 1$.

Theorem 6.5: For any $m > 1$, let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a cycle of length $\geq 2m + 1$.

Let $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ c_2 \geq c_1 & if \quad i \quad is\ even \end{cases}$ then G is an (m, k)-regular fuzzy graph.

Proof: Let $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ c_2 \geq c_1 & if \quad i \quad is\ even \end{cases}$

Case-1: Let G: (σ, μ) be a fuzzy graph on G^* :(V, E) an even cycle of length $\geq 2m + 2$. Then by theorem 6.3, G is an $(m, 2c_1)$-regular fuzzy graph.

Case-2: Let G: (σ, μ) be a fuzzy graph on G^*: (V,E) an odd cycle of length $\geq 2m + 1$. For any $m > 1$, $d_m(v) = 2c_1$, for all $v \in V$. Hence G is an $(m, 2c_1)$-regular fuzzy graph.

Remark 6.6: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), a cycle of length $\geq 2m + 1$.

Even if $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ c_2 \geq c_1 & if \quad i \quad is\ even \end{cases}$

then G need not be totally (m, k)-regular fuzzy graph, since if σ is not constant function then G is not totally (m, k)-regular fuzzy graph.

Theorem 6.7: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), an odd cycle of length $\geq 2m + 1$.

Let $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ Membership \quad value \quad x \geq c_1 & if \quad i \quad is\ even \end{cases}$

where x is not constant. Then G is (m, k)-regular fuzzy graph.

Proof: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), an odd cycle of length $\geq 2m + 1$.

Let $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ Membership \quad value \quad x \geq c_1 & if \quad i \quad is\ even \end{cases}$

where x is not constant.

$d_m(v) = min\ \{c_1, c_2\} + min\ \{c_1, c_2\} = c_1 + c_1 = 2c_1$, for all $v \in V$. Hence G is an $(m, 2c_1)$-regular fuzzy graph. Hence G is an $(m, 2c_1)$-regular fuzzy graph.

Remark 6.8: Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), an odd cycle of length $\geq 2m + 1$.

Even if $\mu(e_i) = \begin{cases} c_1 & if \quad i \quad is\ odd \\ Membership \quad value \quad x \geq c_1 & if \quad i \quad is\ even \end{cases}$

where x is not constant. Then G need not be a totally (m, k)-regular fuzzy graph, since if σ is not constant function then G is not totally (m, k)-regular fuzzy graph.

ACKNOWLEDGEMENT

This work is supported by F.No.4-4/2014-15, MRP- 5648/15 of the University Grant Commission, SERO, Hyderabad, India.

REFERENCES

1. Mini Tom and M.S.Sunitha, *Sum Distance in fuzzy graphs*, Annals of Pure and Applied Mathematics Vol.7, No.2, 2014, 73-89.
2. A. Nagoor Gani, and M. Basheer Ahamed, Order and Size in Fuzzy Graph, *Bulletin of Pure and Applied Sciences*, 22E(1) (2003), 145{ 148.
3. A. Nagoor Gani and S. R. Latha, *On Irregular Fuzzy graphs*, Applied Mathematical Sciences, 6, (2012), 517{523.
4. A. Nagoor Gani and K. Radha, *On Regular Fuzzy graphs*, Journal of Physical Science, 12 (2008), 33(40).
5. A. Rosenfeld, *Fuzzy Graphs*, In. L.A.Zadeh, K.S.Fu, M.Shimura, Eds., Fuzzy Sets and Their Applications, Academic press (1975), 77(95).
6. N. R. Santhi Maheswari and C. Sekar, *(r, 2, r(r-1)-regular graphs*, International Journal of Mathematics and soft Computing, 2(2) (2012), 25-33.

7. N. R. Santhi Maheswari and C. Sekar, *On* (2, *k*)-*regular fuzzy graphs and totally* (2, *k*)-*regular fuzzy graphs*, International Journal of Mathematics and Soft Computing, 4(2) (2014), 59-69.
8. N. R. Santhi Maheswari and C. Sekar, On (*r*, 2, *k*)-regular fuzzy graphs, accepted in JCMCC.
9. N. R. Santhi Maheswari and C. Sekar, *On m-Neighbourly Irregular fuzzy graphs*, International Journal of Mathematics and Soft computing, 5(2) (2015). ISSN 2249-3328.
10. N. R. Santhi Maheswari and C. Sekar, *A Study on Distance d-regular and Neighbourly Irregular fuzzy graphs*, Ph.D Thesis, Manonmaniam Sundaranar University, Tirunelveli \ (2014).
11. M.S.Sunitha and Sunil Mathew, *Fuzzy Graph Theory*. *A survey*, Annals of Pure and Applied Mathematics Vol. 4, No.1, 2013, 92-110.

Source of support: Nil, Conflict of interest: None Declared