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MODELING TRUST IN SOCIAL NETWORK

DAVIS NTWIGA[1*], PATRICK WEKE[2], MOSES MANENE[3], AND JOSEPH MWANIKI[4]

[1-4] School of Mathematics, University of Nairobi, PO Box 30197 -00100, Nairobi, Kenya.

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ABSTRACT

We rely on trust in our day to day interactions and activities with each other. It is not easy to estimate it but we offer a simple and powerful method for estimating trust levels of agents in a social network using data from the agents' reputation matrix. The reputation resultant method (RRM) is based on the mean values of the reputation rating matrix and the reputation resultant matrix. Reputation ratings are derived from the agents' peer to peer ratings and the resultant reputation data is the relative reputation ratings by the agents. A comparison is made between the results of Singular value decomposition (SVD) and our new method, the RRM. The two methods offer results that are highly comparative with the RRM being simple, powerful and easy to understand and implement.

Keywords: *SVD, social network, trust, RRM.*

Mathematics Subject Classification: *91D30.*

1. INTRODUCTION

Social networks permeate our lives and play a central role in the transmission of information. The analysis of social networks has continued to provide a significant role in domains of security, terrorism, biology, sales, disease spreading models, economic and marketing to secure higher profits, finance and many others. Due to the large amount of available social networking data, studies and simulation of different nature are possible. These contribute significantly to understanding the properties and the behaviour of social networks [9]. A network embeds dynamic, complex and flexible agents activities and behaviour where the agents act and possibly react to external stimuli, interact with the environment and other agents [6]

When agents interact with one another over time, the history of past interactions informs them about their abilities and dispositions. A good reputation system collects, distributes and aggregates the feedback about the agents past behaviour [11]. Reputation is a collective measure of trustworthiness. In social networks, it is based on the ratings from members in that community. The reputation of an agent is an important factor in performing trust decisions. We rely on trust in our everyday activities and thus it is easy to recognize. It can manifest itself in many different forms, making it quite challenging to define. Reputation is what is generally said or believed about a person's or thing's character or standing. Therefore, trust and reputation are closely linked [8].

Trust information is widely exploited to improve various online applications such as recommender systems, spammer detection, finding high quality user generated content and viral marketing [14]. In social networks, reputation is a quantity derived from the underlying network and the agent's reputation is visible to all agents. A good example is the online trading communities where the seller reputation has significant influences on online auction process. Trust plays an important role in the formation of connections as it determines how information flows and to assess the quality and the credibility of information in the network [3]. The peer-peer agents' reputation ratings are used to model the trust levels of agents in a social network through a simple and powerful method the RRM. Results from the method are compared to those of matrix approximation using SVD.

2. TRUST AND SOCIAL NETWORKS

A social network dynamics provides a platform to study agents and their collective behaviour on a large scale. Social interactions on networks affect agent activity and these activities should be incorporated in social network models to develop optimal models [10]. Social effects of influence on a user's activity from the activity of a user's neighbor to increase the model explanatory and predictive power were incorporated.

Corresponding Author: Davis Ntwiga[1]*

A dynamic social network model by [13] considers individual agents who interact at random with the interactions modeled as games. The payoff from the games determines which interactions are reinforced and the social network structures emerge from these stochastically evolving social networks. The social interaction structures that emerge tend to separate the agents into small interaction groups.

Even though trust and reputation are closely linked, where reputation is the rating a member of a group receives from others, trust is a more complex social relationship than reputation [2]. Trust and distrust are referred to as positive and negative trust. A method for computing trust is developed by [2] and acknowledges that computing distrust is complex. Distrust is incorporated by resolving conflicting trust/distrust information through a non- linear optimization.

The work of [7] represents trust in the interval $(0,1)$ where 0 represents complete distrust and value 1 blind trust. The model reflects members of social network and differentiates them according to their disposition to trusting somebody. A value of 1 indicates that the agent is highly trusted and hence blind trust. Trust and similarity have a strong and significant correlation [14]. Users with trust relations are more similar than those without. The asymmetry of trust suggests that for two people involved in a relation, trust is not necessarily identical in both directions.

Reputation and trust rating systems have wide applications with many different types of mechanisms. [4] Summarizes the basic criteria for judging the quality and soundness of reputation computation engines. No single solution is known to exist that is suitable in all contexts and applications. They further discuss the number of reputation computational engines that exist. One of the engines is the flow model that is used in the reputation ratings of the agents in this study.

We model the trust levels of agents in the social network using reputation rating based on two techniques; the SVD method which has been used extensively in many applications though it is not easy in computations algorithms; and a new method that is simple and powerful, the RRM. We have singled out the SVD because it has a wide appeal, highly versatile and used in many applications in different areas of research.

Section 3 introduces the RRM and the rank application based on SVD. The results are in section 4 and we conclude in section 5.

3. SOCIAL NETWORKS

We assume that reputation ratings about current interactions are captured and distributed and agents are willing to provide the ratings. Consider a set $\mathrm{N} = \{1, 2, ..., N\}$ of agents whose state and interactions in a social network evolve in discrete time *t*. We assume that the agents are connected to each other at any given time $t \in [0,T]$ and thus we have a peer to peer review system for the agents' reputation ratings in the network. Let $R_i = \{r_{1i}, r_{2i}, ..., r_{(n-1)i}\}$ be the reputation ratings agent i receives from the other $N-1$ agents in the social network. This peer to peer reputation rating is based on the five start scale: 1 - lowest, 2 - low, 3 - medium, 4 - good and 5 - high, that is, $R \in [1,2,3,4,5]$

$$R = \begin{cases} r_{ij} = 5 & If \quad i = j \\ 1 \le r_{ij} \le 5 & If \quad i \ne j \end{cases}$$

Each agent is expected to rate the other $N-1$ agents. As would be ideal in real life situations, if we were to rate ourselves, we would likely give ourselves a maximum score of 5. The ratings form the entries of a real valued matrix, R and are bidirectional. These entries of matrix R are assumed to be the 'raw' trust values of the agents as [2] notes that trust and reputation are closely linked.

3.1 Reputation resultant method

We outline a new method that is simple and powerful, the RRM. The results from this method compares well with those of the SVD. Let $\hat{R}$ be a resultant reputation matrix which is extracted from the reputation ratings matrix. Then, $\hat{R}_{ij} = r_{ij} - r_{ji}$, with $r_{ij} = 0$ for all $i = j$. The matrix $\hat{R}$ shows the perceived differences in the reputation ratings of the agents in the network. It shows the strength of agent reputation ratings in reference to the other agents.

We compute the mean value for the reputation ratings matrix *R* by first excluding the values on the main diagonal that are the agents self rating of 5. Then, compute the mean for the resultant reputation matrix $\hat{R}$ by also excluding the zero values on the main diagonal of the matrix as this does not give us any information. Let Θ be defined as the mean of the reputation ratings matrix, that is,

$$\Theta_i = \frac{\sum_{j=1}^{N}\left(R_{ji} - R_{ii}\right)}{N-1}, \quad i, j = 1,2,...,N$$

Let $\hat{\Theta}$ be defined as the mean of the resultant reputation matrix, that is,

$$\hat{\Theta}_i = \frac{\sum_{j=1}^{N}\left(\hat{R}_{ji} - \hat{R}_{ii}\right)}{N-1}, \quad i, j = 1,2,...,N$$

We combine Θ and $\hat{\Theta}$ to have a new equation for the computation of the raw trust values. Let $\overline{\Theta}$ be the raw trust values at time $t \in [0,T]$, then

$$\overline{\Theta} = \frac{\hat{\Theta}_i}{\Theta_i}, \quad i = 1,2,...,N$$

The values of $\overline{\Theta}$ can be scaled on $(0,1]$ to estimate the trust levels of the agents.

3.2 Singular value decomposition

The SVD is a matrix factorization method that has been used widely in different applications ever since an efficient algorithm for its computation was developed [8]. SVD is a powerful and important technique in matrix computations and analysis as it reduces high dimensional and highly variable set of data to a lower dimensional space that exposes the substructure of the original data more clearly [1]. Let R be a $N \times N$ matrix which can be represented as the product of two orthonormal matrices U and V and a diagonal matrix S. Note *R* is the reputation ratings matrix of the agents in the network. We express the R matrix as

$$R = U_{N\times N} S_{N\times K} V_{K\times N}$$

The columns of U are the eigenvectors of RR^T, and the columns of V are the eigenvectors of $R^T R$. The singular values on the diagonal of S are the square roots of the nonzero eigenvalues of both RR^T and $R^T R$, which are ordered decreasingly. The best rank one approximation by the singular value decomposition is used [1, 5, 12]

$$\| R - R^T VV^T \|_2$$

The low rank matrix factorization method is widely employed in various applications such as collective filtering and document clustering [14]. SVD has a variety of applications in engineering, chemistry, ecology, geology, biomedical, scientific computing, geophysics, automatic control and many other areas [12]. We apply this technique to extract the trust levels of the agents from the reputation rating matrix of the agents. The columns of matrix V are the eigenvectors generated from the columns of matrix R. These are the values used to estimate the agents trust levels in our analysis.

The data from the RRM and SVD are scaled in the interval $(0,1]$ to estimate the individual agent trust levels in the social network. The results of the reputation resultant method are then compared to those from the SVD method. We do not have benchmark data set available for this study and simulation is implemented to test the system with the reputation matrix, $R \sim U(0,1)$. Simulation and analysis is based on Matlab version $7.0.1$. The analysis is achieved through descriptive statistics.

4. RESULTS

We compare the trust levels results from the RRM and compare it to those of the SVD method using the low rank approximation. Table 1 shows the descriptive statistics comparing the results from our model, the RRM and those from the low rank approximation based on the SVD. The correlation between the two methods is very high showing that the two methods emit results that are similar. Other statistical values show the same trend. Evidently, the proposed method is as powerful as the traditional SVD method from these results.

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Table 1: Comparison between the RRM and SVD

Method	Agents N	Correlation	Mean	Standard Error	Standard Deviation	Skewness	Coefficient of variation
RRM	8	0.993	0.524	0.117	0.331	0.037	0.632
SVD			0.500	0.127	0.359	0.298	0.717
RRM	15	0.998	0.632	0.069	0.266	-0.755	0.421
SVD			0.611	0.070	0.271	-0.571	0.444
RRM	20	0.999	0.468	0.061	0.272	0.064	0.581
SVD			0.444	0.059	0.264	0.187	0.461
RRM	30	0.998	0.594	0.050	0.274	-0.280	0.461
SVD			0.565	0.051	0.276	-0.114	0.489

Figure 1 highlights the close link between the results obtained from the RRM compared to the traditional method of the SVD. Similarity is evident even when we have many or few agents in the social network.

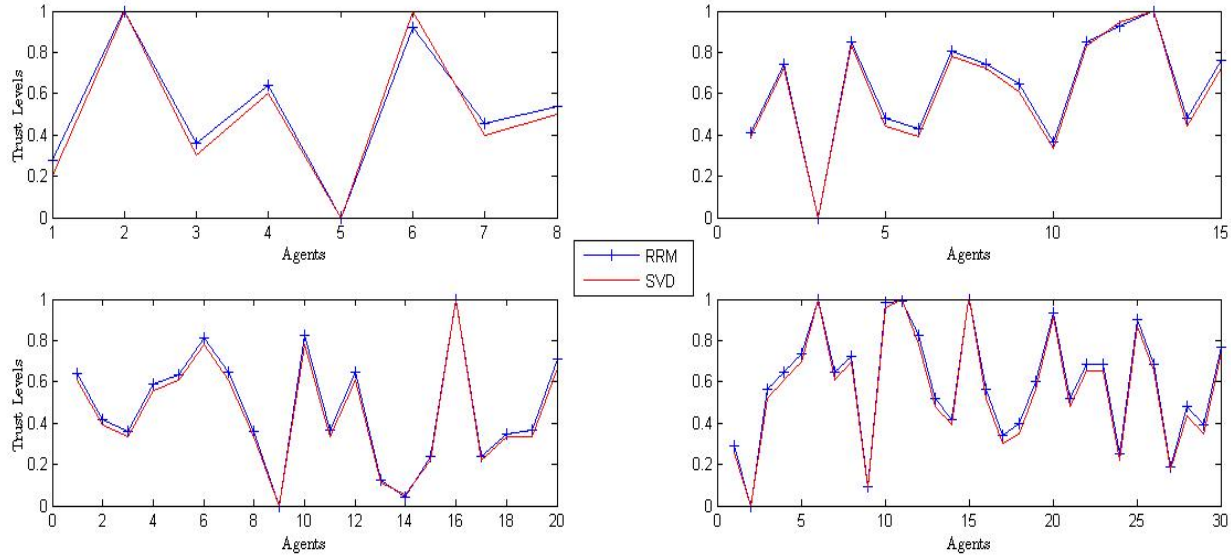

Figure 1: Comparison between RRM and SVD in modeling trust in social network with varying number of agents in the network

5. CONCLUSION

Trust levels in a social network based on the reputation ratings of agents can be estimated using the traditional SVD. This is due to the availability of the ratings in matrix form. We have shown that the RRM offers the same trust level estimations using a reputation matrix similar to those of the singular value decomposition. In this scenario for estimation of trust levels using the reputation matrix, this new method is simple, powerful, easy to implement, uses less computing resources and easy to understand compared to the traditional SVD. Our work invites an extension of the use of RRM to estimate trust with reputation ratings that are undirected and for matrices with sparse entries.

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