ON SOME PROPERTIES OF (1, 2)*-\(gab\)CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of closed sets called (1, 2)*-\(gab\)-closed sets in bitopological spaces is introduced. Several properties of this class and its inclusion relationship with other known classes of closed sets are analyzed. This class contains the class of all (1, 2)*-\(ag\)-closed sets and is contained in the class of all (1, 2)*-\(ga\)-closed sets. Also several new classes of spaces induced by the class of (1, 2)*-\(gab\)-closed sets are defined and their properties are investigated.

Keywords: (1,2)*-\(a\)-closed sets, (1,2)*-\(b\)-closed sets, (1,2)*-\(g\)-closed sets, (1,2)*-\(ab\)-closed sets, (1,2)*-\(gb\)-closed sets, (1,2)*-\(gb\) continuous and (1,2)*-\(T_{gab}\) space.

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1. INTRODUCTION

Njastad [17] introduced and investigated the concept of alpha open sets (briefly \(a\)-open sets). Maki et.al [16] defined generalized alpha and alpha generalized closed sets (briefly \(g\alpha\) and \(ag\) closed sets) in 1993 and 1994 respectively in topological spaces as an extension of alpha and generalized closed sets. Andrijevic [2] in 1996 exhibited a new class called b-open sets in a topological space. This class is contained in the class of semi pre-opens sets and contains all semi-open and pre-open sets. Norman Levine introduced the concept of generalized closed sets [13] (briefly \(g\)-closed set) in topological spaces in 1963. As an extension of \(g\)-closed sets, Veera Kumar [21] defined \(g\)-closed sets in topological spaces in 2003. Subasree and Maria Singam [20] defined a new class namely \(gb\)-closed sets in topological spaces which is a subclass of \(gb\)-closed sets and contains \(b\)-closed set. Followed by this, Mary and Nagajothi [18] [19] defined and characterized the class \(ab\)-closed sets which is a subclass of \(ba\)-closed sets and contains \(a\)-closed sets.

In 1963 Kelly [16] introduced the concept of Bitopological Spaces. A set \(X\) equipped with two topologies \(\tau_1\) and \(\tau_2\) is called Bitopological spaces and it is denoted by \((X, \tau_1, \tau_2)\). The concept and various class of closed sets defined in topological spaces \((X, \tau)\) have been extended to bitopological spaces \((X, \tau_1, \tau_2)\). Fukutake [8], [9] defined generalized closed sets and semi open sets in bitopological space in 1986 and 1989 respectively. In 1990, Jelic [11] introduced the concept of alpha open sets in bitopological space. El-Tantawy and Abu-Donia [7] extend the class of \(a\)-closed set to alpha generalized closed sets in bitopological spaces. Recently the authors introduced (1,2)*-\(ab\)-closed sets and analyzed their properties[20].

In this paper another new class of closed sets namely (1,2)*-\(gab\)-closed sets is introduced in bitopological spaces that satisfies the inclusion relation given below:

\[
\{(1,2)^*\text{-closed sets}\} \subset \{(1,2)^*\text{-gab\-closed sets}\} \subset \{(1,2)^*\text{-ag\-closed sets}\}
\]

Based on the definition of (1,2)*-\(gab\)-closed sets, a new space namely (1,2)*-\(T_{gab}\) space is defined and further several theorems on its relationship with other known bitopological spaces are proved.

2. PRELIMINARIES:

Throughout this paper, \((X, \tau_1, \tau_2)\) denote a bitopological space with the topologies \(\tau_1\) and \(\tau_2\).
Definition 2.1.5: [6] A topology on a set $X$ is a collection $\tau$ of subsets of $X$ having the following properties:
1) $\phi$ and $X$ are in $\tau$.
2) The union of the elements of any sub collection of $\tau$ is in $\tau$.
3) The intersection of the elements of any finite sub collection of $\tau$ is in $\tau$.
A set $X$ for which a topology $\tau$ has been specified is called a topological space.

Definition 2.1.8: [6] The intersection of the elements of any finite sub collection of $\tau_1 \cup \tau_2$ is denoted by $(1,2)^*\cap \tau_1 \cup \tau_2$.

Definition 2.1.2: [12] A set $X$ with two topologies $\tau_1$ and $\tau_2$ is said to be a bitopological space and it is denoted by $(X, \tau_1, \tau_2)$.

Definition 2.1.3: [6] A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $\tau_{1,2}$-open set if $A \in \tau_1 \cup \tau_2$. The complement of a $\tau_{1,2}$-open set is $\tau_{1,2}$-closed set.

Definition 2.1.4: [6] Let $A$ be a subset of a bitopological space $(X, \tau_1, \tau_2)$, then
1. The $\tau_{1,2}$-interior of $A$ in $(X, \tau_1, \tau_2)$, denoted by $\tau_{1,2}$-int$(A)$, is defined as $\cup \{F/F \subseteq A$ and $F$ is $\tau_{1,2}$-open set $\}.$
2. The $\tau_{1,2}$-closure of $A$ in $(X, \tau_1, \tau_2)$, denoted by $\tau_{1,2}$-cl$(A)$, is defined as $\cap \{G/A \subseteq G$ and $G$ is $\tau_{1,2}$-closed set $\}.$

Definition 2.1.5: [6] A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called
1. a $(1,2)^*$-semi open set if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A))$ and a $(1,2)^*$-semi closed set if $\tau_{1,2}$-cl$(A)(\tau_{1,2}$-int$(A)) \subseteq A$.
2. a $(1,2)^*$-pre open set if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$ and a $(1,2)^*$-pre closed set if $\tau_{1,2}$-cl$(A)(\tau_{1,2}$-int$(A)) \subseteq A$.
3. a $(1,2)^*$-a-open set if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$, and a $(1,2)^*$-a-closed set if $\tau_{1,2}$-cl$(\tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))) \subseteq A$.
4. a $(1,2)^*$-b-open set if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A)) \cup \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$, and a $(1,2)^*$-b-closed set if $\tau_{1,2}$-int$(\tau_{1,2}$-cl$(A)) \cap \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A)) \subseteq A$.
5. a $(1,2)^*$-semi pre open set if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(\tau_{1,2}$-cl$(A)))$ and a $(1,2)^*$-semi pre closed set if $\tau_{1,2}$-int$(\tau_{1,2}$-cl$(\tau_{1,2}$-int$(A))) \subseteq A$.

Definition 2.1.6: [6] The family of all $(1,2)^*$-open sets, $(1,2)^*$-a-open sets, $(1,2)^*$-b-open sets, $(1,2)^*$-semi open sets in $X$ are denoted by $(1,2)^*$-O$(X)$, $(1,2)^*$-aO$(X)$, $(1,2)^*$-BO$(X)$ and $(1,2)^*$-SO$(X)$ respectively.

Definition 2.1.7: [6] A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ or $X$ is called
1. a $(1,2)^*$-generalized closed set (briefly $(1,2)^*$-g-closed set) if $\tau_{1,2}$-cl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
2. a $(1,2)^*$-generalized semi closed set (briefly $(1,2)^*$-gs-closed set) if $\tau_{1,2}$-scl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
3. a $(1,2)^*$-semi generalized closed set (briefly $(1,2)^*$-sg-closed set) if $\tau_{1,2}$-cl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-SO$(X)$.
4. a $(1,2)^*$-a generalized closed set (briefly $(1,2)^*$-ag-closed set) if $\tau_{1,2}$-acl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
5. a $(1,2)^*$-generalized a closed set (briefly $(1,2)^*$-ga-closed set) if $\tau_{1,2}$-acl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
6. a $(1,2)^*$-generalized pre closed set (briefly $(1,2)^*$-gp-closed set) if $\tau_{1,2}$-pcl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
7. a $(1,2)^*$-generalized semi pre closed set (briefly $(1,2)^*$-gsp-closed set) if $\tau_{1,2}$-spcl$(A) \subseteq U$, whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
8. a $(1,2)^*$-strongly generalized closed set (briefly $(1,2)^*$-strongly-g-closed set) if $\tau_{1,2}$cl$(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)^*$-O$(X)$.
Definition 2.1.10: The family of all (1,2) *-g-open sets, (1,2) *-g- open sets, (1,2) *-a-g- open sets and (1,2) *-b- open sets in X are denoted by (1,2) *-G0(X), (1,2) *-G0(X), (1,2) *-aG0(X) and (1,2) *-bG0(X) respectively.

Definition 2.1.11: A function $f: (X, τ_1, τ_2) → (Y, σ_1, σ_2)$ is called
1. a (1,2)*-g-continuous function if $f^{-1}(V)$ is (1,2)*-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
2. a (1,2)*-g-continuous function if $f^{-1}(V)$ is (1,2)*-g-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
3. a (1,2)*-a generalized continuous function (briefly (1,2)*-ag-continuous) if $f^{-1}(V)$ is (1,2)*-ag-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
4. a (1,2)*-generalized a continuous function (briefly (1,2)*-ga-continuous) if $f^{-1}(V)$ is (1,2)*-ga-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
5. a (1,2)*-generalized semi continuous function (briefly (1,2)*-gs-continuous) if $f^{-1}(V)$ is (1,2)*-gs-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
6. a (1,2)*-generalized semi continuous function (briefly (1,2)*-sg-continuous) if $f^{-1}(V)$ is (1,2)*-sg-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
7. a (1,2)*-generalized semi-pre continuous function (briefly (1,2)*-gsp-continuous) if $f^{-1}(V)$ is (1,2)*-gsp-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
8. a (1,2)*-generalized pre continuous function (briefly (1,2)*-gp-continuous) if $f^{-1}(V)$ is (1,2)*-gp-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.
9. a (1,2)*-gb-continuous function if $f^{-1}(V)$ is (1,2)*-gb-closed set in $(X, τ_1, τ_2)$ for every (1,2)*-closed set $V$ of $(Y, σ_1, σ_2)$.

Definition 2.1.12: A bitopological space $(X, τ_1, τ_2)$ is called
1. a (1,2)*-T_{1/2}-space if every (1,2)*-g-closed set in it is (1,2)*-closed set.
2. a (1,2)*-T_{2}-space if every (1,2)*-gs-closed set in it is (1,2)*-closed set.
3. a (1,2)*-gT_b-space if every (1,2)*-ag-closed set in it is (1,2)*-closed set.
4. a (1,2)*-gT_{ba}-space if every (1,2)*-ba- closed set in it is (1,2)*-closed set.

3. (1,2)*-gab- CLOSED SETS

In this section we introduce a new class of closed sets called (1,2)*-gab- closed sets ((1,2)*-generalized ab- closed sets) which lie between the class of (1,2)*-closed sets and the class of (1,2)*-ag-closed sets.

Definition 3.1: A subset $A$ of a bitopological space $(X, τ_1, τ_2)$ is said to be (1,2)*-gab- closed set if $τ_{1,2}-cl(A) ⊆ U$, whenever $A ⊆ U$ and $U ∈ (1,2)*-ag$-open set in $(X, τ_1, τ_2)$. The family of all (1,2)*-gab- open sets in $X$ is denoted by (1,2)*-gab-0(X).

3.1 Relationship of (1,2)*- gab- closed sets with other classes of (1,2)*-closed sets:

Theorem 3.1.1:
(i) Every (1,2)*-closed set is (1,2)*-gab- closed set. But the converse need not be true.
(ii) If a (1,2)*- gab- closed set is (1,2)*- ab- closed set, then it is (1,2)*-closed set.

Proof:
(i) Let $A$ be an (1,2)*-closed set and $U ∈ (1,2)*-ab$-open set such that $A ⊆ U$. Since $A$ is (1,2)*-closed set, we have $τ_{1,2}-cl(A) = A ⊆ U$. Therefore, $τ_{1,2}-cl(A) ⊆ U$, whenever $A ⊆ U$ and $U ∈ (1,2)*-ab$-open set. Hence $A$ is (1,2)*-gab- closed set. The converse of the above theorem need not be true. This is proved in the following example:
Example 3.1.1: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a, c\}$ is a (1,2)*-gab$\tilde{g}$-closed set, but not (1,2)*-closed set.

(ii) Let $A$ be (1,2)*-ab$\tilde{g}$-open set and (1,2)*-gab$\tilde{g}$-closed set. Since $A \subseteq A$, and by our hypothesis, we have $\tau_{1,2}\text{cl}(A) \subseteq A$. It is obvious that $A \subseteq \tau_{1,2}\text{cl}(A)$. Hence $A$ is a (1,2)*-closed set.

Theorem 3.1.2: Let $A$ be a (1,2)*-gab$\tilde{g}$-closed set in a bitopological space $(X, \tau_1, \tau_2)$, Then $A$ is a) (1,2)*-g-closed set, b) (1,2)*-$\alpha$-g-closed set, c) (1,2)*-g$^*$-closed set, d) (1,2)*-gp-closed set, e) (1,2)*-gsp-closed set, f) (1,2)*-gb-closed set, g) (1,2)*-g$\alpha$-closed set.

Proof: Let $A$ be a (1,2)*-gab$\tilde{g}$-closed set. Then by definition $\tau_{1,2}\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ be a (1,2)*-ab$\tilde{g}$-closed set.

a) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set, and hence $\tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-g-closed set.

b) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. Always $\tau_{1,2}\text{acl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{acl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-g$^*$-closed set.

c) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. Always $\tau_{1,2}\text{cl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{cl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-gp-closed set.

d) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. It is well known that, $\tau_{1,2}\text{pcl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{pcl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-gsp-closed set.

e) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. It is well known that, $\tau_{1,2}\text{spcl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{spcl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-gb-closed set.

f) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. It is well known that, $\tau_{1,2}\text{bcl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{bcl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is (1,2)*-g$\alpha$-closed set.

g) Let $U$ be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*-ab$\tilde{g}$-open set [20], $U$ is a (1,2)*-ab$\tilde{g}$-open set. Always $\tau_{1,2}\text{cl}(A) \subseteq \tau_{1,2}\text{cl}(A)$. Hence by hypothesis $\tau_{1,2}\text{cl}(A) \subseteq \tau_{1,2}\text{cl}(A) \subseteq U$. Thus $A$ is a (1,2)*-g$\alpha$-closed set.

Remark 3.1.1: Note that the converse of the above Theorem need not be true. The following examples prove this statement:

Example 3.1.2: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a, c\}$ is (1,2)*-g-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.3: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a, c\}$ is (1,2)*-$\alpha$-g-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.4: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{b\}$ is (1,2)*-g$\alpha$-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.5: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{b\}$ is (1,2)*-gs-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.6: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a, c\}$ is (1,2)*-gp-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.7: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a\}$ is (1,2)*-gsp-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Example 3.1.8: Let $X = \{a, b, c\}$ with $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Clearly $A = \{a\}$ is (1,2)*-gb-closed set, but not (1,2)*-gab$\tilde{g}$-closed set.

Remark 3.1.2: From Theorem 3.1.1 and Theorem 3.1.2 it is observe that the following inclusion relations holds: 
\{(1,2)*-closed sets\} \subset \{(1,2)*-gab$\tilde{g}$-closed sets\} \subset \{(1,2)*-$\alpha$g-closed sets\}
Remark 3.1.3: The following examples reveal that the class of (1,2)*-$gab\hat{g}$-closed set are independent from the class of (1,2)*-$\alpha$-closed sets, class of (1,2)*-$ab\hat{g}$-closed sets and class of (1,2)*-semi closed sets.

Example 3.1.9:
(i) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$. Clearly $A = \{b\}$ is (1,2)*-$\alpha$-closed set, but not (1,2)*-$gab\hat{g}$-closed set.
(ii) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{b, c\}\}$. Clearly $A = \{a, b\}$ is (1,2)*-$gab\hat{g}$-closed set, but not (1,2)*-$\alpha$-closed set.

Example 3.1.10:
(i) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$. Clearly $A = \{a\} \in (1,2)*-ab\hat{g}$-closed set, but not (1,2)*-$ab\hat{g}$-closed set.
(ii) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$. Clearly $A = \{b\}$ is (1,2)*-$ab\hat{g}$-closed set, but not (1,2)*-$\alpha$-closed set.

Example 3.1.11:
(i) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$. Clearly $A = \{b\} \in (1,2)*-\alpha$-closed set, but not (1,2)*-$\alpha$-closed set.

(ii) Let $X = \{a, b, c\}$ with $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$. Clearly $A = \{a\}$ is (1,2)*-$gab\hat{g}$-closed set, but not (1,2)*-$\alpha$-closed set.

Relationships of (1,2)*-$gab\hat{g}$-closed sets with other closed sets are represented by the following diagram:

![Diagram of Relationships](image-url)

In the above diagram, $A \rightarrow B$ denotes A implies B, $A \leftarrow B$ denotes B implies A, $A \leftrightarrow B$ denotes A and B are independent.

3.2 PROPERTIES OF (1,2)*-$gab\hat{g}$-CLOSED SETS

Theorem 3.2.1: If A and B are (1,2)*-$gab\hat{g}$-closed sets in $(X,\tau_1, \tau_2)$ then $A \cup B$ is (1,2)*-$gab\hat{g}$-closed set.

Proof: Let $A$ and $B$ be (1,2)*-$gab\hat{g}$-closed sets in $(X,\tau_1, \tau_2)$ and $U$ be any (1,2)*-$ab\hat{g}$-open set containing $A \cup B$. Since $A \subseteq U$ and $B \subseteq U$, we have $\tau_{1,2}$-$cl(A) \subseteq U$ and $\tau_{1,2}$-$cl(B) \subseteq U$. Also $\tau_{1,2}$-$cl(A \cup B) = \tau_{1,2}$-$cl(A) \cup \tau_{1,2}$-$cl(B) \subseteq U$. Hence $A \cup B$ is (1,2)*-$gab\hat{g}$-closed set.

Theorem 3.2.2: If a set $A$ is (1,2)*-$gab\hat{g}$-closed set then $\tau_{1,2}$-$cl(A) \backslash A$ contains no non empty (1,2)*-closed set in $(X,\tau_1, \tau_2)$.

Proof: Suppose $F$ is a (1,2)*-closed subset of $\tau_{1,2}$-$cl(A) \backslash A$. Then $F \subseteq \tau_{1,2}$-$cl(A)$ and $A \subseteq F^c$. Since $A$ is (1,2)*-$gab\hat{g}$-closed set and $F^c$ is (1,2)*-$ab\hat{g}$-open set, $F^c$ is (1,2)*-$ab\hat{g}$-open set such that $A \subseteq F^c$, we have $\tau_{1,2}$-$cl(A) \subseteq F^c$. Hence $F \subseteq \tau_{1,2}$-$cl(A)$ and $\tau_{1,2}$-$cl(A)$ is (1,2)*-$ab\hat{g}$-open set and hence F is empty. Therefore $\tau_{1,2}$-$cl(A) \backslash A$ contains no non empty (1,2)*-closed set in $(X,\tau_1, \tau_2)$.
Remark 3.2.1: The converse of the above Theorem need not be true. This is proved in the following example:

Example 3.2.1: Let X = {a, b, c} with τ₁ = {ϕ, X, {a}} and τ₂ = {ϕ, X, {a, b}}. If A = {a, c}, τ₁₂-cl(A) = A = X - {a, c} = {b} does not contain non empty closed set then A is not (1,2)*-gabĝ-closed set.

Theorem 3.2.3: A set A is (1,2)*-gabĝ-closed set if and only if τ₁₂-cl(A)\A contains no non empty (1,2)*-abĝ-closed set.

Proof: Suppose that A is (1,2)*-gabĝ-closed set. Let F be a (1,2)*-abĝ-closed subset of τ₁₂-cl(A)\A. Thus, A ⊆ F^c. Since A is (1,2)*-gabĝ-closed set, we have τ₁₂-cl(A) ⊆ F^c. Therefore F ⊆ τ₁₂-cl(A))^c. Consequently F ⊆ τ₁₂-cl(A)∩ τ₁₂-cl(A))^c = ϕ. Thus F is empty. Hence τ₁₂-cl(A)\A contains no non empty (1,2)*-abĝ-closed set.

Conversely, Suppose that τ₁₂-cl(A)\A contains no non empty (1,2)*-abĝ-closed set. Let A ⊆ G and G be (1,2)*-abĝ-open set. If τ₁₂-cl(A) is not a subset of a G, then τ₁₂-cl(A) ∩ G^c, then τ₁₂-cl(A) ∩ G^c is a non empty subset of a τ₁₂-cl(A)\A. Since τ₁₂-cl(A) is a closed set and G^c is a (1,2)*-abĝ-closed set, τ₁₂-cl(A) ∩ G^c is a non empty (1,2)*-abĝ-closed subset of τ₁₂-cl(A)\A which is a contradiction. Therefore τ₁₂-cl(A) ∩ G and hence A is (1,2)*-gabĝ-closed set.

Corollary 3.2.1: If A is (1,2)*-gabĝ-closed set in (X, τ₁, τ₂) and A ⊆ B ⊆ τ₁₂-cl(A), then B is (1,2)*-gabĝ-closed set in (X, τ₁, τ₂).

Proof: Since B ⊆ τ₁₂-cl(A), we have τ₁₂-cl(B) ⊆ τ₁₂-cl(A). Then τ₁₂-cl(B)\B ⊆ τ₁₂-cl(A)\A. By theorem 3.2.3, τ₁₂-cl(A)\A contains no non empty (1,2)*-abĝ-closed subset of (X, τ₁, τ₂) and hence τ₁₂-cl(B)\B contains no non empty (1,2)*-abĝ-closed subset of (X, τ₁, τ₂). Hence again by Theorem 3.2.3, B is (1,2)*-gabĝ-closed set in (X, τ₁, τ₂).

Theorem 3.2.4: Suppose Y is a subspace of (X, τ₁, τ₂) and A ⊆ Y is a (1,2)*-gabĝ-closed set in (X, τ₁, τ₂) then A is (1,2)*-gabĝ-closed relative to Y.

Proof: Let A ⊆ Y ∩ G where G ∈ (1,2)*-abĝ-open set in (X, τ₁, τ₂). Then A ⊆ G and since A is a (1,2)*-gabĝ-closed set, we have τ₁₂-cl(A) ⊆ G. This implies that Y ∩ τ₁₂-cl(A) ⊆ Y ∩ G whenever A ⊆ Y ∩ G where G ∈ (1,2)*-abĝ-open set in (X, τ₁, τ₂). Thus A is (1,2)*-gabĝ-closed relative to Y.

Theorem 3.2.5: In a bitopological space (X, τ₁, τ₂), let F denotes the class of (1,2)*-closed sets in X, then (1,2)*-abĝO(X) = F if and only if every subset of X is (1,2)*-gabĝ-closed set in (X, τ₁, τ₂).

Proof: Suppose (1,2)*-abĝO(X) = F. Let A be a subset of X such that A ⊆ G where G ∈ (1,2)*-aBÔO(X) then τ₁₂-cl(G) = G. Also τ₁₂-cl(A) ⊆ τ₁₂-cl(G) = G. Hence A is (1,2)*-gabĝ-closed set in (X, τ₁, τ₂).

Conversely, suppose every subset of X is (1,2)*-gabĝ-closed set in X.

Let G ∈ (1,2)*-aBÔO(X). Since G ⊆ G and G is (1,2)*-gabĝ-closed set in X, we have τ₁₂-cl(G) ⊆ G. Thus τ₁₂-cl(G) = G. Therefore (1,2)*-aBÔO(X) ⊆ F.

If S ⊆ F , then S^c is (1,2)*-open set and hence it is (1,2)*-abĝ-open set. Therefore S^c ∈ (1,2)*-aBÔO(X) ⊆ F and hence S ∈ F^c. This implies that S is (1,2)*-open set and hence it is (1,2)*-abĝ-open set. Therefore F ⊆ (1,2)*-aBÔO(X). Thus (1,2)*-aBÔO(X)=F.

3.3 Characterization of (1,2)*-gabĝ-Open Sets

Theorem 3.3.1: A set A is (1,2)*-gabĝ-open set if and only if F ∈ τ₁₂-int(A), where F is (1,2)*-abĝ-closed set and F ⊆ A.

Proof: Suppose A is (1,2)*-gabĝ-open set, F ⊆ A and F is (1,2)*-abĝ-closed set. Then F^c is (1,2)*-abĝ-open set and A^c ⊆ F^c. Since A^c is (1,2)*-gabĝ-closed set, we have τ₁₂-cl(A^c) ⊆ F^c. Hence F ⊆ τ₁₂-int(A).

Conversely, Suppose F ∉ τ₁₂-int(A), where F is (1,2)*-abĝ-closed set and F ⊆ A. Let A^c ⊆ G where G = F^c is (1,2)*-abĝ-open set. Then G^c ⊆ τ₁₂-int(A). This implies that, τ₁₂-cl(A^c) ⊆ G. Thus A^c is (1,2)*-gabĝ-closed set. Hence A is (1,2)*-gabĝ-open set.
Theorem 3.3.2: In a bitopological space \((X, \tau_1, \tau_2)\), if \(A \subseteq B \subseteq X\) where \(A\) is \((1,2)^*-\text{gab}\)-open set relative to \(B\) and \(B\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\), then \(A\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\).

Proof: Let \(F\) be a \((1,2)^*-\text{ab}\)-closed set in \(X\) and suppose that \(F \subseteq A\). Then \(F = F \cap B\) is \((1,2)^*-\text{ab}\)-closed set in \(B\). Since \(A\) is \((1,2)^*-\text{gab}\)-open set relative to \(B\), we have \(F \subseteq \tau_{1,2}\)-int\(B\)(\(A\)). Since \(\tau_{1,2}\)-int\(B\)(\(A\)) is an \((1,2)^*-\text{open}\) set relative to \(B\), we have \(F \subseteq G \cap B \subseteq A\) for some \((1,2)^*-\text{open}\) set \(G\) in \(X\). Since \(B\) is \((1,2)^*-\text{gab}\)-open set in \(X\), we have \(F \subseteq \tau_{1,2}\)-int\(B\) \(\subseteq B\). Therefore \(F \subseteq \tau_{1,2}\)-int\(B\) \(\cap G \subseteq B \cap G \subseteq A\). It follows that, \(F \subseteq \tau_{1,2}\)-int\(A\). Hence \(A\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\).

Theorem 3.3.3: If \(\tau_{1,2}\)-int\(A\) \(\subseteq B \subseteq A\) and if \(A\) is \((1,2)^*-\text{gab}\)-open set in \(X\), then \(B\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\).

Proof: Suppose that \(\tau_{1,2}\)-int\(A\) \(\subseteq B \subseteq A\) and if \(A\) is \((1,2)^*-\text{gab}\)-open set in \(X\) then \(A' \subseteq B' \subseteq \tau_{1,2}\)-cl\(A'\). Since \(A'\) is \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\), by corollary 3.2.1 \(B\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\).

Theorem 3.3.4: A set \(A\) is \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\) if and only if \(\tau_{1,2}\)-cl\(A\) \(\subseteq A\).

Proof: Suppose that \(A\) is \((1,2)^*-\text{gab}\)-closed set. Let \(F \subseteq \tau_{1,2}\)-cl\(A\)\(\backslash A\) where \(F\) is \((1,2)^*-\text{ab}\)-closed. By theorem 3.2.3, \(F\) is empty. Therefore \(F \subseteq \tau_{1,2}\)-int\(\tau_{1,2}\)-cl\(A\)\(\backslash A\). By theorem 3.3.1, we have \(\tau_{1,2}\)-cl\(A\)\(\backslash A\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\).

Conversely, Suppose that \(\tau_{1,2}\)-cl\(A\)\(\backslash A\) is \((1,2)^*-\text{gab}\)-open set in \((X, \tau_1, \tau_2)\). Let \(A \subseteq G\) where \(G\) be \((1,2)^*-\text{ab}\)-open set. Then \(\tau_{1,2}\)-cl\(A\)\(\backslash A\) \(\subseteq \tau_{1,2}\)-cl\(A\) \(\cap A'\). Since \(\tau_{1,2}\)-cl\(A\) \(\cap A'\) is \((1,2)^*-\text{abg}\)-closed set and by hypothesis, we have \(\tau_{1,2}\)-cl\(A\) \(\cap A'\) \(\subseteq \tau_{1,2}\)-int\(\tau_{1,2}\)-cl\(A\)\(\backslash A\) = \(\emptyset\). So \(\tau_{1,2}\)-cl\(A\) \(\subseteq G\). Hence \(A\) is \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\).

3.4 \((1,2)^*-\text{gab}\)-CONTINUOUS FUNCTION

We introduce the following definition.

Definition 3.4: A function \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is called \((1,2)^*-\text{gab}\)-continuous if \(f^{-1}(V)\) is a \((1,2)^*-\text{gab}\)-closed set of \((X, \tau_1, \tau_2)\) for every closed set \(V\) of \((Y, \sigma_1, \sigma_2)\).

Theorem 3.4.1: Every continuous map \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is \((1,2)^*-\text{gab}\)-continuous.

Proof: Let \(V\) be a \((1,2)^*-\text{closed}\) set in \((Y, \sigma_1, \sigma_2)\), then \(f^{-1}(V)\) is a \((1,2)^*-\text{closed}\) set in \((X, \tau_1, \tau_2)\). Since every \((1,2)^*-\text{closed}\) set is \((1,2)^*-\text{gab}\)-closed set, \(f^{-1}(V)\) is \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\). Hence \(f\) is \((1,2)^*-\text{gab}\)-continuous.

Remark 3.4.1: The converse of the above theorem need not be true. This is proved in the following example:

Example 3.4.1: Let \(X = \{a, b, c\} = Y\) with \(\tau_{1,2} = \{\emptyset, X, \{a\}, \{b, c\}\}\) and \(\sigma_{1,2} = \{\emptyset, Y, \{a\}, \{b\}\}\). Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be the identity map, then \(f\) is \((1,2)^*-\text{gab}\)-continuous but not \((1,2)^*-\text{continuous}\). For the \((1,2)^*-\text{closed}\) set \(\{c\}\) in \((Y, \sigma_1, \sigma_2)\), \(f^{-1}(\{c\}) = \{c\}\) is \((1,2)^*-\text{gab}\)-closed set, but not \((1,2)^*-\text{closed}\) set in \((X, \tau_1, \tau_2)\).

Theorem 3.4.2: Every \((1,2)^*-\text{gab}\)-continuous map is a \((1,2)^*-\text{g-continuous}\), \((1,2)^*-\text{ag-continuous}\), \((1,2)^*-\text{gs-continuous}\), \((1,2)^*-\text{gp-continuous}\), \((1,2)^*-\text{gsp-continuous}\), \((1,2)^*-\text{gb-continuous}\), \((1,2)^*-\text{g-continuous}\).

Proof: Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be a \((1,2)^*-\text{gab}\)-continuous map. Let \(V\) be a \((1,2)^*-\text{closed}\) set in \((Y, \sigma_1, \sigma_2)\), then \(f^{-1}(V)\) is a \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\). Then by Theorem 3.1.2 a), b), c), d), e), f) and g), \(f\) is an \((1,2)^*-\text{g-continuous}\), \((1,2)^*-\text{ag-continuous}\), \((1,2)^*-\text{gs-continuous}\), \((1,2)^*-\text{gp-continuous}\), \((1,2)^*-\text{gsp-continuous}\), \((1,2)^*-\text{gb-continuous}\) and \((1,2)^*-\text{g-continuous}\) respectively.

Remark 3.4.2: The converse of the above theorem need not be true. This is proved in the following example:

Example 3.4.2: Let \(X = \{a, b, c\} = Y\) with \(\tau_{1,2} = \{\emptyset, X, \{a\}, \{b\}\}\) and \(\sigma_{1,2} = \{\emptyset, Y, \{a\}, \{b\}\}\). Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be the identity map, then \(f\) is \((1,2)^*-\text{g-continuous}\) but not \((1,2)^*-\text{gab}\)-continuous. For the \((1,2)^*-\text{closed}\) set \(\{a, c\}\) in \((Y, \sigma_1, \sigma_2)\), \(f^{-1}(\{a, c\}) = \{a, c\}\) is \((1,2)^*-\text{g-continuous}\), but not \((1,2)^*-\text{gab}\)-closed set in \((X, \tau_1, \tau_2)\).
Example 3.4.3: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map, then $f$ is (1,2)*-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a, c\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a, c\})=\{a, c\}$ is (1,2)*-ag-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

Example 3.4.4: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Define $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a)=\{c\}, f(b)=\{a\}, f(c)=\{b\}$ and $f^{-1}(\{a\})=\{a\}$, $f^{-1}(\{b\})=\{b\}$, then $f$ is (1,2)*-ag-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a\})=\{b\}$ is (1,2)*-ag-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

Example 3.4.5: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map, then $f$ is (1,2)*-gs-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a, c\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a, c\})=\{a, c\}$ is (1,2)*-gs-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

Example 3.4.6: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Define $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a)=\{c\}, f(b)=\{a\}, f(c)=\{b\}$ and $f^{-1}(\{a\})=\{a\}$, $f^{-1}(\{b\})=\{b\}$, then $f$ is (1,2)*-gp-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a\})=\{a\}$ is (1,2)*-gp-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

Example 3.4.7: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an identity map, then $f$ is (1,2)*-gp-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a\})=\{a\}$ is (1,2)*-gp-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

Example 3.4.8: Let $X=\{a, b, c\}=Y$ with $\tau_{1,2}=\{\phi, X, \{a\},\{a, b\}\}$ and $\sigma_{1,2}=\{\phi, Y, \{a\},\{a, b\}\}$. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an identity map, then $f$ is (1,2)*-gb-continuous but not (1,2)*-gab$g$-continuous. For the (1,2)*-closed set $\{a\}$ in $(Y, \sigma_1, \sigma_2)$, $f^{-1}(\{a\})=\{a\}$ is (1,2)*-gb-closed set, but not (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$.

3.5 APPLICATIONS OF (1,2)*-gab$g$-CLOSED SETS

As an application of (1,2)*-gab$g$-closed sets we introduce a new space namely (1,2)*-$T_{gab\ g}$-space.

Definition 3.5: A space $(X, \tau_1, \tau_2)$ is called (1,2)*-$T_{gab\ g}$-space if every (1,2)*-gab$g$-closed sets in it is (1,2)*-closed set.

Theorem 3.5.1: For a space $(X, \tau_1, \tau_2)$ the following are equivalent:

a) $(X, \tau_1, \tau_2)$ is a (1,2)*-$T_{gab\ g}$-space.

b) Every singleton of $(X, \tau_1, \tau_2)$ is either (1,2)*-ab$g$-closed set or (1,2)*-open set.

Proof:

(a) $\Rightarrow$ (b): Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{gab\ g}$-space. Assume that for some $x \in X$, the set $\{x\}$ is not (1,2)*-ab$g$-closed set in $(X, \tau_1, \tau_2)$. Then the only (1,2)*-ab$g$-open set containing $\{x\}$ is the space $X$ itself and so $\{x\}$ is (1,2)*-gab$g$-closed set in $(X, \tau_1, \tau_2)$. By our assumption $\{x\}$ is (1,2)*-closed set in $(X, \tau_1, \tau_2)$ and hence $\{x\}$ is (1,2)*-open set.

(b) $\Rightarrow$ (a): Let $A$ be a (1,2)*-gab$g$-closed subset of $(X, \tau_1, \tau_2)$ and let $x \in \tau_{1,2}$-$cl(A)$. By assumption $\{x\}$ is either (1,2)*-ab$g$-closed set or (1,2)*-open set.

CASE 1: Suppose $\{x\}$ is (1,2)*-ab$g$-closed set.

If $\{x\} \not\subseteq A$, then $\tau_{1,2}$-$cl(A) \setminus A$ contains a non empty (1,2)*-ab$g$-closed set $\{x\}$, which is a contradiction. Therefore $\{x\} \subseteq A$. This implies that $\tau_{1,2}$-$cl(A) \subseteq A$. Hence $A$ is (1,2)*-closed set.

CASE 2: Suppose $\{x\}$ is (1,2)*-open set.

Since $x \in \tau_{1,2}$-$cl(A)$, $\{x\} \cap A \neq \emptyset$ and therefore $\tau_{1,2}$-$cl(A) \subseteq A$. Hence $A$ is (1,2)*-closed set.
Theorem 3.5.2:
a) Every (1,2)*-$T_{1/2}$-space is a (1,2)*-$T_{gbg}$-space.
b) Every (1,2)*-$T_{g}$-space is a (1,2)*-$T_{gbg}$-space.
c) Every (1,2)*-$aT_{g}$-space is a (1,2)*-$T_{gbg}$-space.

Proof:
a) Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{1/2}$-space, and let $A$ be a (1,2)*-$gabg$-closed subset of $(X, \tau_1, \tau_2)$. By theorem 3.1.2(a), $A$ is (1,2)*-$g$-closed. Since $(X, \tau_1, \tau_2)$ is a (1,2)*-$T_{1/2}$-space, $A$ is (1,2)*-closed set in $(X, \tau_1, \tau_2)$. Hence $(X, \tau_1, \tau_2)$ is (1,2)*-$T_{gbg}$-space.
b) Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{g}$-space, and let $A$ be a (1,2)*-$gabg$-closed subset of $(X, \tau_1, \tau_2)$. By theorem 3.1.2(d), $A$ is (1,2)*-$g$-closed. Since $(X, \tau_1, \tau_2)$ is a (1,2)*-$T_{g}$-space, $A$ is (1,2)*-closed set in $(X, \tau_1, \tau_2)$. Hence $(X, \tau_1, \tau_2)$ is (1,2)*-$T_{gbg}$-space.
c) Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$aT_{g}$-space, and let $A$ be a (1,2)*-$gabg$-closed subset of $(X, \tau_1, \tau_2)$. By theorem 3.1.2(b), $A$ is (1,2)*-$ag$-closed. Since $(X, \tau_1, \tau_2)$ is a (1,2)*-$aT_{h}$-space, $A$ is (1,2)*-closed set in $(X, \tau_1, \tau_2)$. Hence $(X, \tau_1, \tau_2)$ is (1,2)*-$T_{gbg}$-space.

Remark 3.5.1: The converse of the above Theorem need not be true. This is proved in the following examples:

Example 3.5.1: Let $X=\{a, b, c\}$ with $\tau_1=\{\phi, X, \{a\}\}$ and $\tau_2=\{\phi, X, \{a, b\}\}$. Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{gbg}$-space. Here $A=\{a, c\}$ is (1,2)*-$g$-closed set but not (1,2)*-closed set. Hence $(X, \tau_1, \tau_2)$ is not (1,2)*-$T_{1/2}$-space.

Example 3.5.2: Let $X=\{a, b, c\}$ with $\tau_1=\{\phi, X, \{a\}\}$ and $\tau_2=\{\phi, X, \{a, b\}, \{a, b\}\}$. Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{gbg}$-space. Here $A=\{a\}$ is (1,2)*-$g$-closed set but not (1,2)*-closed set. Hence $(X, \tau_1, \tau_2)$ is not (1,2)*-$T_{g}$-space.

Example 3.5.3: Let $X=\{a, b, c\}$ with $\tau_1=\{\phi, X, \{a\}\}$ and $\tau_2=\{\phi, X, \{a, b\}\}$. Let $(X, \tau_1, \tau_2)$ be a (1,2)*-$T_{gbg}$-space. Here $A=\{a, c\}$ is (1,2)*-$ag$-closed set but not (1,2)*-closed set. Hence $(X, \tau_1, \tau_2)$ is not (1,2)*-$aT_{h}$-space.

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