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A SINGLE SERVER MARKOVIAN QUEUEING SYSTEM WITH DISCOURAGED ARRIVALS RETENTION OF RENEGED CUSTOMERS AND CONTROLLABLE ARRIVAL RATES

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ABSTRACT

In this paper a Finite Waiting Space Markovian Single – Server Queueing Model with discouraged arrivals, retention of reneged customers and controllable arrival rates is considered. The steady state solutions and the system characteristics are derived for this model. The measures of effectiveness of the queueing model are also obtained. The analytical results are numerically illustrated.

Keywords: *Finite Capacity, Probability of Customers Retention, Reneging, Discouraged arrivals, single Server, Bivariate Poisson process.*

1. INTRODUCTION

A Customer who may enter the queue and decides to leave the queue due to impatience after waiting for a long time is said to have reneged. Customer who tend to be impatient may not always be discouraged by excessive queue size, but may instead join the queue to see how long they have to wait, all the time retaining the prerogative to renege if their estimate of their total wait is intolerable. A reneged customer may be convinced to stay in the queueing system for his further service with probability, say q and he may abandon the queue without receiving the service with a probability $p(=1-q)$.

Queues with discouraged arrivals have applications in computers with batch job processing where job submissions are discouraged when the system is used frequently and arrivals are modelled as a Poisson process with state dependent arrival rate. The discouragement affects the arrival rate of the queueing system. Morse [1] considered discouragement in which the arrival rate falls according to a negative exponential law. Ancker and Gafarian [2] have studied M/M/1/N queueing system with balking and reneging and performed its steady state analysis. Multi-server queueing system with customer impatient can be found in many real life situations such as in hospitals, retail stores, computer communication etc.

Queueing with reneging was firstly studied by Haight [3]. He concentrated on how to make rational decision while waiting in the queue and the probable effect of this decision. Kapodistria [4] has studied a single server Markovian queue with impatient customers and considered the situations, where customers abandoned the system simultaneously. Kumar and Sharma [5] have applied M/M/1/N queueing model for modelling supply chain situations facing customer impatience. Queueing models where potential customers are discouraged by queue length are studied by many researchers in their research work.

Much work has been reported in literature regarding interdependent standard queueing model with controllable arrival rates. Srinivasa Rao *et. al* [6] have discussed M/M/1/∞ interdependent queueing model with controllable arrival rates. Srinivasan and Thiagarajan [7, 8] have analysed M/M/1/K interdependent queueing model with controllable arrival rates and M/M/C/K/N interdependent queueing model with controllable arrivals rates, balking, reneging and spares. Rakeshkumar and Sumeet Kumar sharma [9] have analyzed a single server Markovian queueing system with discouraged arrivals and retention of reneged customers.

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A single server finite capacity retention of reneged customers queueing system M/M/1/K is considered with the assumption that the arrival and service processes were correlated and followed a Bivariate Poisson process. In addition to this interdependence, the system has two arrivals rates, λ_0 - faster arrival rate and λ_1 - slower arrival rate which control the arrivals. It is also assumed that whenever the queue size reaches a prescribed number R, the arrival rate reduces from λ_0 to λ_1 and it continues with that rate as long as the content in the queue was greater than some prescribed integer r[(r ≥ 0) and r < R]. When the content reaches R, the arrival rate changes back to λ_0 and the same process is repeated. In section 2, the description of the model is given. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived. In section 5, the analytical results are numerically illustrated.

2. DESCRIPTION OF THE MODEL

Consider a single server, finite capacity, discouraged arrivals and retention of reneged customers queuing model where arrivals occur according to the Poisson flow of rate λ_0 and λ_1 that depends on the number of customers present in the system at that time and service times are independently, identically and exponentially distributed with rate μ.

The customers are served in order of their arrival. Each customer upon joining the queue will wait a certain length of time for his service to begin. If it does not begin by then, he will get impatient and may leave the queue without getting service with probability p and may remain in the queue for his service with probability $q\left(=1-p\right)$. The reneging time follows exponential distribution with parameter ξ.

The arrival process $\left\{X_1\left(t\right)\right\}$ and the service process $\left\{X_2\left(t\right)\right\}$ of the system are correlated and follow a Bivariate Poisson process given by

$$P\left[X_1(t)=x_1 \quad \mathrm{X}_2(t)=x_2\right]$$
$$=e^{-(\lambda_i+\mu-\in)t}\sum_{\mathrm{j}=0}^{\min(x_1,x_2)}\frac{(\in t)^j[(\lambda_i-\in)t]^{X_1-j}[(\mu\text{-}\in)t]^{X_2-j}}{j!(x_1-j)!(x_2-j)!} \tag{1}$$

where $x_1,x_2=0,1,2,...,\ \ \lambda_1>0,\ \ i=0,1;\ \ \mu>0,\ 0\leq\in<\min(\lambda_{\mathrm{i}},\mu),i=0,1$. with parameters λ_0,λ_1,μ and $\in$ as mean faster rate of arrivals, mean slower rate of arrivals, mean service rate and mean dependence rate (co-variance between arrival and service processes) respectively.

3. STEADY STATE EQUATIONS

We observe that $P_n(0)$ exists when $n=0,1,2,....r-1$; both $P_n(0)$ and $P_n(1)$ exist when $n=r+1,r+2,....R-1$; only $P_n(1)$ exists when $n=R,R+1,....K$.

The Steady state equations are

$$-(\lambda_0-\in)P_0(0)+(\mu-\in)P_1(0)=0 \tag{3.1}$$

$$-\left[\frac{\lambda_0-\in}{n+1}+(\mu-\in)+(n-1)\xi p\right]P_n(0)+\left[(\mu-\in)+n\xi\ \mathrm{p}\right]P_{n+1}(0)+\frac{\lambda_0-\in}{n}\ P_{n-1}(0)=0,\ \mathrm{n}=1,2,3,...r-1 \tag{3.2}$$

$$-\left[\frac{\lambda_0-\in}{r+1}+(\mu-\in)+(r-1)\xi p\right]P_r(0)+\left[(\mu-\in)+r\xi\ p\right]P_{r+1}(0)$$
$$+\frac{\lambda_0-\in}{r}\ P_{r-1}(0)+\left[(\mu-\in)+r\xi\ p\right]P_{r+1}(1)=0 \tag{3.3}$$

$$-\left[\frac{\lambda_0-\in}{n+1}+(\mu-\in)+(n-1)\xi p\right]P_n(0)+\left[(\mu-\in)+n\xi\ p\right]P_{n+1}(0)+\frac{\lambda_0-\in}{n}\ P_{n-1}(0)=0,\ \mathrm{n}=r+1,r+2,...R-2 \tag{3.4}$$

$$-\left[\frac{\lambda_0-\in}{R}+(\mu-\in)+(R-2)\xi p\right]P_{R-1}(0)+\left(\frac{\lambda_0-\in}{R-1}\right)P_{R-2}(0)=0 \tag{3.5}$$

$$-\left[\frac{\lambda_1-\in}{r+2}+(\mu-\in)+r\xi p\right]P_{r+1}(1)+\left[(\mu-\in)+(r+1)\xi\ \mathrm{p}\right]P_{r+2}(1)=0 \tag{3.6}$$

$$-\left[\frac{\lambda_1-\in}{n+1}+(\mu-\in)+(n-1)\xi p\right]P_n(1)+\left[(\mu-\in)+n\xi\ \mathrm{p}\right]P_{n+1}(1)$$
$$+\left(\frac{\lambda_1-\in}{n}\right)P_{n-1}(1)=0 \qquad \mathrm{n=r+2,r+3,\ldots,R\text{-}1}$$

$$-\left[\frac{\lambda_1-\in}{n+1}+(\mu-\in)+(n-1)\xi p\right]P_n(1)+\left[(\mu-\in)+n\xi\ \mathrm{p}\right]P_{n+1}(1)$$
$$+\left(\frac{\lambda_1-\in}{n}\right)P_{n-1}(1)=0 \qquad \mathrm{n=r+2,r+3,\ldots,R\text{-}1} \tag{3.7}$$

$$-\left[\frac{\lambda_1-\in}{R+1}+(\mu-\in)+(R-1)\xi p\right]P_R(1)+\left[(\mu-\in)+R\xi\ \mathrm{p}\right]P_{R+1}(1)$$
$$+\left(\frac{\lambda_1-\in}{R}\right)P_{R-1}(1)+\left(\frac{\lambda_0-\in}{R}\right)P_{R-1}(0)=0 \tag{3.8}$$

$$-\left[\frac{\lambda_1-\in}{n+1}+(\mu-\in)+(n-1)\xi p\right]P_n(1)+\left[(\mu-\in)+n\xi\ p\right]\mathrm{P_{n+1}}(1)$$
$$+\left(\frac{\lambda_1-\in}{n}\right)P_{n-1}(1)=0 \qquad \mathrm{n=R+1,R+2,\ldots,K\text{-}1} \tag{3.9}$$

$$-\left[(\mu-\in)+(K-1)\,\xi\ \mathrm{p}\right]P_K(1)+\left(\frac{\lambda_1-\in}{K}\right)P_{K-1}(1)=0 \tag{3.10}$$

From (3.1) and (3.2) we get

$$P_n(0)=\left[\frac{1}{\mathrm{n!}}\prod_{\mathrm{k=1}}^{\mathrm{n}}\frac{(\lambda_0-\in)}{[(\mu-\in)+(k-1)\,\xi\ \mathrm{p}]}\right]\mathrm{P_0}(0)\ ,\ \mathrm{n=0,1,2\ldots r} \tag{3.11}$$

Using (3.11) in (3.3) we get

$$P_{r+1}(0)=\left[\frac{1}{(\mathrm{r+1})!}\prod_{\mathrm{k+1}}^{\mathrm{r+1}}\frac{(\lambda_0-\in)}{[(\mu-\in)+(k-1)\,\xi\ \mathrm{p}]}\right]\mathrm{P_0}(0)\ \text{-}\ \mathrm{P_{r+1}}\ (1)$$

Using the above result in (3.4), We recursively derive

$$P_n(0)=\left[\frac{1}{\mathrm{n!}}\ \underset{\mathrm{k=1}}{\overset{n}{\pi}}\frac{(\lambda_0-\in)}{[(\mu-\in)+(k-1)\,\xi\ \mathrm{p}]}\right]\mathrm{P_0}(0)-\frac{P_{r+1}(1)}{\underset{\mathrm{l=r+1}}{\overset{n-1}{\pi}}[\,(\mu-\in)+l\,\xi\ \mathrm{p}]}.\mathrm{A} \tag{3.12}$$

where

$$A=\frac{(\lambda_0-\in)^{n-(r+1)}}{n\,\mathrm{P}_{n-(r+1)}}+\frac{(\lambda_0-\in)^{n-r+2}}{n\,\mathrm{P}_{n-(r+2)}}\ \ [(\mu\text{-}\in)+\mathrm{r}\ \xi\ \mathrm{p}]+\ldots+$$
$$[(\mu\text{-}\in)+\mathrm{r}\ \xi\ \mathrm{p}][(\mu\text{-}\in)+(\mathrm{r}+1)\ \xi\ \mathrm{p}\ ]\ldots\ldots[(\mu\text{-}\in)+(n-2)\ \xi\ \mathrm{p}][\ (\mu\text{-}\in)+(n-3)\ \xi\ \mathrm{p}]$$
$$\mathrm{n=r+1,r+2,\ldots,R\text{-}1}$$

Using (3.12) in (3.5) we get

$$\mathrm{p}_{r+1}(1)=\left[\frac{\dfrac{(\lambda_0-\in)^R}{\mathrm{R!}}\ \underset{\mathrm{K=1}}{\overset{r+1}{\pi}}\dfrac{1}{[(\mu-\in)+(k-1)\,\xi\ \mathrm{p}]}}{B}\right]\mathrm{P_0}(0) \tag{3.13}$$

where

$$B=\frac{(\lambda_0-\in)^{R-(r+1)}}{R\,\mathrm{P}_{R^{-(r+1)}}}+\frac{(\lambda_0-\in)^{R-(r+2)}}{R\mathrm{P}_{R-(r+2)}}\ \ (\mu\text{-}\in+\mathrm{r}\ \xi\ \mathrm{p})+\ldots+$$
$$[\ (\mu\text{-}\in)+\mathrm{r}\ \xi\ \mathrm{p}][(\mu\text{-}\in)+(\mathrm{r}+1)\ \xi\ \mathrm{p}\ ]\ldots\ldots[(\mu\text{-}\in)+(R-2)\ \xi\ \mathrm{p}]$$

Using (3.13) in (3.7) we get

$$P_n(1)=\frac{P_{r+1}(1)}{\underset{\mathrm{l=r+1}}{\overset{n-1}{\pi}}[(\mu-\in)+l\,\xi\,\mathrm{p}]}\ .\mathrm{C} \tag{3.14}$$

and $$\mathrm{C}=\frac{(\lambda_1-\in)^{n-(r+1)}}{n\,\mathrm{P}_{n-(r+1)}}+\frac{(\lambda_1-\in)^{n-(r+2)}}{n\,\mathrm{P}_{n-(r+2)}}\ [\ (\mu\text{-}\in)+\mathrm{r}\,\xi\,\mathrm{p})]+\ldots$$

$$+[(\mu\text{-}\in)+r\ \xi\ \mathrm{p}\ ][(\mu\text{-}\in)+(r+1)\ \xi\ \mathrm{p}]\ldots[\ (\mu\text{-}\in)+(n-2)\ \xi\ \mathrm{p}\ ]\qquad \mathrm{n}=r+1,\ \mathrm{r}+2,\ \mathrm{r}+3,\ldots\mathrm{R}\text{-}1$$

where $P_{r+1}(1)$ is given by (3.13)

Using (3.12) and (3.14) in (3.8)

$$P_{R+1}(1)=\frac{P_{r+1}(1)}{\underset{\mathrm{l=r+1}}{\overset{R}{\pi}}(\mu-\varepsilon)+l\,\xi\,\mathrm{p})}\left[\frac{(\lambda_1-\in)^{R-r}}{(R+1)\mathrm{P}_{R-r}}+\frac{(\lambda_1-\in)^{R-(r+1)}}{(\mathrm{R}+1)\mathrm{P}_{R-(r+1)}}\ [\ (\mu\text{-}\in+\mathrm{r}\,\xi\,\mathrm{p})]+\cdots\right.$$
$$\left.+\frac{(\lambda_1-\in)}{(R+1)\ \mathrm{P}_1}[(\mu\text{-}\in)+\mathrm{r}\,\xi\,\mathrm{p}]\ldots[(\mu\text{-}\in)+(R-2)\,\xi\,\mathrm{p}]\right] \tag{3.15}$$

where $P_{r+1}(1)$ is given by (3.13)

Using (3.14) and (3.15) in (3.9) we recursively derive

$$P_n(1)=\frac{P_{r+1}(1)}{\underset{l=r+1}{\overset{k}{\pi}}[(\mu-\in)+l\xi\,\mathrm{p}]}D$$

$$\mathrm{D}=\frac{(\lambda_1-\in)^{n-r-1}}{n\ \mathrm{P}_{\text{n-r-1}}}+\frac{(\lambda_1-\in)^{n-r-2}}{n\ \mathrm{P}_{\text{n-r-2}}}(\mu-\in+r\ \ \xi\ \mathrm{p})+\ldots$$

$$+\frac{(\lambda_1-\in)^{K-R}}{n\ \mathrm{P}_{n-R}}[(\mu-\in)+r\ \ \xi\ \ \mathrm{p}]\ [(\mu\text{-}\in)+(r+1)\xi\ \ \mathrm{p}]\ldots\ [(\mu\text{-}\in)+(R-2)\xi\ \ \mathrm{p}] \tag{3.16}$$

$$\mathrm{n}=R+1,\ \mathrm{R}+2,\ \mathrm{R}+3,\ \ldots\mathrm{K}\text{-}1$$

where $\mathrm{P}_{\mathrm{r+1}}(1)$ is given by (3.13)

4. CHARACTERISTICS OF THE MODEL

The probability that the system is in faster rate of arrivals is

$$P(0)=\sum_{n=0}^{K}P_n(0)$$
$$=\sum_{n=0}^{r}P_n(0)+\sum_{n=r+1}^{R-1}\mathrm{P_n}(0)+\sum_{n=R}^{K}P_n(0)$$

Since $P_n(0)$ exist only when $n=0,1,2,3....,r-1,r,r+1,...R-2,R-1$, we get

$$P(0)=\sum_{n=0}^{r}\mathrm{P_n}(0)+\sum_{n=r+1}^{R-1}\mathrm{P_n}(0) \tag{4.1}$$

From (3.11), (3.12) and (3.13), we get

$$P(0)=\sum_{\mathrm{n=o}}^{\mathrm{R\text{-}1}}\left[\frac{1}{\mathrm{n!}}\underset{\mathrm{k=1}}{\overset{n}{\pi}}\frac{(\lambda_0-\in)}{[(\mu-\in)+(k-1)\ \xi\,p]}\right]\mathrm{P}_0(0)\ \text{-}\sum_{\mathrm{n=}r+1}^{\mathrm{R\text{-}1}}A\left[\frac{\frac{(\lambda_0-\in)^R}{\mathrm{R!}}\underset{\mathrm{k=1}}{\overset{R-1}{\pi}}\left[\frac{1}{[(\mu-\in)+(k-1)\ \xi\ \mathrm{p}]}\right]\mathrm{P}_0(0)}{B}\right] \tag{4.2}$$

where A and B are given by (3.12) and (3.13)

The Probability that the system is in slower rate of arrival is

$$P(1) = \sum_{n=0}^{K} \mathrm{P_n}(1)$$

$$= \sum_{n=0}^{r} P_n(1) + \sum_{n=r+1}^{R-1} \mathrm{P_n}(1) + \sum_{n=R}^{K} \mathrm{P_n}(1)$$

Since $\mathrm{P_n}(1)$ exists only when $n = r+1, r+2, ...R-2, R-1, ...K$, we get

$$P(1) = \sum_{n=r+1}^{R} \mathrm{P_n}(1) + \sum_{n=R+1}^{K} \mathrm{P_n}(1)$$

$$P(1) = \sum_{n=r+1}^{\mathrm{R}} C\left[\frac{(\lambda_0 - \in)^R}{R!} \underset{\mathrm{l=1}}{\overset{n-1}{\pi}} \frac{1}{[(\mu - \in) + l\xi \ \mathrm{p}]} \right] \mathrm{P_0}(0)\frac{1}{\mathrm{B}}$$

$$+ \sum_{n=R+1}^{\mathrm{K}} \mathrm{D}\left[\frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{\mathrm{l=1}}{\overset{k-1}{\pi}} \frac{1}{[(\mu - \in) + l\xi \ \mathrm{p}]} \mathrm{P_0}(0) \right] \frac{1}{\mathrm{B}} \qquad (4.3)$$

where C and D are given by (3.14) and (3.16).

The probability $[\mathrm{P_0}(0)]$ that the system is empty can be calculated from the normalizing condition $\mathrm{P}(0) + P(1) = 1$

From (4.2) and (4.3) we get

$$P_0(0) = \frac{1}{\left\{ \begin{array}{l} 1 + \sum_{\mathrm{n\text{-}1}}^{\mathrm{R\text{-}1}} \left[\frac{1}{\mathrm{n}!} \underset{\mathrm{k=1}}{\overset{n}{\pi}} \frac{(\lambda_0 - \in)}{[(\mu - \in) + (k-1)\ \xi\ \mathrm{p}]} \right] - \\ \sum_{n=r+1}^{R-1} A \frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{\mathrm{k=1}}{\overset{r+1}{\pi}} \left(\frac{1}{[(\mu - \in) + (k-1)\ \xi\ \mathrm{p}]} \right) \frac{1}{\mathrm{B}} + \sum_{\mathrm{n=r+1}}^{\mathrm{R}} \mathrm{C} \frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \\ \underset{l=1}{\overset{n-1}{\pi}} \frac{1}{[(\mu - \in) + l\xi\ \mathrm{p}]} \frac{1}{\mathrm{B}} + \sum_{n=R+1}^{K} \mathrm{D} \frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{l=1}{\overset{k-1}{\pi}} \frac{1}{[(\mu - \in) + l\ \xi\ \mathrm{p}]} \frac{1}{\mathrm{B}} \end{array} \right\}} \qquad (4.4)$$

where A, B, C and D are given by (3.12) (3.13) (3.14) and (3.16).

The average number of customers in the system is given by

$L_s = L_{s_o} + L_{s_1}$

where

$$L_{S_0} = \sum_{n=0}^{r} \mathrm{nP_n}(0) + \sum_{n=r+1}^{R-1} \mathrm{nP_n}(0) \quad \text{and} \quad L_{S_1} = \sum_{n=r+1}^{R-1} \mathrm{nP_n}(1) + \sum_{n=R}^{K} \mathrm{nP_n}(1)$$

$$L_{S_0} = \sum_{\mathrm{n\text{-}0}}^{\mathrm{r\text{-}1}} \left[\mathrm{n}.\frac{1}{\mathrm{n}!} \underset{\mathrm{k=1}}{\overset{n}{\pi}} \frac{(\lambda_0 - \in)}{[(\mu - \in) + (k-1)\xi\ \mathrm{p}]} \right] P_0(0) - \sum_{\mathrm{n\text{-}r+1}}^{\mathrm{R\text{-}1}} A\left(\frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{\mathrm{k=1}}{\overset{r+1}{\pi}} \frac{1}{[(\mu - \in) + (k-1)\ \xi\ \mathrm{p}]} \right) \frac{1}{\mathrm{B}}\ P_0(0)$$

where A, B and $\mathrm{P_0}(0)$ are given by (3.12) (3.13) and (4.4) and

$$L_{S_1} = \sum_{\mathrm{n=r+1}}^{\mathrm{R\text{-}1}} C\left[\mathrm{n}\ \frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{l=1}{\overset{n-1}{\pi}} \frac{1}{[(\mu - \in) + l\ \xi\ \mathrm{p}]} \right] \frac{1}{B} P_0(0) + \sum_{\mathrm{n=R}}^{\mathrm{K}} D\left(n\frac{(\lambda_0 - \in)^R}{\mathrm{R}!} \underset{l=1}{\overset{K}{\pi}} \frac{1}{[(\mu - \in) + l\ \xi\ \mathrm{p}]} \right) \frac{1}{\mathrm{B}} P_0(0)$$

where B, C, D and $P_0(0)$ are given by (3.13) (3.14) (3.16) and (4.4).

Using Little's Formula the expected waiting time of the customers in the system is given as $W_S = \frac{L_S}{\overline{\lambda}}$

where $\overline{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$

5. NUMERICAL ILLUSTRATIONS

For Various Values of λ_0, $\lambda_1, \mu, \in, \xi$, r, R, K, p, the values of $P_0(0)$, P(0), P(1), L_S and W_S are computed and tabulated in the following tables.

Table: 5.1

R	R	K	λ_0	λ_1	μ	$\in$	p	ξ	$P_0(0)$	$P(0)$	$P(1)$	L_s	W_s
3	5	8	4	3	5	0	1	0	0.4442	0.98614	0.01393	1.057080	0.2651749
3	5	8	4	3	5	0.5	1	0	0.4552	0.96908	0.01137	6.335146	1.620063
3	5	8	4	3	5	0	1	1	0.4698	0.98836	0.00527	0.737007	0.185679
3	5	8	4	3	6	0.5	1	1	0.5438	0.99430	0.00113	0.594669	0.149392
3	5	8	4	3	8	0.5	1	1	0.6344	0.99679	0.00053	0.449929	0.112799
3	5	8	4	4	5	0.5	1	1	0.4812	0.99219	0.00175	0.708159	0.178119
3	5	8	4	4	5	0	1	1	0.4698	0.98815	0.00252	0.740530	0.186876
3	5	8	4	3	5	0.5	1	1	0.4812	0.99219	0.00172	0.707732	0.178093
3	5	8	3	2	5	0.5	1	1	0.5883	0.99815	0.00039	0.512121	0.170978
3	5	8	2	1	5	0.5	1	1	0.7233	0.99978	0.00003	0.315442	0.157753
3	5	8	6	3	5	0.5	1	1	0.3262	0.95365	0.01656	1.128214	0.195477
3	5	8	6	5	4	0	1	1	0.2614	0.92632	0.01851	1.368569	0.242204
3	5	8	5	5	5	0	1	1	0.3928	0.97062	0.00639	0.9359104	0.191586
3	5	8	4	3	5	1	2	2	0.5255	0.99938	0.00005	0.5620376	0.140591
3	5	8	4	3	6	1	2	1	0.5733	0.99848	0.00022	0.520854	0.130390
3	5	8	4	3	6	0.5	2	1	0.554	0.99728	0.00038	0.554791	0.139036
3	5	8	4	4	5	0.5	2	2	0.5118	0.99888	0.00010	0.587448	0.1470113

6. CONCLUSIONS

It is observed from the table 5.1 that when the mean dependence rate increases and the other parameters are kept fixed L_s and W_s decrease. When the service rate increases and the other parameters are kept fixed, $P_0(0)$ and $P(0)$ increase and $P(1)$, L_s and W_s decrease. When the arrival rate decreases $P_0(0)$, $P(0)$ increase and $P(1), L_s$ and W_s decrease.

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