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RELATIONS INVOLVING STIRLING NUMBERS OF THE FIRST KIND WITH FINITE SUMS

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ABSTRACT.

The stirling numbers were conjectured in the end of 18th century by stirling. In mathematics Stirling numbers of the first kind arise in the study of permutations. In particular, the Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one).

In this paper I am proving some finite sums which are involving stirling numbers of first kind.

Keywords: *Stirling 1st kind numbers, finite sums*

1. INTRODUCTION

The stirling numbers were conjectured in the end of 18th century by stirling. In mathematics Stirling numbers of the first kind arise in the study of permutations. In particular, the Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one). [1]

The original definition of Stirling numbers of the first kind was algebraic. These numbers, usually written $s(n, k)$, are signed integers whose sign, positive or negative, depends on the parity of $n - k$. Afterwards, the absolute values of these numbers, $|s(n, k)|$, which are known as unsigned Stirling numbers of the first kind, were found to count permutations, so in combinatorics the (signed) Stirling numbers of the first kind, $s(n, k)$, are often defined as counting numbers multiplied by a sign factor [2][4].

The unsigned Stirling numbers of the first kind are denoted in various ways by different authors. Common notations are $c(n,k), |s(n,k)|$ and $\begin{bmatrix} n \\ k \end{bmatrix}$

The unsigned Stirling numbers also arise as coefficients of the rising factorial,
i.e., $(x)^{(n)} = x(x+1)(x+2)\ldots\ldots\ldots..(x+n-1) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} x^k$ [4]

In this paper Binomial function is denoted with C_r^k. This stirling numbers had relation with finite sums of integers. From That relations three are proving in this paper.

Table of values for small k and r of stirling 1st kind numbers s(k, r)

Below is a triangular array of unsigned values for the Stirling numbers of the first kind, similar in form to Pascal's triangle. These values are easy to generate using the recurrence relation in the given below.

k \ r	1	2	3	4	5	6			
1	1								
2	1	1							
3	1	3	2						
4	1	6	11	6					
5	1	10	35	50	24				
6	1	…..	…..	……	……	……			
….	……	….	….	…..	……	…..	……		

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The Recurrence Relation:

S(k+1,r+1)= S(k,r)+[k×S(k,r+1)]

Where 1<=r<=K

Theorem 1.1: Prove that $\frac{s(k,1)\sum_{n=1}^{N} n^k+s(k,2)\sum_{n=1}^{N} n^{k-1}+s(k,3)\sum_{n=1}^{N} n^{k-2}+\cdots\ldots+s(k,k)\sum_{n=1}^{N} n^1}{k!}=\frac{(N)^{(k+1)}}{(k+1)!}$

Proof:
Given is $\frac{s(k,1)\sum_{n=1}^{N} n^k+s(k,2)\sum_{n=1}^{N} n^{k-1}+s(k,3)\sum_{n=1}^{N} n^{k-2}+\cdots\ldots+s(k,k)\sum_{n=1}^{N} n^1}{k!}$

We know that the corresponding Bernoulli numbers B_m. Written explicitly in terms of a sum of powers,
$\sum_{n=1}^{N} n^k=\sum_{n=1}^{k+1} b_{k\,n}N^n$ [5]

Where $b_{k\,n}=\frac{(-1)^{k-n+1}B_{k-n+1}k!}{n!(k-n+1)!}$

$$=> \frac{1}{k!}[s(k,1)\sum_{n=1}^{k+1} b_{k\,n}N^n+s(k,2)\sum_{n=1}^{k} b_{k\,n}N^n+s(k,3)\sum_{n=1}^{k-1} b_{k\,n}N^n+\cdots\ldots+s(k,k)\sum_{n=1}^{2} b_{k\,n}N^n]$$
$$=\frac{1}{k!}[N^1\sum_{r=1}^{k} s(k,r)b_{(k+1-r)1}+N^2\sum_{r=1}^{k} s(k,r)b_{(k+1-r)2}$$
$$+N^3\sum_{r=1}^{k-1} s(k,r)b_{(k+1-r)3}+\cdots\ldots\ldots+N^k\sum_{r=1}^{2} s(k,r)b_{(k+1-r)k}+N^{k+1}\sum_{r=1}^{1} s(k,r)b_{(k+1-r)(k+1)}]$$

But $\sum_{\substack{r=1\\p=1}}^{\substack{r=k\\p=k+1}} s(k,r)\mathrm{b}_{(k+1-r)p}=\frac{s(k+1,k+2-p)}{k+1}$

$$=> \frac{1}{k!}[N^1*\frac{s(k+1,k+1)}{k+1}+N^2*\frac{s(k+1,k)}{k+1}+N^3*\frac{s(k+1,k-1)}{k+1}+\cdots\ldots\ldots+N^k*\frac{s(k+1,2)}{k+1}+N^{k+1}*\frac{s(k+1,1)}{k+1}]$$
$$=\frac{1}{(k+1)!}[s(k+1,1)N^{k+1}+s(k+1,2)N^k+s(k+1,3)N^{k-1}+\cdots\ldots+s(k+1,k+1)N^1]$$
$$=\frac{(N)^{(k+1)}}{(k+1)!}$$

Theorem 1.2: Prove that $\frac{s(k,1)\sum_{n=1}^{N} n^k-s(k,2)\sum_{n=1}^{N} n^{k-1}+s(k,3)\sum_{n=1}^{N} n^{k-2}-\cdots\ldots s(k,k)\sum_{n=1}^{N} n^1}{k!}=C_{k+1}^{N+1}$

Proof:

$$\frac{s(k,1)\sum_{n=1}^{N} n^k-s(k,2)\sum_{n=1}^{N} n^{k-1}+s(k,3)\sum_{n=1}^{N} n^{k-2}-\cdots\ldots s(k,k)\sum_{n=1}^{N} n^1}{k!}$$
$$=\frac{1}{k!}[s(k,1)\sum_{n=1}^{k+1} b_{k\,n}N^n-s(k,2)\sum_{n=1}^{k} b_{k\,n}N^n+s(k,3)\sum_{n=1}^{k-1} b_{k\,n}N^n-\cdots\ldots s(k,k)\sum_{n=1}^{2} b_{k\,n}N^n]$$
$$=\frac{1}{k!}[N^{k+1}\sum_{r=1}^{1}(-1)^{r+1}s(k,r)b_{(k-r+1)(k+1)}+N^k\sum_{r=1}^{2}(-1)^{r+1}s(k,r)b_{(k-r+1)(k)}$$
$$+N^{k-1}\sum_{r=1}^{3}(-1)^{r+1}s(k,r)b_{(k-r+1)(k-1)}+N^{k-2}\sum_{r=1}^{4}(-1)^{r+1}s(k,r)b_{(k-r+1)(k-2)}$$
$$+\cdots\ldots\ldots+N^1\sum_{r=1}^{k}(-1)^{r+1}s(k,r)b_{(k-r+1)1}$$
$$=\frac{1}{k!}\left[N^{k+1}*\frac{s(k+1,1)}{(N-K)!(k+1)}-N^k*\frac{s(k+1,2)}{(N-K)!(k+1)}+N^{k-1}*\frac{s(k+1,3)}{(N-K)!(k+1)}-\cdots\ldots\ldots N^1*\frac{s(k+1,k+1)}{(N-K)!(k+1)}\right]$$
$$=\frac{1}{(N-K)!(k+1)!}[s(k+1,1)N^{k+1}-s(k+1,2)N^k+s(k+1,3)N^{k-1}-\cdots\ldots s(k+1,k+1)N^1$$
$$=\frac{1}{(N-K)!(k+1)!}[(N+1)_k]$$
$$=\frac{(N+1)!}{(N-K)!(k+1)!}$$
$$=C_{k+1}^{N+1}$$

Theorem 1.3: Prove that $g(x)=f(y)\quad if\ y=x+k-1$
Where

$$g(x)=s(k,1)x^k+s(k,2)x^{k-1}+s(k,3)x^{k-2}+\cdots\ldots+s(k,k)x^1$$
$$f(y)=s(k,1)y^k-s(k,2)y^{k-1}+s(k,3)y^{k-2}-\cdots\ldots s(k,k)y^1$$

Proof:
Let us take

$$f(y)=s(k,1)y^k-s(k,2)y^{k-1}+s(k,3)y^{k-2}-\cdots\ldots s(k,k)y^1$$

Substitute $y=x+k-1$ on above eq^n
$$f(x+k-1)=s(k,1)(x+k-1)^k-s(k,2)(x+k-1)^{k-1}+s(k,3)(x+k-1)^{k-2}-\cdots\ldots s(k,k)(x+k-1)^1$$

Let us take n=k-1

$$f(x+n) = s(k,1)(x+n)^k - s(k,2)(x+n)^{k-1} + s(k,3)(x+n)^{k-2} - \cdots \ldots . s(k,k)(x+n)^1$$

Using binomial formula we can write above eq^n as

$$\begin{aligned} f\,(x+n) &= s(k,1)\textstyle\sum_{r=0}^{k} c_r^k x^{k-r} n^r - s(k,2)\sum_{r=0}^{k-1} c_r^{k-1} x^{k-r} n^r \\ &\quad + s(k,3)\textstyle\sum_{r=0}^{k-2} c_r^{k-2} x^{k-r} n^r - \cdots \ldots \ldots s(k,k)\sum_{r=0}^{1} c_r^1 x^{k-r} n^r \\ &= (s(k,1)x^k + s(k,3)x^{k-2} + s(k,5)x^{k-4} + \cdots \ldots) - (s(k,2)x^{k-1} + s(k,4)x^{k-3} + \cdots \ldots \ldots) \\ &\quad + (s(k,1)\textstyle\sum_{r=1}^{k} c_r^k x^{x-r} n^r - s(k,2)\sum_{r=1}^{k-1} c_r^{k-1} x^{x-r} n^r \\ &\quad - s(k,3)\textstyle\sum_{r=1}^{k-2} c_r^{k-2} x^{x-r} n^r + \cdots \ldots . . s(k,k)\sum_{r=1}^{1} c_r^1 x^{x-r} n^r) \end{aligned}$$

We know that

$$\begin{aligned} & s(k,1)\textstyle\sum_{r=1}^{k} c_r^k x^{x-r} n^r - s(k,1)\sum_{r=1}^{k-1} c_r^{k-1} x^{x-r} n^r + s(k,1)\sum_{r=1}^{k-2} c_r^{k-2} x^{x-r} n^r - \cdots \ldots . . s(k,1)\sum_{r=1}^{1} c_r^1 x^{x-r} n^r \\ &= 2(s(k,2)x^{k-1} + s(k,4)x^{k-3} + \cdots \ldots \ldots) \\ &= (s(k,1)x^k + s(k,3)x^{k-2} + s(k,5)x^{k-4} + \cdots \ldots) - (s(k,2)x^{k-1} + s(k,4)x^{k-3} + \cdots \ldots \ldots) \\ &\qquad + 2(s(k,2)x^{k-1} + s(k,4)x^{k-3} + \cdots \ldots \ldots) \\ &= s(k,1)x^k + s(k,2)x^{k-1} + s(k,3)x^{k-2} + \cdots \ldots . + s(k,k)x^1 \\ &= g(x) \end{aligned}$$

Hence $g(x) = f(y) \quad if\ y = x + k - 1$

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