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STRONGLY EDGE IRREGULAR INTERVAL-VALUED FUZZY GRAPHS

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ABSTRACT

In this paper, strongly edge irregular interval-valued fuzzy graphs and strongly edge totally irregular interval-valued fuzzy graphs are introduced. Comparative study between strongly edge irregular interval-valued fuzzy graph and strongly edge totally irregular interval-valued fuzzy graph is done. Some properties of strongly edge irregular interval-valued fuzzy graphs are studied and they are examined for strongly edge totally irregular interval-valued fuzzy graphs. Strongly edge irregularity on interval-valued fuzzy graphs whose underlying graphs are cycle, path, star and barbell graph are studied.

Key words: *interval-valued fuzzy graphs, edge degree, total edge degree, strongly irregular, highly irregular,*

AMS subject classification: *05C12, 03E72, 05C72.*

1. INTRODUCTION

In 1736, Euler first introduced the concept of graph theory. Graph theory is a very useful tool for solving combinatorial problems in different areas such as operations research, optimization, topology, geometry, number theory and computer science. Fuzzy set theory was first introduced by Zadeh in 1965. Interval-valued fuzzy graphs are introduced by Akram and Dudec in 2011. Infact interval-valued fuzzy graph and interval-valued intuitionistic fuzzy graphs are two models that extend theory of fuzzy graphs. Interval-valued fuzzy sets provide more adequate description of uncertainity than traditional fuzzy sets. It is therefore important to use interval-valued fuzzy set in application such as fuzzy control.

1.1 Review of Literature

M.Akram and Wieslaw A.Dudek introduced the concept of interval-valued fuzzy graphs[1]. A. Nagoorgani and S. R. Latha introduced irregular fuzzy graphs [5]. S.P. Nandhini and E. Nandhini introduced strongly irregular fuzzy graphs[8]. K. Radha and N. Kumaravel introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs [9]. S. Ravi Narayanan and N.R. Santhi Maheswari introduced edge regular interval-valued fuzzy graphs [11]. N.R. Santhi Maheswari and C.Sekar introduced edge irregular fuzzy graphs [12]. These motivates us to introduce an edge irregular interval-valued fuzzy graphs and edge totally irregular interval-valued fuzzy graphs and discussed some of its properties. Throughout this paper, the vertices take the membership value $A = (\mu_A^-, \mu_A^+)$ and edges take the membership value $B = (\mu_B^-, \mu_B^+)$.

2. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1: A fuzzy graph denoted by G: (σ, μ) on the graph G^*: (V,E), is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a set V and $\mu : V X V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \Lambda \sigma(v)$ is satisfied. A fuzzy graph G is complete if $\mu(u, v) = \mu(uv) = \sigma(u) \Lambda \sigma(v)$ for all $u, v \in V$ where uv denotes the line joining u and v. $G^* : (V,E)$ is called the underlying crisp graph of the fuzzy graph $G : (\sigma, \mu)$, where σ and μ are called membership function[4].

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Definition 2.2: An interval-valued fuzzy graph with an underlying set V is defined to be the pair (A, B), where $A = (\mu_A-, \mu_A+)$ is an interval-valued fuzzy set on V and $B = (\mu_B- \mu_B+)$ is an interval-valued fuzzy set on E such that $\mu_B-(x, y)\leq min\{(\mu_A-(x), \mu_A-(y)\}$ and $\mu_B+(x, y)\leq max\{(\mu_A+(x), \mu_A+(y)\}$, for all $(x,y) \in E$. Here, A is called interval-valued fuzzy vertex set on V and B is called interval-valued fuzzy edge set on E.[1]

Definition 2.3: Let G: (A, B) be an interval-valued fuzzy graph, where $A = (\mu_A-, \mu_A+)$ and $B = (\mu_B- , \mu_B+)$. The positive degree of a vertex u in G is defined as $d^+(u) = \sum \mu_B+ (uv)$, for $uv \in E$. The negative degree of a vertex u in G is defined as $d^-(u) = \sum \mu_B- (uv)$, for $uv \in E$ and $\mu_B- (uv)= \mu_B+ (uv) = 0$ if uv not in E. The degree of a vertex u is defined as $d(u)=(d^-(u),d^+(u))$.[1]

Definition 2.4: Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$.

The positive degree of an edge is defined as $d_G^+(uv)=d_G^+(u)+d_G^+(v)-2\mu_B^+ (uv)$.

The negative degree of an edge is defined as $d_G^-(uv)=d_G^-(u)+d_G^-(v)-2 \mu_B^- (uv)$.

The degree of an edge is defined as $d(uv)=(d_G^-(uv),d_G^+(uv))$.

The minimum degree of an edge is $\delta_E(G)=\Lambda\{d(uv):uv \in E\}$

The maximum degree of an edge is $\Delta_E(G)=V\{d(uv):uv \in E\}$[11]

Definition 2.5: Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$. The total positive degree of an edge is defined as $td_G^+(uv)=d_G^+(u)+d_G^+(v)- \mu_B^+ (uv)$.

The total negative degree of an edge is defined as $td_G^-(uv)=d_G^-(u)+d_G^-(v)- \mu_B^- (uv)$.

The total edge degree is defined as $td_G(uv)=(td_G^-(uv),td_G^+(uv))$.

It can also be defined as $td_G(uv)=d_G(uv)+B(uv)$, where $B(uv)=(\mu_B^- (uv), \mu_B^+ (uv))$.

The minimum total degree of an edge is $\delta_{tE}(G)=\Lambda\{td_G(uv):uv \in E\}$

The maximum total degree of an edge is $\Delta_{tE}(G)=V\{td_G(uv):uv \in E\}$ [11]

Definition 2.6: Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$. If each edge in G has the same degree (k_1, k_2), then G is said to be an (k_1, k_2)- edge regular interval-valued fuzzy graph. If there exists a vertex which is adjacent to vertices with distinct degrees then G is said to be an irregular interval-valued fuzzy graph.[11].

Definition 2.7: Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$. If each edge in G has the same total degree (c_1, c_2), then G is said to be totally (c_1, c_2)- edge regular interval-valued fuzzy graph. If there exists a vertex which is adjacent to vertices with distinct total degrees then G is said to be totally irregular interval-valued fuzzy graph.[11].

Definition 2.8: Let $G: (\sigma, \mu)$ be a connected fuzzy graph on $G^*: (V, E)$. Then G is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees[8].

Definition 2.9: Let $G: (\sigma, \mu)$ be a connected fuzzy graph on $G^*: (V, E)$. Then G is said to be a highly irregular fuzzy graph if every vertex is adjacent to vertices having distinct degrees[8].

Definition 2.10: Star $K_{1,n}$ with n spokes(having $n+1$ vertices with n pendant edges)[12].

Definition 2.11: Barbell graph $B_{n,m}$ is defined by n pendant edges attached with one end of K_2 and m pendant edges attached with other end of K_2.[12].

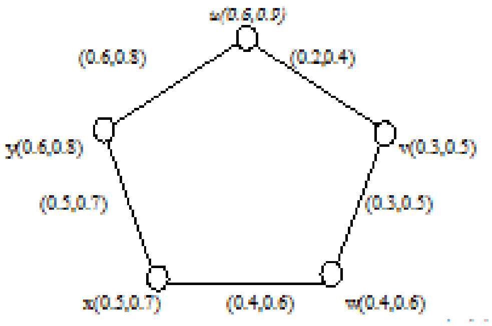
3. STRONGLY EDGE IRREGULAR INTERVAL-VALUED FUZZY GRAPHS

In this section, we define an strongly edge irregular interval-valued fuzzy graph and strongly edge totally irregular interval-valued fuzzy graph. Some properties of strongly edge irregular and strongly edge totally irregular interval-valued fuzzy graphs are studied.

Definition 3.1: Let $G: (A,B)$ be a connected interval-valued fuzzy graph on $G^*(V,E)$, where $A = (\mu_{A^-}, \mu_A+)$ and $B = (\mu_{B^-}, \mu_B+)$ be two interval-valued fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a strongly edge irregular interval-valued fuzzy graph if every pair of edges having distinct degrees.

Definition 3.2: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V,E)$, where $A = (\mu_{A^-}, \mu_A+)$ and $B = (\mu_{B^-}, \mu_B+)$ be two interval-valued fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a strongly edge totally irregular interval-valued fuzzy graph if every pair of edges having distinct total degrees.

Example 3.3: Consider an interval-valued fuzzy graph on $G^*(V, E)$, a cycle of length 5.



Here, $d_G(u) = (0.8, 1.2)$, $d_G(v) = (0.5, 0.9)$, $d_G(w) = (0.7, 1.1)$, $d_G(x) = (0.9, 1.3)$ and $d_G(y) = (1.1, 1.5)$

$d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2\mu_B^-(uv) = 0.8 + 0.5 - 2(0.2) = 0.9$
$d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2\,\mu_B^+(uv) = 1.2 + 0.9 - 2(0.4) = 1.3$
$d_G(uv) = (d_G^-(uv), d_G^+(uv)) = (0.9, 1.3).$

$d_G^-(vw) = d_G^-(v) + d_G^-(w) - 2\,\mu_B^-(vw) = 0.5 + 0.7 - 2(0.3) = 0.6$
$d_G^+(vw) = d_G^+(v) + d_G^+(w) - 2\,\mu_B^+(vw) = 0.9 + 1.1 - 2(0.5) = 1$
$d_G(vw) = (d_G^-(vw), d_G^+(vw)) = (0.6, 1).$

$d_G^-(wx) = d_G^-(w) + d_G^-(x) - 2\,\mu_B^-(wx) = 0.7 + 0.9 - 2(0.4) = 0.8.$
$d_G^+(wx) = d_G^+(w) + d_G^+(x) - 2\,\mu_B^+(wx) = 1.1 + 1.3 - 2(0.6) = 1.2.$
$d_G(wx) = (d_G^-(wx), d_G^-(wx)) = (0.8, 1.2).$

$d_G^-(xy) = d_G^-(x) + d_G^-(y) - 2\,\mu_B^-(xy) = 0.9 + 1.1 - 2(0.5) = 1$
$d_G^+(xy) = d_G^+(x) + d_G^+(y) - 2\,\mu_B^+(xy) = 1.3 + 1.5 - 2(0.7) = 1.4$
$d_G(xy) = (d_G^-(xy), d_G^+(xy)) = (1, 1.4).$

$d_G^-(yu) = d_G^-(y) + d_G^-(u) - 2\,\mu_B^-(yu) = 1.1 + 0.8 - 2(0.6) = 0.7.$
$d_G^+(yu) = d_G^+(y) + d_G^+(u) - 2\,\mu_B^+(yu) = 1.5 + 1.2 - 2(0.8) = 1.1$
$d_G(yu) = (d_G^-(yu), d_G^-(yu)) = (0.7, 1.1).$

Here, $d_G(uv) = (0.9, 1.3)$, $d_G(vw) = (0.6, 1)$, $d_G(wx) = (0.8, 1.2)$, $d_G(xy) = (1, 1.4)$, $d_G(yu) = (0.7, 1.1)$.

It is noted that G is strongly edge irregular interval-valued fuzzy graph.

Also, $td_G(uv) = (1.1, 1.7)$, $td_G(vw) = (0.9, 1.5)$, $td_G(wx) = (1.2, 1.8)$, $td_G(xy) = (1.5, 2.1)$, $td_G(yu) = (1.3, 1.9)$.

It is noted that G is strongly edge totally irregular interval-valued fuzzy graph.

Remark 3.4: A strongly edge irregular interval-valued fuzzy graph need not be a strongly edge totally irregular interval-valued fuzzy graph.

Remark 3.5: A strongly edge totally irregular interval-valued fuzzy graph need not be a strongly edge irregular interval-valued fuzzy graph.

Theorem 3.6: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge irregular interval-valued fuzzy graph, then G is strongly edge totally irregular interval-valued fuzzy graph.

Proof: Assume that B is a constant function, let $B(uv) = (c_1, c_2)$, for all $uv \in E$, where c_1 and c_2 are constant. Let uv and xy be any pair of edges in E. Suppose that G is an edge irregular interval-valued fuzzy graph. Then $d_G(uv) \neq d_G(xy)$, where uv , xy are any pair of edges in E .Consider $d_G(uv) \neq d_G(xy)$

$\Rightarrow (d_G^-(uv), d_G^+(uv)) \neq (d_G^-(xy), d_G^+(xy))$
$\Rightarrow (d_G^-(uv), d_G^+(uv)) + (c_1, c_2) \neq (d_G^-(xy), d_G^+(xy)) + (c_1, c_2)$
$\Rightarrow d_G(uv) + B(uv) \neq d_G(xy) + B(uv) \Rightarrow td_G(uv) \neq td_G(xy)$,
where uv, xy are any pair of edges in E. Hence G is strongly edge totally irregular interval-valued fuzzy graph.

Theorem 3.7: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge totally irregular interval-valued fuzzy graph, then G is a strongly edge irregular interval-valued fuzzy graph.

Proof: Proof is similar to the above theorem 3.6.

Remark 3.8: Theorems 3.6 and 3.7 jointly yield the following result. Let $G:(A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If B is constant function, then G is strongly edge irregular interval-valued fuzzy graph if and only if G is strongly edge totally irregular interval-valued fuzzy graph.

Remark 3.9: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If G is both strongly edge irregular interval-valued fuzzy graph and strongly edge totally irregular interval-valued fuzzy graph. Then B need not be a constant function.

Theorem 3.10: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If G is a strongly edge irregular interval-valued fuzzy graph, then G is neighbourly edge irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Let us assume that G is a strongly edge irregular interval-valued fuzzy graph $\Rightarrow$ every pair of edges in G have distinct degrees$\Rightarrow$ every pair of adjacent edges have distinct degrees. Hence G is neighbourly edge irregular interval-valued fuzzy graph.

Theorem 3.11: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If G is strongly edge totally irregular interval-valued fuzzy graph, then G is neighbourly edge totally irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Let us assume that G is strongly edge totally irregular interval-valued fuzzy graph $\Rightarrow$ every pair of edges in G have distinct total degrees $\Rightarrow$ every pair of adjacent edges have distinct total degrees. Hence G is neighbourly edge totally irregular interval-valued fuzzy graph.

Remark 3.12: Converse of the above two theorems 3.10 and 3.11 need not be true.

Definition 3.13: Let $G: (A, B)$ be interval-valued fuzzy graph on $G^*(V, E)$. Then G is said to be a highly edge irregular interval-valued fuzzy graph if every edge is adjacent to the edges having distinct degrees.

Definition 3.14: Let $G: (A, B)$ be interval-valued fuzzy graph on $G^*(V, E)$. Then G is said to be a highly edge totally irregular interval-valued fuzzy graph if every edge is adjacent to the edges having distinct total degrees.

Theorem 3.15: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If G is strongly edge irregular interval-valued fuzzy graph, then G is highly edge irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Let us assume that G is a strongly edge irregular interval-valued fuzzy graph $\Rightarrow$ every pair of edges in G have distinct degrees $\Rightarrow$ every edge is adjacent to the edges having distinct degrees. Hence G is highly edge irregular interval-valued fuzzy graph.

Theorem 3.16: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. If G is a strongly edge totally irregular interval-valued fuzzy graph, then G is a highly edge totally irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Let us assume that G is a strongly edge totally irregular interval-valued fuzzy graph $\Rightarrow$ every pair of edges in G have distinct total degrees$\Rightarrow$ every edge is adjacent to the edges having distinct total degrees. Hence G is highly edge totally irregular interval-valued fuzzy graph.

Remark 3.17: Converse of the above two theorems 3.15 and 3.16 need not be true.

Theorem 3.18: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is a strongly edge irregular interval-valued fuzzy graph, then G is an irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2)$, for all $uv \in E$, where c_1 and c_2 are constant. Let us suppose that G is a strongly edge irregular interval-valued fuzzy graph. Then every pair of edges have distinct degrees. Let uv and vw are adjacent edges in G having distinct degrees. Then $d_G(uv) \neq d_G(vw) \Longrightarrow (d_G^-(uv), d_G^+(uv)) \neq (d_G^-(vw), d_G^+(vw))$
$\Longrightarrow (d_G^-(u)+d_G^-(v)-2\mu_B^-(uv), d_G^+(u)+d_G^+(v)-2\mu_B^+(uv)) \neq (d_G^-(v)+d_G^-(w)-2\mu_B^-(vw), d_G^+(v)+d_G^+(w)-2\mu_B^+(vw))$
$\Longrightarrow d_G^-(u) + d_G^-(v) - 2c_1 \neq d_G^-(v) + d_G^-(w) - 2c_1$ (or) $d_G^+(u) + d_G^+(v) - 2c_2 \neq d_G^+(v) + d_G^+(w) - 2c_2$
$\Longrightarrow d_G^-(u)+d_G^-(v) \neq d_G^-(v)+d_G^-(w)$ (or) $d_G^+(u)+d_G^+(v) \neq d_G^+(v) + d_G^+(w)$
$\Longrightarrow d_G^-(u) \neq d_G^-(w)$ (or) $d_G^+(u) \neq d_G^+(w) \Longrightarrow (d_G^-(u), d_G^+(u)) \neq (d_G^-(w), d_G^+(w))$
$\Longrightarrow d_G(u) \neq d_G(w)$
$\Longrightarrow$ there exists a vertex v which is adjacent to the vertices u and w having distinct degrees. Hence G is an irregular interval-valued fuzzy graph.

Theorem 3.19: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is a strongly edge totally irregular interval-valued fuzzy graph, then G is an irregular interval-valued fuzzy graph.

Proof: Proof is similar to above Theorem 3.18

Remark 3.20: Converse of above two Theorems 3.18 and 3.19 need not be true.

Theorem 3.21: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is a strongly edge irregular interval-valued fuzzy graph, then G is a highly irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2)$, for all $uv \in E$,where c_1 and c_2 are constant. Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G. Let us suppose that G is a strongly edge irregular interval-valued fuzzy graph
$\Longrightarrow$ every pair of edges have distinct degrees
$\Longrightarrow d_G(uv) \neq d_G(vw) \neq d_G(vx)$
$\Longrightarrow (d_G^-(uv), d_G^+(uv)) \neq (d_G^-(vw), d_G^+(vw)) \neq (d_G^-(vx), d_G^+(vx))$
$\Longrightarrow d_G^-(u)+d_G^-(v)-2\mu_B^-(uv) \neq d_G^-(v)+d_G^-(w)-2\mu_B^-(vw)$ (or) $d_G^+(u)+d_G^+(v)-2\mu_B^+(uv) \neq d_G^+(v) + d_G^+(w) - 2\mu_B^+(vw)$
$\Longrightarrow d_G^-(u) + d_G^-(v) - 2c_1 \neq d_G^-(v)+d_G^-(w) - 2c_1$ (or) $d_G^+(u)+d_G^+(v) - 2c_2 \neq d_G^+(v)+d_G^+(w) - 2c_2$
$\Longrightarrow d_G^-(u)+d_G^-(v) \neq d_G^-(v)+d_G^-(w)$ (or) $d_G^+(u)+d_G^+(v) \neq d_G^+(v)+d_G^+(w)$
$\Longrightarrow d_G^-(u) \neq d_G^-(w)(or)d_G^+(u) \neq d_G^+(w)$
$\Longrightarrow (d_G^-(u), d_G^+(u)) \neq (d_G^-(w), d_G^+(w))$
$\Longrightarrow d_G(u) \neq d_G(w)$

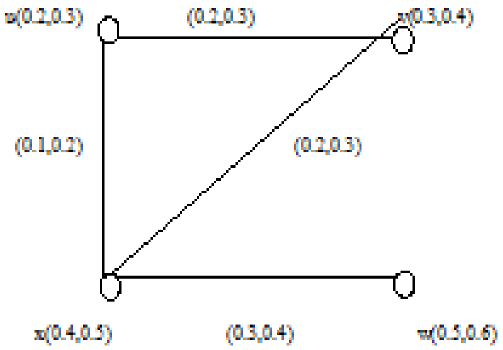
Similarly by taking $(d_G^-(vw), d_G^+(vw)) \neq (d_G^-(vx), d_G^+(vx))$, we get $d_G(w) \neq d_G(x)$
$\Longrightarrow d_G(u) \neq d_G(w) \neq d_G(x)$. So, every vertex is adjacent to vertices having distinct degrees. Hence G is a highly irregular interval-valued fuzzy graph.

Theorem 3.22: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is a strongly edge totally irregular interval-valued fuzzy graph, then G is a highly irregular interval-valued fuzzy graph.

Proof: Proof is similar to the above Theorem 3.21

Remark 3.23: Converse of above two theorems 3.21 and 3.22 need not be true.

Example 3.24: Consider an interval-valued fuzzy graph on $G^*(V, E)$.



Here, $d(u) = (0.3, 0.5)$, $d(v) = (0.4, 0.6)$, $d(w) = (0.6, 0.9)$, $d(x) = (0.3, 0.4)$.

G is an irregular interval-valued fuzzy graph and highly irregular interval-valued fuzzy graph.

The degree of the edges are $d_G(uv) = (0.3, 0.5)$, $d_G(vw) = (0.6, 0.9)$, $d_G(wx) = (0.3, 0.5)$ and $d_G(uw) = (0.7, 1)$.

It is noted that $d_G(uv) = (0.3, 0.5)$ and $d_G(wx) = (0.3, 0.5)$. Hence G is not strongly edge irregular interval-valued fuzzy graph.

The total degree of the edges are $td_G(uv) = (0.5, 0.8)$, $td_G(vw) = (0.8, 1.2)$, $td_G(wx) = (0.6, 0.9)$ and $td_G(uw) = (0.8, 1.2)$.

Also, it is noted that $td_G(vw) = (0.8, 1.2)$ and $td_G(uw) = (0.8, 1.2)$. Hence G is not strongly edge totally irregular interval-valued fuzzy graph.

Definition 3.25: Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$. If no two edges have the same edge degree and each edge e_i have edge degree (c_i, k_i) with $c_i = |k_i|$, then G is called an equally strongly edge irregular interval-valued fuzzy graph. Otherwise it is unequally strongly edge irregular interval-valued fuzzy graph.

Remark 3.26: An equally strongly edge irregular interval-valued fuzzy graph is strongly edge irregular interval-valued fuzzy graph.

Remark 3.27: A strongly edge irregular interval-valued fuzzy graph need not be equally strongly edge irregular interval-valued fuzzy graph.

4. EDGE IRREGULARITY ON PATH, CYCLE, STAR AND BARBELL GRAPH WITH SOME SPECIFIC MEMBERSHIP FUNCTIONS

In this section, we study about the properties of edge irregular and edge totally irregular interval-valued fuzzy graphs on a path, a cycle, a star and a barbell graph.

Theorem 4.1: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$, a path on $2m$ vertices. If the membership values of the edges $e_1, e_2, \ldots, e_{2m-1}$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \ldots, (c_{2m-1}, k_{2m-1})$ such that $(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_{2m-1}, k_{2m-1})$, then G is both strongly edge irregular and strongly edge totally irregular interval-valued fuzzy graph.

Proof: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$, a path on $2m$ vertices. Let $e_1, e_2, \ldots, e_{2m-1}$ be the edges of G^* in that order. Let the membership values of the edges $e_1, e_2, \ldots, e_{2m-1}$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \ldots, (c_{2m-1}, k_{2m-1})$ such that $(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_{2m-1}, k_{2m-1})$

For $i = 2, 3, 4, \ldots, 2m - 1$
$d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$
$d_G(v_1) = (c_1, k_1)$
$d_G(v_{2m}) = (c_{2m-1}, k_{2m-1})$

For $i = 2, 3, 4, \ldots, 2m - 2$
$d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$
$d_G(e_1) = (c_2, k_2)$
$d_G(e_{2m}) = (c_{2m-2}, k_{2m-2})$

Hence G is strongly edge irregular interval-valued fuzzy graph.

For $i = 2, 3, 4, \ldots, 2m - 2$
$td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$
$td_G(e_1) = (c_2, k_2) + (c_1, k_1)$
$td_G(e_{2m}) = (c_{2m-2}, k_{2m-2}) + (c_{2m-1}, k_{2m-1})$

Hence G is strongly edge totally irregular interval-valued fuzzy graph.

Theorem 4.2: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$, a cycle on n vertices. If the membership values of the edges $e_1, e_2, \ldots, e_n$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \ldots, (c_n, k_n)$ such that $(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_n, k_n)$, then G is both strongly edge irregular and strongly edge totally irregular interval-valued fuzzy graph.

Proof: Let $G: (A,B)$ be a connected interval-valued fuzzy graph on $G^*(V,E)$, a cycle on n vertices. Let $e_1, e_2, \ldots, e_n$ be the edges of G^* in that order. Let the membership values of the edges $e_1, e_2, \ldots, e_n$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \ldots, (c_n, k_n)$ such that $(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_n, k_n)$

For $i = 2, 3, 4, \ldots, n$
$d_G(v_i) = (c_{i-1}, k_{i-1})+(c_i, k_i)$
$d_G(v_1) = (c_1, k_1) + (c_n, k_n)$

For $i = 2, 3, 4, \ldots, n - 1$
$d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$
$d_G(e_1) = (c_2, k_2) + (c_n, k_n)$
$d_G(en) = (c_1, k_1) + (c_{n-1}, k_{n-1})$

Hence G is a strongly edge irregular interval-valued fuzzy graph.

For $i = 2, 3, 4, \ldots, n - 1$
$td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$
$td_G(e_1) = (c_2, k_2) + (c_n, k_n) + (c_1, k_1)$
$td_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1}) + (c_n, k_n)$

Hence G is a strongly edge totally irregular interval-valued fuzzy graph.

Theorem 4.3: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$, a star on $K_{1,n}$ vertices. If the membership values of all the edges are distinct then G is a strongly edge irregular interval-valued fuzzy graph and G is a totally edge regular interval-valued fuzzy graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent to the vertex x. Let $e_1, e_2, \ldots, e_n$ be the edges of a star G^* in that order have membership values (c_1, k_1) , (c_2, k_2) , (c_3, k_3) , . . . , (c_n, k_n) such that$(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_n, k_n)$, then
$d_G(e_i) =(c_1, k_1)+ (c_2, k_2) + (c_3, k_3)+ \ldots + (c_n, k_n)+(c_i, k_i)-2(c_i, k_i)(1 \leq i \leq n)$
$d_G(e_i) =(c_1, k_1) + (c_2, k_2)+ (c_3, k_3)+ \ldots + (c_n, k_n) - (c_i, k_i)(1 \leq i \leq n)$

All the edges e_i have distinct degrees. Hence G is a strongly edge irregular interval-valued fuzzy graph. Also,
$td_G(e_i) =(c_1, k_1) , (c_2, k_2) , (c_3, k_3) , \ldots , (c_n, k_n) + (c_i, k_i) - (c_i, k_i)(1 \leq i \leq n)$
$td_G(e_i) =(c_1, k_1) , (c_2, k_2) , (c_3, k_3) , \ldots , (c_n, k_n)(1 \leq i \leq n)$

All the edges e_i have same total degrees. Hence G is totally edge regular interval-valued fuzzy graph.

Theorem 4.4: Let $G: (A, B)$ be a connected interval-valued fuzzy graph on $G^*(V, E)$, a Barbell graph $B_{n,m}$. If the membership values of all the edges are distinct then G is a strongly edge irregular interval-valued fuzzy graph and G is not strongly edge totally irregular interval-valued fuzzy graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent to the vertex x. Let $e_1, e_2, \ldots, e_n$ be the edges incident with vertex x in that order have membership values (c_1, k_1) , (c_2, k_2) , (c_3, k_3) , . . . , (c_n, k_n) such that$(c_1, k_1) < (c_2, k_2), < (c_3, k_3) < \cdots < (c_n, k_n)$. Let $u_1, u_2, \ldots, u_m$ be the vertices adjacent with vertex y. Let $f_1, f_2, \ldots, f_m$ be the edges incident with vertex y in that order have membership values (h_1, j_1) , (h_2, j_2) , (h_3, j_3) , . . . , (h_m, j_m) such that $(h_1, j_1) <(h_2, j_2), < (h_3, j_3) < \cdots < (h_m, j_m) < (c, k)$ where (c, k) is the membership value of the edge xy. Then
$d_G(xy) = (c_1, k_1)+(c_2, k_2)+ \cdots +(c_n, k_n)+(c, k)+(h_1, j_1)+(h_2, j_2)+ \cdots + (h_m, j_m) + (c, k) - 2(c, k)$
$d_G(xy) = (c_1, k_1)+(c_2, k_2)+ \cdots +(c_n, k_n)+(h_1, j_1)+(h_2, j_2)+ \cdots + (h_m, j_m)$
$td_G(xy) = (c_1, k_1)+(c_2, k_2)+ \cdots +(c_n, k_n)+ (h_1, j_1)+(h_2, j_2)+ \cdots + (h_m, j_m)+(c, k)$
$d_G(e_i) = (c_1, k_1) + (c_2, k_2) + \cdots + (c_n, k_n) + (c, k) + (c_i, k_i) - 2(c_i, k_i)$
$d_G(f_i) = (h_1, j_1) + (h_2, j_2) + \cdots + (h_m, j_m) + (c, k) + (h_i, j_i) - 2(h_i, j_i)$

Note that every pair of edges have distinct degrees. Hence G is a strongly edge irregular interval-valued fuzzy graph.
$td_G(e_i) = (c_1, k_1)+(c_2, k_2)+ \cdots +(c_n, k_n) + (c, k) + (c_i, k_i) - (c_i, k_i)$
$td_G(e_i) =(c_1, k_1)+(c_2, k_2)+ \cdots +(c_n, k_n)+ (c, k)$
$td_G(f_i) = (h_1, j_1) + (h_2, j_2) + \cdots + (h_m, j_m) + (c, k) + (h_i, j_i) - (h_i, j_i)$
$td_G(f_i) = (h_1, j_1) + (h_2, j_2) + \cdots + (h_m, j_m) + (c, k)$

Note that all $e_i(1 \leq i \leq n)$ and all $f_i(1 \leq i \leq m)$ have same total degrees. Hence G is not strongly edge totally irregular interval-valued fuzzy graph.

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